**Econ 101A – Midterm 2**
**Th 27 October 2004.**

You have approximately 1 hour and 20 minutes to answer the questions in the midterm. We will collect the exams at 12.30 sharp. Show your work, and good luck!

**Problem 1. Uncertainty (21 points)**. Consider the case of transportation accidents.

1. Mary is worried about car accidents next year. At the beginning of the year Mary has \$10,000 in wealth, with no additional earnings for the year. With probability 2/3 Mary has an accident and suffers a loss of \$7,500. (Mary is not hurt, just the car) With probability 1/3 there is no accident leading to zero loss. What is Mary's expected wealth? (3 points)

2. From now on, assume that Mary's utility function over wealth is $u(w) = w^{1/2}$, where $w$ is the wealth left over after the accident. What is her expected utility? (3 points)

3. Mary can purchase an insurance with a premium \$5,100. This insurance will fully reimburse the damage (\$7,500) if an accident occurs and wil give no payment in case of no accident. What is Mary's expected wealth if she takes the insurance? Is the insurance premium fair? (5 points)

4. Will Mary take up the insurance? (Compute the expected utility and compare to expected utility in point 2) (3 points)

5. Angela is a friend of Mary. She hears Mary talk about her decision and she exclaims "*I* would not have purchased the insurance!" Given an example of a utility function such that Mary would *not* have purchased the insurance given a wealth of \$10,000 and the accident probabilities given above. (4 points)

6. Angela adds: "Mary, you are so risk-averse, relax!". Let's leave aside the 'relaxing' issue. Provide intuition on why risk-aversion translates into a concave utility function, like $u(w) = w^{1/2}$. (3 points)

**Problem 2. Production. (43 points)** In this exercise, we consider a firm producing product $y$ using two inputs, labor $L$ and capital $K$. The production function is $y = f(L, K) = (L + K)^{\alpha}$. Assume that the wage of a worker is $w$ and the cost of capital is $r$. Assume $L \geq 0$, $K \geq 0$, and $\alpha > 0$.

1. Draw a picture of the isoquants. What is the unusual feature of this production function? (5 point)

2. For which values of $\alpha$ does the function exhibit decreasing returns to scale (that is, $f(tL) < tf(L)$ for all $t > 1$ and all $L \geq 0$)? (3 points)

3. Consider now the first step of the cost minimization problem. The firm solves

$$\min wL + rK$$
$$s.t. f(L, K) \geq y$$

   for $y > 0$. What are the solutions for $L^*(w, r, y|\alpha)$ and $K^*(w, r, y|\alpha)$? (This notation stresses that the solution depends also on the parameter $\alpha$. Hint: You are better off not using Lagrangeans. The pictures you drew in point 1 may be helpful) (9 points)

4. Write down the implied cost function $c(w, y|\alpha)$. (4 points)

5. Derive an expression for the average cost $c\left(w, y|\bar{L}, \alpha\right)/y$ and the marginal cost $c'_y\left(w, y|\bar{L}, \alpha\right)$ for $y > 0$ and $w < r$. Graph the average cost and marginal cost for $\alpha = .5$, $w = 1$, and $r = 2$. Graph the supply function for the same values of the parameters. [remember, $y$ is on the horizontal axis]. (5 points)

6. Now that we graphically solved for the supply function, we also derive it formally for all $\alpha > 0$. Consider the second step of cost minimization

$$\max_y py - c\left(w, y|\bar{L}, \alpha\right).$$

Write down the first order condition and the second order conditions. Solve for $y^*\left(w, p|\bar{L}, \alpha\right)$. (here do *not* assume $w < r$) For what values of $\alpha$ is the second order condition satisfied? (5 points)

7. From now on, assume $\alpha < 1$. Take the solution for $y^*\left(w, p|\bar{L}, \alpha\right)$ in point 6 and consider what happens to $y^*$ as the wage $w$ increases. Obtain the sign of $\partial y^*/\partial w$ for the cases $w < r$ and $w > r$. Provide intuition on this result. (5 points)

8. Consider now what happens to the supply function as price of output $p$ increase. Obtain the sign of $\partial y^*/\partial p$ and provide intuition on the result. (3 points)

9. Does the company make, negative, or positive profits for $p > 0$? Provide an argument for your answer. (4 points)

**Problem 3. (Exercise with less guidance than usual) (18 points)** Consider now the decision making of a governor that has limited funds to spend and wants to minimize the accidents on freeways and railways. Each freeway accident occurs with probability $p_F$ and a railway accident occurs with probability $p_R$. Either accident generates a social loss of $L$. If there is no loss, the social utility is 0. The expected social utility therefore is $-p_F L - p_R L$, with $L > 0$. The governor maximizes social utility by spending the State Budget $M$ on improving streets ($M_F$) and railways ($M_R$), with $M_F + M_R \leq M$. In particular, the probabilities of accident depend on the funds spent as follows: $p_F(M_F) = \exp(-M_F)$ and $p_R(M_R) = \exp(-M_R)$

1. Graph $p_F(M_F) = \exp(-M_F)$. Comment briefly on how increased expenditure affects the probability of an accident. (3 points)

2. Solve for the optimal levels of spending $M_F^*$ and $M_R^*$, as well as for $p_F^*$ and $p_R^*$. Comment on the solution you found. Will the governor reduce the probability of accidents to zero if the budget $M$ is very large? (15 points)