Econ 101A – Midterm 2
Th 7 November.

Do not turn page until instructed to.

Problem 1. Cost functions, demand functions, market equilibrium. (41 points) In this exercise, we consider the market equilibrium of an economy of which we know the cost function of firms, as well as the demand function of consumers. We consider first the short-run equilibrium, and then the long-run equilibrium. Each firm in the industry has the same technology with cost function

$$c(y) = k^2 + y^2$$

if $y > 0$ and $c(0) = 0$, where y is the quantity produced and k is some fixed cost that the firm pays only if it produces anything. You should think of this cost function as coming out of a standard cost minimization problem with respect to the inputs.

1. Derive the average cost $c(y)/y$ and the marginal cost $c'_y(y)$ for $y > 0$. Graph the average cost and marginal cost [remember, p is on the vertical axis]. (3 points)

2. Draw the supply function in the graph, and write down the equation that represents the supply function $y(p)$. [If possible, write y as a function of p and not viceversa] (5 points)

3. Derive the aggregate supply function $Y^S(p)$ by summing the supply $y(p)$ over the J firms that are in the market. Write down the expression for $y^S(p)$ (2 points)

4. Consider now the demand side of the market. For simplicity, assume a linear demand function: $Y^D(p) = a - bp$ where Y^D is the total quantity demanded in the industry. Assume that this comes from aggregation of the individual demand functions derived from maximization. Find the short-run equilibrium price p^* by equating Y^D and Y^S. Assume that a and b are such that we are on the increasing part of the supply function (i.e., the firm produces a positive quantity.) Find the short-run equilibrium industry production $Y^* = Y^S(p^*) = Y^S(p^*)$. (4 points)

5. Under what condition for a, J, b, k the firms will indeed produce a positive quantity of output? We maintain this assumption for points 6 and 7. (3 points)

6. Assuming positive production, we consider several comparative statics predictions. What happens to Y^* and p^* as the number of firms J increases? What is the intuition? [You do not even have to take derivatives, as long as you can infer the sign of the effect from the equations you derived at point 5]. If this was Ec10, and you could not do any algebra but only shift curves, how would you prove the same result graphically? What happens to Y^* and p^* as the demand coefficient a increase? What is the intuition? (4 points)

7. What happens to Y^* and p^* as the fixed costs k increase? What is the intuition? (3 points)

8. Consider now the long-run equilibrium, in which firms are allowed to enter the market. Solve for the number of firms J^* that will enter into the market [You can assume that the number that you find is integer] (5 points)

9. How does the number of firms J^* depend on k and a? [Give the sign only] (2 points)

10. What is the equation for the long-run supply curve? Is it horizontal, increasing or decreasing? (4 points)

11. What does the answer to the previous question tell you about incidence of a tax t? You can argue intuitively, or using the expression we derived in class on $\partial p/\partial t$. In the long-run, who bears the burden of the tax, the consumers or the producers, or both? [you may find a graph helpful] (6 points)

Problem 2. Uncertainty. (20 points) In the world, we observe many individuals that purchase both insurance and that gamble, a puzzling behavior. Define the problem as follows. An agent has utility function $u(w)$ defined over wealth w, with $u' > 0$. The agent has wealth w.

1. Consider the following stylized Las Vegas gamble: the agent wins \$10 with probability 1/10 and loses \$2 with probability 9/10. Write the expected value and the expected utility associated with this gamble. (5 points)

2. Try to show that a risk-averse agent (concave utility, $u'' < 0$) will prefer not to take this gamble using Jensen's inequality. Risk-averse people do not go to Las Vegas. If you do not remember Jensen's inequality, it's ok! Try a graphical or verbal argument. (5 points)

3. In class, we also showed that risk-averse agents purchase insurance. To sum up, risk-averse agents purchase insurance, but do not gamble. To reconcile the theory with the evidence that people do both, Friedman and Savage in 1950 proposed that the utility function over wealth is as in Figure 1: concave for low levels of wealth, and convex for high levels of wealth. Explain verbally why this theory predicts that we should observe in the world both insurance and gambling. (4 points)

4. Do you find this explanation convincing? What kind of evidence would imply that this theory is the wrong explanation? (6 points)

Problem 3. Economics of crime (Becker). (18 points) Consider a risk-neutral agent that files taxes. She puts effort e, $e \in [0, 1]$, to file taxes correctly. Effort e has cost $e^2/2$. The benefit of effort is that it reduces the probability of errors: the agent makes an error with probability $(1 - e)$. If the agent makes an error, she is discovered with probability p, at which point she has to pay a fine f, so wealth w goes down to $w - f$. The maximization problem of the individual is

$$\max_e p(1-e)(w-f) + [1 - p(1-e)]\, w - \frac{e^2}{2}.$$

1. Write down the first order conditions and solve for the optimal level of effort e^* (3 points)

2. Why is effort e^* increasing in the probability of being caught p? Why is it increasing in the fine f? (3 points)

3. (Hard) Here is now the interesting part: what is the optimal choice of p and f for the goverment? Suppose that the government has to pay wages x to agents that audit the taxes. Moreover, the government cares about deviations from the optimal level of effort $e = 1$. The government then maximizes

$$\begin{aligned}
&\max_{f,p} -px - (1 - e^*(p, f))^2 \\
&s.t.\ 0 \le e^*(p, f) \le 1, \\
&s.t.\ 0 \le p \le 1
\end{aligned}$$

   You should substitute the expression for $e^*(p, f)$ that you found in point 1. Now compute the optimal levels of fine f^* and auditing probability p^*. What are the solutions? [Hint: Do not use the Lagrangeans, there are corner solutions. Use your intuition. Technically, there is no optimum, but there are sups] (8 points)

4. Why does the government adopt this enforcement strategy? (4 points)