Economics 101A
(Lecture 21)

Stefano DellaVigna

November 18, 2004

Outline

1. Oligopoly: Cournot
2. Oligopoly: Bertrand
3. Second-price Auction
4. Dynamic Games

1 Oligopoly: Cournot

- Nicholson, Ch. 14, pp. 418–419, 421–422 [OLD: p. 531, 534–535].

- Back to oligopoly maximization problem

- Assume 2 firms, cost $c_i(y_i) = cy_i$, $i = 1, 2$

- Firms choose simultaneously quantity y_i

- Firm i maximizes:

$$\max_{y_i} p(y_i + y_{-i}) y_i - cy_i.$$

- First order condition with respect to y_i:

$$p'_Y \left(y_i^* + y_{-i}^*\right) y_i^* + p - c = 0, \; i = 1, 2.$$

- Nash equilibrium:
  - y_1 optimal given y_2;
  - y_2 optimal given y_1.
- Solve equations:

$$p'_Y\left(y_1^* + y_2^*\right) y_1^* + p - c = 0 \text{ and}$$

$$p'_Y\left(y_2^* + y_1^*\right) y_2^* + p - c = 0.$$

- Cournot -> Pricing above marginal cost

2 Oligopoly: Bertrand

- Previously, we assumed firms choose quantities

- Now, assume firms first choose prices, and then produce quantity demanded by market

- 2 firms

- Profits:

$$\pi_i(p_i, p_{-i}) = \begin{cases} (p_i - c)\,Y(p_i) & \text{if } p_i < p_{-i} \\ (p_i - c)\,Y(p_i)/2 & \text{if } p_i = p_{-i} \\ 0 & \text{if } p_i > p_{-i} \end{cases}$$

- First show that $p_1 = c = p_2$ is Nash Equilibrium

- Does any firm have a (strict) incentive to deviate?

- Show that this equilibrium is unique

- Case 1. $p_1 > p_2 > c$

- Case 2. $p_1 = p_2 > c$

- Case 3. $p_1 > c \geq p_2$

- Case 4. $c > p_1 \geq p_2$

- Case 5. $p_1 = c > p_2$

- Case 6. $p_1 = c = p_2$

- It is unique!

- Marginal cost pricing

- Two firms are enough to guarantee perfect competition!

- Price wars

3 Second-price Auction

- Sealed-bid auction

- Highest bidder wins object

- Price paid is second highest price

- Two individuals: $I = 2$

- Strategy s_i is bid b_i

- Each individual knows value v_i

- Payoff for individual i is

$$u_i(b_i, b_{-i}) = \begin{cases} v_i - b_{-i} & \text{if } b_i > b_{-i} \\ (v_i - b_{-i})/2 & \text{if } b_i = b_{-i} \\ 0 & \text{if } b_i < b_{-i} \end{cases}$$

- Show: weakly dominant to set $b_i^* = v_i$

- To show:

$$u_i(v_i, b_{-i}) \geq u_i(b_i, b_{-i})$$

for all b_i, for all b_{-i}, and for $i = 1, 2$.

1. Assume $b_{-i} > v_i$

   - $u_i(v_i, b_{-i}) = 0 = u_i(b_i, b_{-i})$ for any $b_i < b_{-i}$
   - $u_i(b_{-i}, b_{-i}) = (v_i - b_{-i})/2 < 0$
   - $u_i(b_i, b_{-i}) = (b_i - b_{-i}) < 0$ for any $b_i > b_{-i}$

2. Assume now $b_{-i} = v_i$

3. Assume now $b_{-i} < v_i$

4 Dynamic Games

- Nicholson, Ch. 15, pp. 449–454.[OLD: Ch. 10, pp. 256–259]

- Dynamic games: one player plays after the other

- Decision trees
  - Decision nodes
  - Strategy is a plan of action at each decision node

- Example: battle of the sexes game

She \ He	Ballet	Football
Ballet	$2, 1$	$0, 0$
Football	$0, 0$	$1, 2$

- Dynamic version: she plays first

- **Subgame-perfect equilibrium.** At each node of the tree, the player chooses the strategy with the highest payoff, given the other players' strategy

- Backward induction. Find optimal action in last period and then work backward

- Solution

- Example 2: Entry Game

$1 \setminus 2$	Enter	Do not Enter
Enter	$-1, -1$	$10, 0$
Do not Enter	$0, 5$	$0, 0$

- Exercise. Dynamic version.

- Coordination games solved if one player plays first

- Can use this to study finitely repeated games

- Suppose we play the prisoner's dilemma game ten times.

$1 \setminus 2$	D	ND
D	$-4, -4$	$-1, -5$
ND	$-5, -1$	$-2, -2$

- What is the subgame perfect equilibrium?

5 Next lecture

- Stackelberg duopoly
- General equilibrium
- Edgeworth Box