Econ 101A – Problem Set 6
Due on Tuesday 9 December in Ethan's office (2521 Channing Way Room 315) before 5pm.
No late Problem Sets accepted, sorry!

This Problem set tests the knowledge that you accumulated mainly in lectures 24 to 26. The problem set is focused on dynamic games and general equilibrium. General rules for problem sets: show your work, write down the steps that you use to get a solution (no credit for right solutions without explanation), write legibly. If you cannot solve a problem fully, write down a partial solution. We give partial credit for partial solutions that are correct. Do not forget to write your name on the problem set!

Problem 1. Dynamic Games (48 points) Two companies produce the same good. In the first period, firm1 seels its product as a monopolist on the West Coast. In the second period, firm 1 competes with firm 2 on the East Coast as a Cournot duopolist. There is no discounting between the two periods. Firm 1 produces quantity $x_W \leq c/\alpha$ at the West at cost cx_W. On the East Coast, and that's what makes this problem interesting, firm 1 produces quantity x_E at cost $(c - \alpha x_W)\, x_E$, where $0 < \alpha < c < 1/2$. The parameter α captures a form of learning by doing. The more firm 1 produces on the West Coast, the lower the marginal costs are going to be on the East Coast. As for firm 2, it produces in the East market with cost cx_2. The inverse demand functions are $p_W(x_W) = 1 - x_W$ and $p_E(x_E, x_2) = 1 - x_E - x_2$. Each firm maximizes profit. *In particular, firm 1 maximizes the total profits from its West and East coast operations.*

1. Consider first the case of **simultaneous choice**. Assume that firm 2 does not observe x_W before making its production decision. This means that, although formally firm 1 chooses output x_W first, that you should analyze the game as a symultaneous game between firm 1 and firm 2. Use Nash Equilibrium. Write down the profit function that firm 1 maximizes (careful here) and the profit function that firm 2 maximizes (5 points)
2. Write down the first order conditions of firm 1 with respect to x_W and x_E, and the first order condition of firm 2 with respect to x_2. Solve for x_W^*, x_E^*, and x_2^*. (4 points)
3. Check the second order conditions for firm 1 and for firm 2. (3 points)
4. What is the comparative statics of x_W^* and x_E^* with respect to α? Does it make sense? How about the comparative statics of x_2^* with respect to α? (4 points)
5. Compute the profits of firm 2 in equilibrium. How do they vary as α varies? (compute the comparative statics) Why are firm 2's profits affected by α even though the parameter α does not directly affect the costs of firm 2? (5 points)
6. Now consider the case of **sequential choice**. Assume that firm 2 observes x_W before making its production decision x_2. This means that you should analyze the game as a dynamic game between firm 1 and firm 2, and use the concept of subgame-perfect equilibrium. Remember, we start from the last period. Write down the profit functions that firm 1 and firm 2 maximize on the East Coast (4 points)
7. Write down the first order conditions of firm 1 with respect to x_E, and the first order condition of firm 2 with respect to x_2. Solve for x_E^* and x_2^* as a function of x_W^* (4 points)
8. Compute the comparative statics of x_E^* and x_2^* with respect to x_W^*. Do these results make sense? (3 points)
9. Compute the profits of firm 1 on the East Coast as a function of x_W^*. (2 points)
10. Using the answer to point 9, write down the maximization problem of firm 1 in the first period, that it, when it decides the production on the West Coast. (3 points)
11. Write down the first order conditions of firm 1 with respect to x_W. Solve for x_W^* and then, using the solution for x_W^*, find the solution for x_E^* and x_2^*. (5 points)

12. Compare the solutions for x_W^* under simultaneous and under sequential choice. What can you conclude? Under which conditions the firm does more preemption, that is, produces more on the West Coast in order to reduce the production in equilibrium of firm 2? (6 points)

Problem 2. General Equilibrium (25 points) Consider the case of pure eaxchange with two consumers. Both consumers have Cobb-Douglas preferences, but with different parameters. Consumer 1 has utility function $u(x_1^1, x_2^1) = (x_1^1)^\alpha (x_2^1)^{1-\alpha}$. Consumer 2 has utility function $u(x_1^2, x_2^2) = (x_1^2)^\beta (x_2^2)^{1-\beta}$. The endowment of good j owned by consumer i is ω_j^i. The price of good 1 is p_1, while the price of good 2 is normalized to 1 without loss of generality.

1. Only for point 1, assume $\omega_1^1 = 1, \omega_1^2 = 3, \omega_2^1 = 3, \omega_2^2 = 1$. (that is, total endowment of each good is 4). Assume further $\alpha = 1/2$, $\beta = 1/2$. Draw the Pareto set and the contract cuve for this economy in an Edgeworth box. (you do not need to give the exact solutions, only a graphical representation) What is the set of points that could be the outcome under barter in this economy? (5 points)

2. For each consumer, compute the utility maximization problem. Solve for x_j^{i*} for $j = 1, 2$ and $i = 1, 2$ as a function of the price p_1 and of the endowments. [By now, this part should bo sooo familiar to you, you should be able to solve this problem with closed eyes] (5 points)

3. Now comes the general equilibrium part. Require now that the total sum of the demands for good 1 equals the total sum of the endowments, that is, that $x_1^1 + x_1^2 = \omega_1^1 + \omega_1^2$. Solve for the general equilibrium price p_1^*. (6 points)

4. What is the comparative statics of p_1^* with respect to the endowment of good 1, that is, with respect to ω_1^i for $i = 1, 2$? What about with respect to the endowment of the other good? Does this make sense? What is the comparative statics of p_1^* with respect to the taste for good 1, that is, with respect to α and β? Does this make sense? (4 points)

5. Now require the same general equilibrium condition in market 2. Solve for p_1^* again, and check that this solution is the same as the one you found in the point above. In other words, you found a property that is called Walras'Law.In an economy with n markets, if $n - 1$ markets are in equilibrium, the nth market will be in equilibrium as well. (5 points)