Econ 101A – Midterm 1
Th 30 September 2004.

You have approximately 1 hour and 20 minutes to answer the questions in the midterm. I will collect the exams at 12.30 sharp. Show your work, and good luck!

Problem 1. Utility maximization. (57 points) In class, we have considered the case of maximization of utility with Cobb-Douglas utility function with goods x (baseball) and y (cupcakes) as arguments. We have also considered the case of intertemporal maximization of utility with consumption of goods today, c_0, and in the future, c_1. This problem asks you to combine the two elements and reconsider some of the properties we had found in the two separate cases. Consider a consumer with utility function

$$u(x_0, y_0, x_1, y_1) = (x_0)^\alpha (y_0)^\beta + \frac{1}{1+\delta} (x_1)^\alpha (y_1)^\beta \tag{1}$$

where x_t is the quantity of baseball x consumed at time $t = 0, 1$. Similarly, y_t is the quantity of cupcakes y consumed at time $t = 0, 1$. For reasons that we will see below, we will assume $\alpha + \beta < 1$, so do not substitute $\beta = 1 - \alpha$. As usual, $\alpha > 0$ and $\beta > 0$ also hold.

1. What is the interpretation of δ in the utility function? Interpret in particular the special cases $\delta = 0$ and $\delta \to \infty$. (6 points)
2. Explain the procedure to determine if $u(x_0, y_0, x_1, y_1)$ is concave in (x_0, y_0, x_1, y_1). You should not do the computations. (4 points)
3. We now consider the budget cfsonstraint. Both in period 0 and in period 1 the price of music x is p_x and the price of cupcakes y is p_y. We denote income at time 0 by M_0 and income at time 1 as M_1. Consumers can borrow across periods 0 and 1 at interest rate r. Show that this implies that the (intertemporal) budget constraint is:

$$p_x x_0 + p_y y_0 + \frac{1}{1+r} (p_x x_1 + p_y y_1) \leq M_0 + \frac{1}{1+r} M_1. \tag{2}$$

   (6 points)
4. The consumer maximizes utility (1) with respect to the four consumption variables (x_0, y_0, x_1, y_1), subject to budget constraint (2). Assume that the budget constraint is satisfied with equality (it is) and write the Lagrangean function. Derive the set of first order conditions. (5 points).
5. Show that the first order conditions with respect to (x_0, y_0, x_1, y_1) lead to the familiar conditions

$$\frac{\alpha}{\beta} \frac{y_0}{x_0} = \frac{p_x}{p_y} \tag{3}$$

   and

$$\frac{\alpha}{\beta} \frac{y_1}{x_1} = \frac{p_x}{p_y}. \tag{4}$$

   Give an economic interpretation of these conditions. Are you surprised that the same conditions that hold in the one-period Cobb-Douglas utility maximization also hold in the 2-period one? (5 points)
6. [Harder] If you can, show that the first order conditions imply

$$\left(\frac{y_1}{y_0}\right)^{1-(\alpha+\beta)} = \left(\frac{x_1}{x_0}\right)^{1-(\alpha+\beta)} = \frac{1+r}{1+\delta} \tag{5}$$

   (You may want to leave this for last – you can continue answering the rest of Problem 1 without solving this point) (10 points)

7. Keep assuming $\alpha + \beta < 1$ and consider equation (5). How does relative consumption in period 1 compared to period 0 ($y_1/y_0 = x_1/x_0$) vary as r increases? Explain intuitively why an increase in the interest rate increase consumption in the future. Similarly, how does relative consumption in period 1 compared to period 0 ($y_1/y_0 = x_1/x_0$) vary as δ increases? Provide intuition. (6 points)

8. What does equation (5) imply about y_1/y_0 for $r = \delta$? Provide intuition on why at the optimum the agent smoothes consumption of goods x and y across periods. (6 points) Extra credit: Why is it crucial for this answer that $\alpha + \beta < 1$?

9. For $r = \delta$, solve for the optimal value of x_0^*. Hint: use the budget constraint and equation (3). (5 points)

10. Use the expression for x_0^* that you obtained in point 9. Differentiate x_0^* with respect to M_0, that is, compute $\partial x_0^*/\partial M_0$. Is music x_0 at time 0 a normal good (for all levels of price and income)? (4 points)

Problem 2. (18 points)

1. Consider the maximization problem

$$\max_x f(x, p)$$

where both x and p are real numbers (that is, not vectors). This leads to the first order condition

$$f'_x(x^*, p) = 0.$$

Use the implicit function theorem to give an expression for dx^*/dp. Argue that, as long as x^* is a maximum, the sign of dx^*/dp is given by $f''_{x,p}(x^*, p)$. (8 points)

2. Consider a consumer with preferences over two goods, music x and cupcakes y, with $x \geq 0$ and $y \geq 0$. The consumer is addicted to cupcakes y and is 'loss averse'to consuming less than $Y > 0$ units of cupcakes. Here are the preferences of the consumer. A bundle (x, y) is weakly preferred ($\succsim$) to a bundle (x', y') if and only if the following is true:

$$\begin{aligned}
(y - Y) + x &\geq (y' - Y) + x' \text{ for } y \geq Y \text{ and } y' \geq Y \\
2(y - Y) + x &\geq (y' - Y) + x' \text{ for } y < Y \text{ and } y' \geq Y \\
(y - Y) + x &\geq 2(y' - Y) + x' \text{ for } y \geq Y \text{ and } y' < Y \\
2(y - Y) + x &\geq 2(y' - Y) + x' \text{ for } y < Y \text{ and } y' < Y
\end{aligned}$$

Can you provide a utility function that represents these preferences? Plot some of the indifference curves. Why could these indifference curves capture an aspect of addiction? (10 points)