Economics 101A
(Lecture 11)

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Outline

1. Intertemporal choice II
2. Altruism and charitable donations
3. Introduction to probability
4. (Expected Utility)

1 Intertemporal choice II

- Maximization problem:

$$\max U(c_0) + \frac{1}{1+\delta}U(c_1)$$
$$s.t.\ c_0 + \frac{1}{1+r}c_1 \leq M_0 + \frac{1}{1+r}M_1$$

- Ratio of f.o.c.s:

$$\frac{U'(c_0)}{U'(c_1)} = \frac{1+r}{1+\delta}$$

- Case $r = \delta$
  - $c_0^* \quad c_1^*$?
  - Substitute into budget constraint using $c_0^* = c_1^* = c^*$:
    $$\frac{2+r}{1+r}c^* = \left[M_0 + \frac{1}{1+r}M_1\right]$$
    or
    $$c^* = \frac{1+r}{2+r}M_0 + \frac{1}{2+r}M_1$$
  - We solved problem virtually without any assumption on U!
  - Notice: $M_0 < c^* < M_1$

- Case $r > \delta$
  - $c_0^* \quad c_1^*$?

- Comparative statics with respect to income M_0

- Rewrite ratio of f.o.c.s as

$$U'(c_0) - \frac{1+r}{1+\delta} U'(c_1) = 0$$

- Substitute c_1 in using $c_1 = M_1 + (M_0 - c_0)(1+r)$ to get

$$U'(c_0) - \frac{1+r}{1+\delta} U'(M_1 + (M_0 - c_0)(1+r)) = 0$$

- Apply implicit function theorem:

$$\frac{\partial c_0^*(r, \mathbf{M})}{\partial M_0} = -\frac{-\frac{1+r}{1+\delta} U''(c_1)(1+r)}{U''(c_0) - \frac{1+r}{1+\delta} U''(c_1) * (-(1+r))}$$

- Denominator is always negative

- Numerator is positive

- $\partial c_0^* (r, \mathbf{M}) / \partial M_0 > 0$ — consumption at time 0 is a normal good.

- Can also show $\partial c_0^* (r, \mathbf{M}) / \partial M_1 > 0$

- Comparative statics with respect to interest rate r

- Apply implicit function theorem:

$$\frac{\partial c_0^*(r,\mathbf{M})}{\partial r} = -\frac{-\frac{1}{1+\delta}U'(c_1)}{U''(c_0)-\frac{1+r}{1+\delta}U''(c_1)*(-(1+r))} - \frac{-\frac{1+r}{1+\delta}U''(c_1)*(M_0-c_0)}{U''(c_0)-\frac{1+r}{1+\delta}U''(c_1)*(-(1+r))}$$

- Denominator is always negative

- Numerator: First term is negative (subsitution effect)

- Second term is income effect:
  - positive if $M_0 > c_0$
  - negative if $M_0 < c_0$.

2 Altruism and Charitable Donations

- Maximize utility = satisfy self-interest?

- No, not necessarily

- 2-person economy:
  - Mark has income M_M and consumes c_M
  - Wendy has income M_W and consumes c_W

- One good: c, with price $p = 1$

- Utility function: $u(c)$, with $u' > 0$, $u'' < 0$

- Wendy is altruistic: she maximizes $u(c_W)+\alpha u\left(c_M\right)$ with $\alpha > 0$

- Mark simply maximizes $u(c_M)$

- Wendy can give a donation of income D to Mark.

- Wendy computes the utility of Mark as a function of the donation D

- Mark maximizes

$$\max_{c_M} u(c_M)$$
$$s.t.\ c_M \leq M_M + D$$

- Solution: $c_M^* = M_M + D$

- Wendy maximizes

$$\max_{c_M, D} u(c_W) + \alpha u\left(M_M + D\right)$$
$$s.t.\ c_W \leq M_W - D$$

- Rewrite as:

$$\max_{D} u(M_W - D) + \alpha u\left(M_M + D\right)$$

- First order condition:

$$-u'(M_W - D^*) + \alpha u'\left(M_M + D^*\right) = 0$$

- Second order conditions:

$$u''(M_W - D^*) + \alpha u''\left(M_M + D^*\right) < 0$$

- Assume $\alpha = 1$.
  - Solution?
  - $u'(M_W - D) = u'(M_M + D^*)$
  - $M_W - D^* = M_M + D^*$ or $D^* = (M_W - M_M)/2$
  - Transfer money so as to equate incomes!
  - Careful: $D < 0$ (negative donation!) if $M_M > M_W$

- Corrected maximization:

$$\max_D u(M_W - D) + \alpha u(M_M + D)$$
$$s.t. D \geq 0$$

- Solution ($\alpha = 1$):

$$D^* = \begin{cases} (M_W - M_M)/2 & \text{if } M_W - M_M > 0 \\ 0 & \text{otherwise} \end{cases}$$

- Assume interior solution. ($D^* > 0$)

- Comparative statics 1 (altruism):

$$\frac{\partial D^*}{\partial \alpha} = -\frac{u'(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0$$

- Comparative statics 2 (income of donor):

$$\frac{\partial D^*}{\partial M_W} = -\frac{-u''(M_W + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0$$

- Comparative statics 3 (income of recipient):

$$\frac{\partial D^*}{\partial M_M} = -\frac{\alpha u''(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} < 0$$

3 Introduction to Probability

- So far deterministic world:
  - income given, known M
  - interest rate known r
- But some variables are unknown at time of decision:
  - future income M_1?
  - future interest rate r_1?
- Generalize framework to allow for uncertainty
  - Events that are truly unpredictable (weather)
  - Event that are very hard to predict (future income)

- Probability is the language of uncertainty

- Example:
  - Income M_1 at $t = 1$ depends on state of the economy
  - Recession ($M_1 = 20$), Slow growth ($M_2 = 25$), Boom ($M_3 = 30$)
  - Three probabilities: p_1, p_2, p_3
  - $p_1 = P(M_1) = P(\text{recession})$

- Properties:
  - $0 \leq p_i \leq 1$
  - $p_1 + p_2 + p_3 = 1$

- Mean income: $EM = \sum_{i=1}^{3} p_i M_i$

- If $(p_1, p_2, p_3) = (1/3, 1/3, 1/3)$,

$$EM = \frac{1}{3}20 + \frac{1}{3}25 + \frac{1}{3}30 = \frac{75}{3} = 25$$

- Variance of income: $V(M) = \sum_{i=1}^{3} p_i (M_i - EM)^2$

- If $(p_1, p_2, p_3) = (1/3, 1/3, 1/3)$,

$$\begin{aligned} V(M) &= \frac{1}{3}(20-25)^2 + \frac{1}{3}(25-25)^2 + \frac{1}{3}(30-25)^2 \\ &= \frac{1}{3}5^2 + \frac{1}{3}5^2 = 2/3 * 25 \end{aligned}$$

- Mean and variance if $(p_1, p_2, p_3) = (1/4, 1/2, 1/4)$?

4 Expected Utility

- Nicholson, Ch. 18, pp. 533–541 [OLD: Ch. 8, pp. 198–206]

- Consumer at time 0 asks: what is utility in time 1?

- At $t = 1$ consumer maximizes

$$\begin{array}{l} \max U(c^1) \\ s.t.\ c_i^1 \leq M_i^1 + (1+r)\left(M^0 - c^0\right) \end{array}$$

with $i = 1, 2, 3$.

- What is utility at optimum at $t = 1$ if $U' > 0$?

- Assume for now $M^0 - c^0 = 0$

- Utility $U\left(M_i^1\right)$

- This is uncertain, depends on which i is realized!

- How do we evaluate future uncertain utility?

- **Expected utility**

$$EU = \sum_{i=1}^{3} p_i U\left(M_i^1\right)$$

- In example:

$$EU = 1/3U(20) + 1/3U(25) + 1/3U(30)$$

- Compare with $U(EC) = U(25)$.

- Agents prefer riskless outcome EM to uncertain outcome M if

$$\begin{aligned} 1/3U(20) + 1/3U(25) + 1/3U(30) &< U(25) \text{ or} \\ 1/3U(20) + 1/3U(30) &< 2/3U(25) \text{ or} \\ 1/2U(20) + 1/2U(30) &< U(25) \end{aligned}$$

- Picture

- Depends on sign of U'', on concavity/convexity

- Three cases:

  - $U''(x) = 0$ for all x. (linearity of U)

    * $U(x) = a + bx$

    * $1/2U(20) + 1/2U(30) = U(25)$

  - $U''(x) < 0$ for all x. (concavity of U)

    * $1/2U(20) + 1/2U(30) < U(25)$

  - $U''(x) > 0$ for all x. (convexity of U)

    * $1/2U(20) + 1/2U(30) > U(25)$

- If $U''(x) = 0$ (linearity), consumer is indifferent to uncertainty

- If $U''(x) < 0$ (concavity), consumer dislikes uncertainty

- If $U''(x) > 0$ (convexity), consumer likes uncertainty

- Do consumers like uncertainty?

- Do *you* like uncertainty?

- **Theorem. (Jensen's inequality)** If a function $f(x)$ is concave, the following inequality holds:

$$f(Ex) \geq Ef(x)$$

where E indicates expectation. If f is strictly concave, we obtain

$$f(Ex) > Ef(x)$$

- Apply to utility function U.

- Individuals dislike uncertainty:

$$U(Ex) \geq EU(x)$$

- Jensen's inequality then implies U concave ($U'' \leq 0$)

- Relate to diminishing marginal utility of income

5 Next Lectures

- Risk Aversion
- Coefficient of risk aversion
- Applications:
  - Insurance
  - Portfolio choice