Econ 101A – Midterm 1
Tu 11 October 2005.

You have approximately 1 hour and 20 minutes to answer the questions in the midterm. Adriana and Vikram will collect the exams at 12.30 sharp. Show your work, and good luck!

Problem 1. Utility maximization and charitable giving. (49 points) Consider Mario, a Californian resident with income M. Mario cares about own consumption c, and about a charitable project G in Louisiana. The welfare effects of the project for Louisiana citizens are measured by $h(g+S)$, where g is Mario's charitable donation and S is the seed money donated by others. Mario derives utility $\alpha h(G)$ from the project, with $\alpha \geq 0$. Therefore, Mario maximizes

$$\begin{aligned} &\max_{c,g} u(c) + \alpha h(g+S) \qquad (1)\\ &s.t.\ c+g \leq M \\ &s.t. c \geq 0 \\ &s.t. g \geq 0 \end{aligned}$$

We assume $u'() > 0$, $u''() < 0$, $h'() > 0$ and $h''() < 0$. That is, both u and h are increasing, concave functions. The function h is concave because there are diminishing returns to the charitable project.

1. What is the interpretation of α in the utility function? Interpret in particular the special case $\alpha = 0$. (3 points)
2. Argue that the problem can be rewritten as

$$\begin{aligned} &\max_{g} u(M-g) + \alpha h(g+S) \\ &s.t. 0 \leq g \leq M. \end{aligned}$$

   (4 points)
3. Neglecting the constraint $0 \leq g \leq M$ for now, derive first order and second order conditions. Argue that the point identified by the first-order condition is a maximum. (6 points)
4. Solve for g^* and c^* for the case $u(c) = \log(c)$ and $h(G) = \log(G)$ using the first-order conditions. (4 points)
5. Keep assuming $u(c) = \log(c)$ and $h(G) = \log(G)$. Now it is time to check the constraint $0 \leq g \leq M$. Is the constraint $g \leq M$ always satisfied? Is the constraint $g \geq 0$ always satisfied? If not, provide an example to the converse. Write the solution for g^* and c^*, taking into account corner solutions (8 points)
6. Keep assuming $u(c) = \log(c)$ and $h(G) = \log(G)$. How do c^* and g^* depend on M, α, and S? Provide intuition for each of these comparative statics, in particular for the latter. (6 points)
7. Now we go back to the general formulation with $u()$ and $h()$ concave. Using the implicit function theorem, and neglecting corner solutions, show that $\partial g^*/\partial S < 0$. Comment on this result. (6 points)
8. (Open-ended) An economist, John List, does an empirical study of the effect of seed money (the money collected early on in a fundraising drive) on charitable donations. He finds that higher seed money is associated with *higher* donations, against the prediction in point 7. What can be an explanation of this finding? (6 points)
9. Keep assuming $u()$ and $h()$ concave. Consider the indirect utility $V(S,M,\alpha) = u(M - g^*(S,M,\alpha)) + \alpha h(g^*(S,M,\alpha) + S)$. Use the envelope theorem to compute $dV(S,M,\alpha)/dS$, that is, how the indirect utility vary as the seed money increases. Under what conditions is $\partial V(S,M,\alpha)/\partial S > 0$ and why? (6 points)

Solution to Problem 1.

1. Parameter δ has the interpretation of an altruism rate, that is, how much the individual cares about the charitable enterprise. For $\alpha = 0$, the individual does not care about the project.

2. Since the utility function is strictly increasing in c, the agent will spend all the money on either c or g, implying $c + g = M$. We can then substitute for $c = M - g$ in the maximization problem (1). The inequality constraint $c \geq 0$ becomes $M - g \geq 0$, or $g \leq M$. This, combined with $g \geq 0$, gives the desired constraint.

3. The first order condition is
$$-u'\left(M - g^*\right) + \alpha h'\left(g^* + S\right) = 0 \tag{2}$$
and the second order condition is
$$u''\left(M - g^*\right) + \alpha h''\left(g^* + S\right) < 0,$$
which is satisfied given that $u''(x) < 0$ for all x and $h''(x) < 0$ for all x.

4. From the first-order condition,
$$-\left(M - g^*\right)^{-1} + \alpha\left(g^* + S\right)^{-1} = 0$$
which implies
$$g^* + S = \alpha\left(M - g^*\right)$$
or
$$g^* = \frac{\alpha M - S}{1 + \alpha}. \tag{3}$$
From this follows
$$c^* = M - g^* = M - \left(\frac{\alpha M - S}{1 + \alpha}\right) = \frac{M + S}{1 + \alpha}.$$

5. It is easy to see from (3) that the constraint $g^* \leq M$ is always satisfied since $g^* = \frac{\alpha M - S}{1+\alpha} \leq M - S/\left(1 + \alpha\right) \leq M$. On the other hand, (3) does not necessarily satisfy $g^* \geq 0$. For example, it does not satisfy it for $\alpha = 0$ and $S > 0$. The intuition is that if the charitable project is sufficiently well-financed (S high), or the individual is sufficiently selfish (α low), the individual would rather channel money out of the charity into his/her own consumption. To take this into account, we write the general solution as
$$\begin{aligned} g^* &= \max\left(\frac{\alpha M - S}{1 + \alpha}, 0\right) \\ c^* &= \min\left(\frac{M + S}{1 + \alpha}, M\right) \end{aligned} \tag{4}$$

6. From (4) we can see that increases in altruism α increase g^* ($\partial g^*/\partial \alpha = \left(M\left(1 + \alpha\right) - \alpha M + S\right)/\left(1 + \alpha\right)^2 = \left(M + S\right)/\left(1 + \alpha\right)^2 > 0$) and decrease c^*, except if the solution is a corner solution. More altruistic people devote more money to the charity. Increases in income M lead to an increase in both the consumption c^* and the charitable donation g^*. Not surprisingly, more income leads to more spending. Finally, a higher seed donation S leads to lower charitable contribution and to more consumption. If other people have already donated into a charity, an individual finds it preferable to channel the money to own consumption.

7. Using the first-order condition (2), we obtain
$$\frac{\partial g^*}{\partial S} = -\frac{\alpha h''\left(g^* + S\right)}{u''\left(M - g^*\right) + \alpha h''\left(g^* + S\right)},$$
which is negative since the denominator is negative (see the second-order condition) and the numerator is negative by concavity of $h\left(\right)$. It follows that $\frac{\partial c^*}{\partial S} < 0$ holds generally, not just for the log case above.

8. A likely explanation for this finding (see List and Lucking-Reiley, *Journal of Political Economy,* 2001) is that individuals use the seed money as a signal for the value of the charity. If others donated, it must be a worthwhile charity. That is, high S signals that the function $h(G)$ is larger than the donor previously thought. To the extent that this force is stronger than the one in point 7, we can explain a positive correlation.

9. Using the envelope theorem, we know that $dV(S,M,\alpha)/dS = \partial(u(M-g^*(S,M,\alpha)) + \alpha h(g^*(S,M,\alpha)+S))/\partial S$, where we can neglect the indirect effects of S on $g^*(S,M,\alpha)$. It follows that $dV(S,M,\alpha)/dS = \alpha h'(g^*(S,M,\alpha)+S)$, which is positive as long as α is positive. Therefore, any altruistic individual is made better off when others contribute to the seed money.

Problem 2. (24 points)

1. You are a consultant for the pumpkin industry. The producers of pumpkins foresee a (small) income increase for the consumers of pumpkins and consult you to predict how this will affect the demand for pumpkins. You observed that, as pumpkin price increases, demand for pumpkin increases. Provide a formal answer to the pumpkin producers and provide intuition. (8 points)

2. Jenny faces a choice set $X = \{A, B, C, D\}$. Her preferences are defined by $A \succsim B, B \succsim D, B \succsim C$. Are these preferences complete? Are these preferences transitive? (8 points)

3. McDo faces a choice set $X = \{fries, apple, sundae\}$. McDo prefers fries to apples, because fries are more appetizing ($fries \succ apple$). McDo prefers apples to sundaes, because a sundae is exceedingly caloric. ($apple \succ sundae$). Finally, McDo prefers sundaes to fries because, once the comparison is between fatty foods, she prefers the sweet one ($sundae \succ fries$). These relations define McDo's preference. Can you represent these preferences with a utility function? (8 points)

Solution to Problem 2.

1. Using the Slutzky equation, we know that

$$\frac{\partial x^* (p, M)}{\partial p} = \frac{\partial h^* (p, v)}{\partial p} - \frac{\partial x^* (p, M)}{\partial M} x^* (p, M)$$

and therefore

$$\frac{\partial x^* (p, M)}{\partial M} = \left[\frac{\partial h^* (p, v)}{\partial p} - \frac{\partial x^* (p, M)}{\partial p} \right] / x^* (p, M)$$

Empirical observation implies $\frac{\partial x^*(p,M)}{\partial p} > 0$. Since $\frac{\partial h^*(p,v)}{\partial p} < 0$ by property of the Hicksian compensated demand, $\frac{\partial x^*(p,M)}{\partial M} < 0$ follows. Intuitively, the empirical observation tells us that pumpkins are Giffen goods, which can only occur if the income effect is negative, that is if the good is inferior at that level of price and income. Therefore, you report that the quantity demanded will go down. The company fires you!

2. Jenny's preferences are not complete because the preferences between A and C are not complete: it is not the case that $A \succsim C$ and it is not the case that $C \succsim A$. That is, Jenny has not stated preferences between A and C, against the completeness axiom. These preferences are also *not* transitive because $A \succsim B$ and $B \succsim D$, but it is not true that $A \succsim D$.

3. McDo's preferences are intransitive: $fries \succsim apple$ and $apple \succsim sundae$, but it is not true that $fries \succsim sundae$. Given that these preference are intransitive, we cannot represent them with a utility function. To see why, assume that we could, and that there existed a utility function u that represents them. Then, it would be the case that $u(fries) > u(apple)$, and $u(apples) > u(sundae)$. However, this would then imply $u(fries) > u(sundae)$ which, by the definition of representing preferences, implies $fries \succ sundae$. This contradicts the given preferences.