Economics 101A
(Lecture 2)

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Outline

1. Who Am I?
2. Questions on Syllabus
3. An Example: Economics of Discrimination
4. Comparative Statics
5. Implicit function theorem
6. Envelope Theorem

1 Who am I?

Stefano DellaVigna

- Assistant Professor, Department of Economics

- Bocconi (Italy) undergraduate (Econ.), Harvard PhD (Econ.)

- Psychology and economics, applied microeconomics, behavioral finance, aging, media

- Evans 515

- OH: We 2-4

2 Questions on Syllabus?

- For questions on enrollment, note:
  - In the past, everyone interndning to take the class managed to
  - I expect (and hope) that this will happen also this year
  - However: No certainty of this
  - Have to wait till end of second week
  - For further questions, see Desiree Schaan. OH: 508-2, 10-12, 1-3 every day till September 7th

3 An Example: Economics of discrimination

- Ok, I need maths. But where is the economics?

- Workers:
  - A and B. They produce 1 widget per hour
  - Both have reservation wage $\bar{u}$

- Firm:
  - sells widgets at price $p > \bar{u}$ (assume p given)
  - dislikes worker B
  - Maximizes profits ($p*$ no of widgets $-$ cost of labor) minus disutility d if employs B

- Wages and employment in this industry?

- Employment
  - Net surplus from employing A: $p - \bar{u}$
  - Net surplus from employing B: $p - \bar{u} - d$
  - If $\bar{u} < p < \bar{u} + d$, Firm employs A but not B
  - If $\bar{u} + d < p$, Firm employs both

- What about wages?

- Case I. Firm monopolist and no worker union
  - Firm maximizes profits and gets all the net surplus
  - Wages of A and B equal $\bar{u}$
- Case II. Firm monopolist and worker union
  - Firm and worker get half of the net surplus each
  - Wage of A equals $\bar{u} + .5 * (p - \bar{u})$
  - Wage of B equals $\bar{u} + .5 * (p - \bar{u} - d)$
- Case III. Perfect competition among firms that discriminate ($d > 0$)
  - Prices are lowered to the cost of production
  - Wage of A equals p
  - B is not employed

- The magic of competition

- Case IIIb. Perfect competition + At least **one** firm does not discriminate ($d = 0$)
  - This firm offers wage p to both workers
  - What happens to worker B?
  - She goes to the firm with $d = 0$!
  - In equilibrium now:
    - Wage of A equals p
    - Wage of B equals p as well!

- Is this true? Any evidence?
- S. Black and P. Strahan, AER 2001.
  - Local monopolies in banking industry until mid 70s
  - Mid 70s: deregulation
  - From local monopolies to perfect competition.
  - Wages?
    * Wages fall by 6.1 percent
  - Discrimination?
    * Wages fall by 12.5 percent for men
    * Wages fall by 2.9 percent for women
    * Employment of women as managers increases by 10 percent

- More evidence on discrimination

- Does black-white and male-female wage back derive from discrimination?

- Field experiment (Betrand and Mullainathan, 2005)

- Send real CV with randomly picked names:

  - Male/Female

  - White/African American

- Measure call-back rate from interview

- Results (Table 1, Handout):

  - Call-back rates 50 percent higher for Whites!

  - No effect for Male-Female call back rates

- Strong evidence of discrimination against African Americans

- Example of Applied Microeconomics

- Not (really) covered in this class: See Ec142 and (partly) Ec152

- If curious: read Steven Levitt and Stephen Dubner, *Freakonomics.*

4 Comparative statics

- Economics is all about 'comparative statics'

- What happens to optimal economic choices if we change one parameter?

- Example: Car production. Consumer:

  1. Car purchase and increase in oil price

  2. Car purchase and increase in income

- Producer:

  1. Car production and minimum wage increase

  2. Car production and decrease in tariff on Japanese cars

- Next two sections

5 Implicit function theorem

- Implicit function: Ch. 2, pp. 32–33 [OLD, 32–34]

- Consider function $y = g(x, p)$

- Can rewrite as $y - g(x, p) = 0$

- **Implicit function** has form: $h(y, x, p) = 0$

- Often we need to go from implicit to explicit function

- Example 3: $1 - xy - e^y = 0$.

- Write x as function of y :

- Write y as function of x :

- **Univariate implicit function theorem (Dini)**: Consider an equation $f(p, x) = 0$, and a point (p_0, x_0) solution of the equation. Assume:

  1. f continuous and differentiable in a neighbourhood of (p_0, x_0);

  2. $f'_x(p_0, x_0) \neq 0$.

- Then:

  1. There is one and only function $x = g(p)$ defined in a neighbourhood of p_0 that satisfies $f(p, g(p)) = 0$ and $g(p_0) = x_0$;

  2. The derivative of $g(p)$ is

$$g'(p) = -\frac{f'_p(p, g(p))}{f'_x(p, g(p))}$$

- Example 3 (continued): $1 - xy - e^y = 0$

- Find derivative of $y = g(x)$ implicitely defined for $(x, y) = (1, 0)$

- Assumptions:

  1. Satisfied?

  2. Satisfied?

- Compute derivative

- **Multivariate implicit function theorem (Dini)**: Consider a set of equations ($f_1(p_1, ..., p_n; x_1, ..., x_s) = 0; ...; f_s(p_1, ..., p_n; x_1, ..., x_s) = 0$), and a point ($p_0$,$x_0$) solution of the equation. Assume:

  1. $f_1, ..., f_s$ continuous and differentiable in a neighbourhood of (p_0,x_0);

  2. The following Jakobian matrix $\frac{\partial \mathbf{f}}{\partial \mathbf{x}}$ evaluated at (p_0,x_0) has determinant different from 0:

$$\frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & & \frac{\partial f_1}{\partial x_s} \\ ... & ... & ... \\ \frac{\partial f_s}{\partial x_1} & ... & \frac{\partial f_s}{\partial xs} \end{pmatrix}$$

- Then:

  1. There is one and only set of functions $x = \mathbf{g}(p)$ defined in a neighbourhood of p_0 that satisfy $\mathbf{f}(p, \mathbf{g}(p)) = \mathbf{0}$ and $\mathbf{g}(p_0) = x_0$;

  2. The partial derivative of x_i with respect to p_k is

$$\frac{\partial g_i}{\partial p_k} = -\frac{\det\left(\frac{\partial(f_1,\ldots,f_s)}{\partial(x_1,\ldots x_{i-1},p_k,x_{i+1}\ldots,x_s)}\right)}{\det\left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)}$$

- Example 2 (continued): Max $h(x_1, x_2) = p_1 * x_1^2 + p_2 * x_2^2 - 2x_1 - 5x_2$

- f.o.c. $x_1 : 2p_1 * x_1 - 2 = 0 = f_1(p,x)$

- f.o.c. $x_2 : 2p_2 * x_2 - 5 = 0 = f_2(p,x)$

- Comparative statics of x_1^* with respect to p_1?

- First compute det $\left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)$

$$\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$$

- Then compute $\det\left(\frac{\partial(f_1,\ldots,f_s)}{\partial(x_1,\ldots x_{i-1},p_k,x_{i+1}\ldots,x_s)}\right)$

$$\begin{pmatrix} \frac{\partial f_1}{\partial p_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial p_1} & \frac{\partial f_2}{\partial x_2} \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$$

- Finally, $\frac{\partial x_1}{\partial p_1} =$

- Why did you compute $\det\left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)$ already?

6 Envelope Theorem

- Ch. 2, pp. 33–37 [OLD, 34–39]

- You now know how x_1^* varies if p_1 varies.

- How does $h\left(\mathbf{x}^*\left(\mathbf{p}\right)\right)$ vary as p_1 varies?

- Differentiate $h(x_1^*(p_1, p_2), x_2^*(p_1, p_2), p_1, p_2)$ with respect to p_1 :

$$\begin{aligned}
& \frac{dh(x_1^*(p_1, p_2), x_2^*(p_1, p_2), p_1, p_2)}{dp_1} \\
= \; & \frac{\partial h(\mathbf{x}^*, \mathbf{p})}{\partial x_1} * \frac{\partial x_1^*(\mathbf{x}^*, \mathbf{p})}{\partial p_1} \\
& + \frac{\partial h(\mathbf{x}^*, \mathbf{p})}{\partial x_2} * \frac{\partial x_2^*(\mathbf{x}^*, \mathbf{p})}{\partial p_1} \\
& + \frac{\partial h(\mathbf{x}^*, \mathbf{p})}{\partial p_1}
\end{aligned}$$

- Notice: First two terms are zero.

- **Envelope Theorem** for unconstrained maximization. Assume that you maximize function $f(\mathbf{x};\mathbf{p})$ with respect to x. Consider then the function f at the optimum, that is, $f(\mathbf{x}^*(\mathbf{p}),\mathbf{p})$. The total differential of this function with respect to p_i equals the partial derivative with respect to p_i:

$$\frac{df(\mathbf{x}^*(\mathbf{p}),\mathbf{p})}{dp_i} = \frac{\partial f(\mathbf{x}^*(\mathbf{p}),\mathbf{p})}{\partial p_i}.$$

- You can disregard the indirect effects. Graphical intuition.

7 Next Class

- Next class:
  - Convexity and Concavity
  - Constrained Maximization
  - Envelope Theorem II

- Going toward:
  - Preferences
  - Utility Maximization (where we get to apply maximization techniques the first time)