Economics 101A
(Lecture 16)

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Outline

1. Aggregation
2. Maket Equilibrium in Short-Run
3. Comparative Statics of Equilibrium
4. Elasticities

1 Aggregation

1.1 Producers aggregation

- J companies, $j = 1, ..., J$, producing good i

- Company j has supply function

$$y_i^j = y_i^{j*}(p_i, w, r)$$

- Industry supply function:

$$Y_i(p_i, w, r) = \sum_{j=1}^{J} y_i^{j*}(p_i, w, r)$$

- Graphically,

1.2 Consumer aggregation

- Nicholson, Ch. 10, pp. 279–282 [OLD: Ch. 7, pp. 172–176]

- *One-consumer economy*

- Utility function $u(x_1, ..., x_n)$, prices $p_1, ..., p_n$

- Maximization $\Longrightarrow$

$$\begin{aligned} x_1^* &= x_1^*(p_1, ..., p_n, M), \\ &\vdots \\ x_n^* &= x_n^*(p_1, ..., p_n, M). \end{aligned}$$

- Good i. Fix prices $p_1, ..., p_{i-1}, p_{i+1}, ..., p_n$ and M

- **Single-consumer demand function**:

$$x_i^* = x_i^*(p_i | p_1, ..., p_{i-1}, p_{i+1}, ..., p_n, M)$$

- What is sign of $\partial x_i^*/\partial p_i$?

- Negative if good i is normal

- Negative or positive if good i is inferior

- Graphically,

- *Aggregation:* J consumers, $j = 1, ..., J$

- Demand for good i by consumer j :

$$x_i^{j*} = x_i^{j*}\left(p_1, ..., p_n, M^j\right)$$

- Market demand X_i:

$$\begin{aligned} & X_i\left(p_1, ..., p_n, M^1, ..., M^J\right) \\ = & \sum_{j=1}^{J} x_i^{j*}\left(p_1, ..., p_n, M^j\right) \end{aligned}$$

- Graphically,

- Notice: market demand function depends on distribution of income M^J

- Market demand function X_i:
  - Consumption of good i as function of prices $\mathbf{p}$
  - Consumption of good i as function of income distribution M^j

2 Market Equilibrium in the Short-Run

- Nicholson, Ch. 10, pp. 283–295 [OLD: Ch. 14, pp. 368–382]

- What is equilibrium price p_i?

- Magic of the Market...

- Equilibrium: No excess supply, No excess demand

- Prices $\mathbf{p}^*$ equates demand and supply of good i:

  $$Y^* = Y_i^S(p_i^*, w, r) = X_i^D\left(p_1^*, ..., p_n^*, M^1, ..., M^J\right)$$

- Graphically,

- Notice: in short-run firms can make positive profits

- Comparative statics exercises with endogenous price p_i :
  - increase in wage w or interest rate r:
  - change in income distribution

3 Comparative statics of equilibrium

- Supply and Demand function of parameter α :
  - $Y_i^S(p_i, w, r, \alpha)$
  - $X_i^D(\mathbf{p}, \mathbf{M}, \alpha)$

- How does α affect p^* and Y^*?

- Comparative statics with respect to α

- Equilibrium:

$$Y_i^S(p_i, w, r, \alpha) = X_i^D(\mathbf{p}, \mathbf{M}, \alpha)$$

- Can write equilibrium as implicit function:

$$Y_i^S(p_i, w, r, \alpha) - X_i^D(\mathbf{p}, \mathbf{M}, \alpha) = 0$$

- What is $dp^*/d\alpha$?

- Implicit function theorem:

$$\frac{\partial p^*}{\partial \alpha} = -\frac{\frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha}}{\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}}$$

- What is sign of denominator?

- Sign of $\partial p^*/\partial \alpha$ is negative of sign of numerator

- Examples:

  1. *Fad.* Good becomes more fashionable: $\frac{\partial X^D}{\partial \alpha} > 0 \Longrightarrow \frac{\partial p^*}{\partial \alpha} > 0$

  2. *Recession in Europe.* Negative demand shock for US firms: $\frac{\partial X^D}{\partial \alpha} < 0 \Longrightarrow \frac{\partial p^*}{\partial \alpha} < 0$

  3. *Oil shock.* Import prices increase: $\frac{\partial Y^S}{\partial \alpha} < 0 \Longrightarrow \frac{\partial p^*}{\partial \alpha} > 0$

  4. *Computerization.* Improvement in technology. $\frac{\partial Y^S}{\partial \alpha} > 0 \Longrightarrow \frac{\partial p^*}{\partial \alpha} < 0$

4 Elasticities

- [Not in midterm]

- Nicholson, Ch.1, pp. 27–28 [OLD: Ch.7, pp. 176–177]

- How do we interpret magnitudes of $\partial p^*/\partial \alpha$?

- Result depends on units of measure.

- Can we write $\partial p^*/\partial \alpha$ in a unit-free way?

- Yes! Use **elasticities.**

- Elasticity of x with respect to parameter p is

$$\varepsilon_{x,p} = \frac{\partial x}{\partial p}\frac{p}{x}$$

- Interpretation: Percent response in x to percent change in p :

$$\varepsilon_{x,p} = \frac{\partial x}{\partial p}\frac{p}{x} = \lim_{dp \to 0} \frac{x(p+dp) - x(p)}{dp}\frac{p}{x} =$$
$$= \lim_{dp \to 0} \frac{dx/x}{dp/p}$$

where $dx \equiv x(p+dp) - x(p)$.

- Now, show

$$\varepsilon_{x,p} = \frac{\partial \ln x}{\partial \ln p}$$

- Notice: This makes sense only for $x > 0$ and $p > 0$

- Proof. Consider function

$$x = f(p)$$

- Rewrite as

$$\ln(x) = \ln f(p) = \ln f\left(e^{\ln(p)}\right)$$

- Define $\hat{x} = \ln(x)$ and $\hat{p} = \ln(p)$

- This implies

$$\hat{x} = \ln f\left(e^{\hat{p}}\right)$$

- Get

$$\begin{aligned} \frac{\partial \hat{x}}{\partial \hat{p}} &= \frac{\partial \ln x}{\partial \ln p} = \\ &= \frac{1}{f\left(e^{\hat{p}}\right)} \frac{\partial f\left(e^{\hat{p}}\right)}{\partial \hat{p}} e^{\hat{p}} = \frac{\partial x}{\partial p} \frac{p}{x} \end{aligned}$$

- Example with Cobb-Douglas utility function

- $U(x, y) = x^{\alpha} y^{1-\alpha}$ implies solutions

$$x^* = \alpha \frac{M}{p_x}, y^* = (1 - \alpha) \frac{M}{p_y}$$

- Elasticity of demand with respect to own price ε_{x,p_x}:

$$\varepsilon_{x,p_x} = \frac{\partial x^*}{\partial p_x} \frac{p_x}{x^*} = -\frac{\alpha M}{(p_x)^2} \frac{p_x}{\alpha \frac{M}{p_x}} = -1$$

- Elasticity of demand with respect to other price $\varepsilon_{x,p_y} = 0$

- Go back to problem above:

$$\frac{\partial p^*}{\partial \alpha} = -\frac{\frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha}}{\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}}$$

- Use elasticities to rewrite response of p to change in α :

$$\frac{\partial p^*}{\partial \alpha}\frac{\alpha}{p} = -\frac{\left(\frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha}\right)\frac{\alpha}{Y}}{\left(\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}\right)\frac{p}{Y}}$$

or (using fact that $X^{D*} = Y^{s*}$)

$$\varepsilon_{p,\alpha} = -\frac{\varepsilon_{S,\alpha} - \varepsilon_{D,\alpha}}{\varepsilon_{S,p} - \varepsilon_{D,p}}$$

- We are likely to know elasticities from empirical studies.

5 Next Lecture

- Midterm and then...

- Taxes and Subsidies

- Long-Run Equilibrium