

Economics 101A

(Lecture 5)

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Outline

1. Pages to study
2. Properties of Preferences (continued)
3. From Preferences to Utility (and viceversa)
4. Common Utility Functions
5. Utility maximization

1 Pages to study

- My mistake: pages in previous lectures referred to old book
- Summary of pages to read:
 - Optimization with 1 variable: Ch. 2, pp. 20–24, 47 [OLD: 22–26, 48–49]
 - Multivariate optimization: Ch. 2, pp. 24–32, but skip elasticities; 48–50 [OLD: 26–32, 50–51]
 - Implicit function: Ch. 2, pp. 32–33 [OLD, 32–34]
 - Envelope theorem: Ch. 2, pp. 33–37 [OLD, 34–39]
 - Constrained maximization: Ch. 2, pp. 38–44 [OLD, 39–46]

- Envelope Theorem II: Ch. 2, pp. 44 [OLD, 46-47]
- ALSO: Read xeroxed pages from Varian, *Microeconomic Analysis*

2 Properties of Preferences (ctd)

- Nicholson, Ch. 3, pp. 69-70 [OLD: 66]
- Indifference relation \sim : $x \sim y$ if $x \succeq y$ and $y \succeq x$
- Strict preference: $x \succ y$ if $x \succeq y$ and not $y \succeq x$
- Exercise. If \succeq is rational,
 - \succ is transitive
 - \sim is transitive
 - Reflexive property of \succeq . For all x , $x \succeq x$.

- Other features of preferences

- Preference relation \succeq is:
 - *monotonic* if $x \geq y$ implies $x \succeq y$.

 - *strictly monotonic* if $x \geq y$ and $x_j > y_j$ for some j implies $x \succ y$.

 - *convex* if for all x, y , and z in X such that $x \succeq z$ and $y \succeq z$, then $tx + (1 - t)y \succeq z$ for all t in $[0, 1]$

3 From preferences to utility

- Nicholson, Ch. 3
- Economists like to use utility functions $u : X \rightarrow R$
- $u(x)$ is 'liking' of good x
- $u(a) > u(b)$ means: I prefer a to b .
- **Def.** Utility function u represents preferences \succeq if, for all x and y in X , $x \succeq y$ if and only if $u(x) \geq u(y)$.
- **Theorem.** If preference relation \succeq is rational and continuous, there exists a continuous utility function $u : X \rightarrow R$ that represents it.

- [Skip proof]

- Example:

$$(x_1, x_2) \succeq (y_1, y_2) \text{ iff } x_1 + x_2 \geq y_1 + y_2$$

- Draw:

- Utility function that represents it: $u(x) = x_1 + x_2$

- But... Utility function representing \succeq is not unique

- Take $3u(x)$ or $\exp(u(x))$

- $u(a) > u(b) \iff \exp(u(a)) > \exp(u(b))$

- If $u(x)$ represents preferences \succeq and f is a strictly increasing function, then $f(u(x))$ represents \succeq as well.

- If preferences are represented from a utility function, are they rational?
 - completeness
 - transitivity

- Indifference curves: $u(x_1, x_2) = \bar{u}$
- They are just implicit functions! $u(x_1, x_2) - \bar{u} = 0$

$$\frac{dx_2}{dx_1} = -\frac{U'_{x_1}}{U'_{x_2}} = MRS$$

- Indifference curves for:
 - monotonic preferences;
 - strictly monotonic preferences;
 - convex preferences

4 Common utility functions

- Nicholson, Ch. 3, pp. 82-86 [OLD: 80–84]

1. Cobb-Douglas preferences: $u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$

- $MRS = -\alpha x_1^{\alpha-1} x_2^{1-\alpha} / (1-\alpha) x_1^\alpha x_2^{-\alpha} = \frac{\alpha}{1-\alpha} \frac{x_2}{x_1}$

2. Perfect substitutes: $u(x_1, x_2) = \alpha x_1 + \beta x_2$

- $MRS = -\alpha/\beta$

3. Perfect complements: $u(x_1, x_2) = \min(\alpha x_1, \beta x_2)$

- MRS discontinuous at $x_2 = \frac{\alpha}{\beta}x_1$

4. Constant Elasticity of Substitution: $u(x_1, x_2) = (\alpha x_1^\rho + \beta x_2^\rho)^{1/\rho}$

- $MRS = -\frac{\alpha}{\beta} \left(\frac{x_1}{x_2}\right)^{\rho-1}$
- if $\rho = 1$, then...
- if $\rho = 0$, then...
- if $\rho \rightarrow -\infty$, then...

5 Utility Maximization

- Nicholson, Ch. 4, pp. 94–105 [OLD: 91–103]
- $X = R_+^2$ (2 goods)
- Consumers: choose bundle $x = (x_1, x_2)$ in X which yields highest utility.
- Constraint: income = M
- Price of good 1 = p_1 , price of good 2 = p_2
- Bundle x is feasible if $p_1x_1 + p_2x_2 \leq M$
- Consumer maximizes

$$\begin{aligned} & \max_{x_1, x_2} u(x_1, x_2) \\ & s.t. \quad p_1x_1 + p_2x_2 \leq M \\ & \quad \quad x_1 \geq 0, \quad x_2 \geq 0 \end{aligned}$$

- Maximization subject to inequality. How do we solve that?
- Trick: u strictly increasing in at least one dimension. (\succeq strictly monotonic)
- Budget constraint always satisfied with equality
- Ignore temporarily $x_1 \geq 0$, $x_2 \geq 0$ and check afterwards that they are satisfied for x_1^* and x_2^* .

- Problem becomes

$$\begin{aligned} \max_{x_1, x_2} u(x_1, x_2) \\ \text{s.t. } p_1 x_1 + p_2 x_2 - M = 0 \end{aligned}$$

- $L(x_1, x_2) = u(x_1, x_2) - \lambda(p_1 x_1 + p_2 x_2 - M)$

- F.o.c.s:

$$\begin{aligned} u'_{x_i} - \lambda p_i &= 0 \text{ for } i = 1, 2 \\ p_1 x_1 + p_2 x_2 - M &= 0 \end{aligned}$$

- Moving the two terms across and dividing, we get:

$$MRS = -\frac{u'_{x_1}}{u'_{x_2}} = -\frac{p_1}{p_2}$$

- Graphical interpretation.

- Example with CES utility function.

$$\begin{aligned} \max_{x_1, x_2} & (\alpha x_1^\rho + \beta x_2^\rho)^{1/\rho} \\ \text{s.t.} & p_1 x_1 + p_2 x_2 - M = 0 \end{aligned}$$

- Lagrangean =

- F.o.c.:

- Special case: $\rho = 0$ (Cobb-Douglas)

6 Next Class

- Utility Maximization – tricky cases
- Indirect Utility Function
- Comparative Statics:
 - with respect to price
 - with respect to income