Econ 101A
Midterm 2

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You have approximately 1 hour and 20 minutes to answer the questions in the midterm. We will collect the exams at 12.30 sharp. Show your work, and good luck!

Problem 1. (Self-control problems) (33 points) Consider an individual with self-control problems, Arnold. Arnold is deciding how much to exercise. The quantity of exercise is e, with $e > 0$. The benefit of exercise is e, which is received one period after the exercise. The effort cost of exercising is $c(e)$, a cost felt immediately. We assume with $c'(e) > 0$ and $c''(e) > 0$ for all $e > 0$.

1. At the moment of exercising, therefore, Arnold maximizes the discounted utility
$$\max_e -c(e) + \frac{\beta}{1+\delta} e.$$
with $\beta < 1$. Compute the first order condition that defines the solution e^*. (3 points)
2. Show that the function $-c(e) + \beta e/(1+\delta)$ is concave. What does this imply about the solution e^*? (5 points)
3. Now consider Arnold one period before the actual exercise decision. In this period Arnold receives no additional payoff. Arnold has a commitment device that allows him to choose the attendance for next period. Write down the discounted utility function that Arnold maximizes and solve for the first-order condition defining e_C^*, the exercise level chosen with commitment. (8 points)
4. Compare e^* and e_C^*. Discuss with reference to self-control problems. (6 points)
5. How do e^* and e_C^* compare when β equals 1? Provide intuition (3 points)
6. (Harder) Suppose now that at each attendance Arnold pays a price p per unit of exercise, that is, he pays $p * e$ overall. (the price could be negative, allowing for a subsidy for attendance) With this additional price, now Arnold chooses the new attendance decision $e^{*\prime}$ to maximize
$$\max_e -c(e) - pe + \frac{\beta}{1+\delta} e.$$
What is the level of price p^* such that the attendance $e^{*\prime}$ with price p^* equals the attendance e_C^* with commitment device? That is, what does the price on exercise need to be to attain the attendance chosen with commitment? Is this price p^* positive or negative? Provide intuition on this result. (8 points)

Problem 2. Production in two locations. (57 points) In this exercise, we consider a farm harvesting papayas y in two locations. Notoriously, papaya harvesting requires no capital, so the production function involves only labor L. Papayas sell at a price $p > 0$.

1. Consider now just the first location. In this location there is ample availability of unskilled workers. The production function is therefore linear in the number of workers: $y = AL$, where L is the number of workers and A is the productivity of each worker. Assume that the wage of a worker is w. Assume also $L \geq 0$, and $A > 0$. Solve the cost minimization problem of a farm that wants to produce y papayas in the first location, that is, determine $L_1^*(w, y|A)$ and the cost function $c_1(w, y|A)$. (6 points)

2. Solve for marginal cost $c'_{y1}(w, y|A)$ and average cost $c_1(w, y|A)/y$. Still assuming that only the first location operates, graph and write out the supply function $y_1^S(p, w|A)$. (6 points)

3. Consider now the second location in isolation. In this second location the very first workers are very capable, but the productivity of the workers declines steeply. The production function is $y = L^{1/3}$, where L is the number of workers. Assume that the wage of a worker is w (same as above). Assume also $L \geq 0$. Solve the cost minimization problem of a farm that wants to produce y papayas in this second location, that is, determine $L_2^*(w, y)$ and the cost function $c_2(w, y)$. (5 points)

4. Solve for marginal cost $c'_{y2}(w, y)$ and average cost $c_2(w, y)/y$ in this second location. Assuming that only this second location operates, graph and write out the supply function $y_2^S(p, w)$. (5 points)

5. Now the company decides that it is more efficient to operate the two locations together. In particular, the farm minimizes the total cost from operating the two locations $c_1(w, y_1|A) + c_2(w, y_2)$, subject to producing a total production y of papayas, where $y = y_1 + y_2$. Set up the problem and solve for the cost-minimizing $y_1^*(p, w, y)$ and $y_2^*(p, w, y)$. That is, find how much a given y will be produced in one location and how much in another location. Assume $y > (1/3A)^{1/2}$ (10 points)

6. Compute $\partial y_1^*(p, w, y)/\partial y$ and $\partial y_2^*(p, w, y)/\partial y$. Use these derivatives to provide intuition on how the overall production of y is divided into the two locations. (Keep assuming $y > (1/3A)^{1/2}$) (6 points)

7. Characterize the solution for $y_1^*(p, w, y)$ and $y_2^*(p, w, y)$ in the case $y < (1/3A)^{1/2}$. (Hint: It is a corner solution) (5 points)

8. (Harder) Use what you did in the previous points to derive the overall cost function for the firm, that is, $c^*(p, w, y)$, where the firm optimally allocates the quantity produced between the two locations. If you cannot do it analytically, try graphically. Provide intuition. (10 points)

9. Even if you were not able to solve point 8 analytically, comment on how using the two locations allows the firm to reduce costs relative to using esclusively one or the other. (4 points)