Economics 101A
(Lecture 9)

Stefano DellaVigna

September 27, 2005

Outline

1. Slutzky equation
2. Complements and substitutes
3. Do utility functions exist?
4. Application 1: Labor Supply

1 Slutsky equation

- Now: go back to Utility Max. in case where p_2 increases to $p_2' > p_2$

- What is $\partial x_2^* / \partial p_2$? Decompose effect:

  1. Substitution effect of an increase in p_i

     – $\partial h_2^* / \partial p_2$, that is change in EMIN point as p_2 descreases

     – Moving along an indifference curve

     – Certainly $\partial h_2^* / \partial p_2 < 0$

2. Income effect of an increase in p_i

   – $\partial x_2^*/\partial M$, increase in consumption of good 2 due to increased income

   – Shift out a budget line

   – $\partial x_2^*/\partial M > 0$ for normal goods, $\partial x_2^*/\partial M < 0$ for inferior goods

- Nicholson, Ch. 5, pp. 135–138 [OLD: 131–136].

- $h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u}))$

- How does the Hicksian demand change if price p_i changes?

$$\frac{dh_i}{dp_i} = \frac{\partial x_i^*(\mathbf{p}, e)}{\partial p_i} + \frac{\partial x_i^*(\mathbf{p}, e)}{\partial M}\frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_i}$$

- What is $\frac{\partial e(\mathbf{p},\bar{u})}{\partial p_i}$? Envelope theorem:

$$\begin{aligned} \frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_i} &= \frac{\partial}{\partial p_i}\left[p_1 h_1^* + p_2 h_2^* - \lambda(u(h_1^*, h_2^*, \bar{u}) - \bar{u})\right] \\ &= h_i^*(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p, \bar{u})) \end{aligned}$$

- Therefore

$$\frac{\partial h_i(\mathbf{p},\bar{u})}{\partial p_i} = \frac{\partial x_i^*(\mathbf{p},e)}{\partial p_i} + \frac{\partial x_i^*(\mathbf{p},e)}{\partial M} x_1^*(p_1,p_2,e)$$

- Rewrite as

$$\begin{aligned}\frac{\partial x_i^*(\mathbf{p},M)}{\partial p_i} &= \frac{\partial h_i(\mathbf{p},v(\mathbf{p},M))}{\partial p_i} \\ &\quad - x_1^*(p_1,p_2,M)\frac{\partial x_i^*(\mathbf{p},M)}{\partial M}\end{aligned}$$

- Important result! Allows decomposition into substitution and income effect

- Two effects of change in price:

  1. Substitution effect negative: $\frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} < 0$

  2. Income effect: $-x_1^*(p_1, p_2, M)\frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}$

     – negative if good i is normal ($\frac{\partial x_i^*(\mathbf{p}, M)}{\partial M} > 0$)

     – positive if good i is inferior ($\frac{\partial x_i^*(\mathbf{p}, M)}{\partial M} < 0$)

- Overall, sign of $\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_i}$?

  – negative if good i is normal

  – it depends if good i is inferior

- Example 1 (ctd.): Cobb-Douglas. Apply Slutsky equation

- $x_i^* = \alpha M / p_i$

- $h_i^* =$

- Derivative of Hicksian demand with respect to price:
$$\frac{\partial h_i\left(\mathbf{p}, \overline{u}\right)}{\partial p_i} =$$

- Rewrite h_i^* as function of m: $h_i\left(\mathbf{p}, v(\mathbf{p}, M)\right)$

- Compute $v(\mathbf{p}, M) =$

- Substitution effect:

$$\frac{\partial h_i\left(\mathbf{p}, v(\mathbf{p}, M)\right)}{\partial p_i} =$$

- Income effect:

$$-x_i^*(p_1, p_2, M)\frac{\partial x_i^*\left(\mathbf{p}, M\right)}{\partial M} =$$

- Sum them up to get

$$\frac{\partial x_i^*\left(\mathbf{p}, M\right)}{\partial p_i} =$$

- It works!

2 Complements and substitutes

- Nicholson, Ch. 6, pp. 161–166 [OLD: 152–158].

- How about if price of another good changes?

- Generalize Slutsky equation

- Slutsky Equation:

$$
\begin{aligned}
\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_j} &= \frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} \\
&\quad - x_j^*(p_1, p_2, M)\frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}
\end{aligned}
$$

- Substitution effect

$$\frac{\partial h_i\,(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} > 0$$

for $n = 2$ (two goods). Ambiguous for $n > 2$.

- Income effect:

$$-x_j^*(p_1, p_2, M)\frac{\partial x_i^*\,(\mathbf{p}, M)}{\partial M}$$

  - negative if good i is normal
  - positive if good i is inferior

- How do we define complements and substitutes?

- Def. 1. Goods i and j are **gross substitutes** at price $\mathbf{p}$ and income M if

$$\frac{\partial x_i^* (\mathbf{p}, M)}{\partial p_j} > 0$$

- Def. 2. Goods i and j are **gross complements** at price $\mathbf{p}$ and income M if

$$\frac{\partial x_i^* (\mathbf{p}, M)}{\partial p_j} < 0$$

- Example 1 (ctd.): $x_1^* = \alpha M/p_1$, $x_2^* = \beta M/p_2$.

- Gross complements or gross substitutes? Neither!

- Notice: $\frac{\partial x_i^*(\mathbf{p},M)}{\partial p_j}$ is usually different from $\frac{\partial x_j^*(\mathbf{p},M)}{\partial p_i}$

- Better definition.

- Def. 3. Goods i and j are **net substitutes** at price $\mathbf{p}$ and income M if

$$\frac{\partial h_i^* \left(\mathbf{p}, v(\mathbf{p}, M)\right)}{\partial p_j} = \frac{\partial h_j^* \left(\mathbf{p}, v(\mathbf{p}, M)\right)}{\partial p_i} > 0$$

- Def. 4. Goods i and j are **net complements** at price $\mathbf{p}$ and income M if

$$\frac{\partial h_i^* \left(\mathbf{p}, v(\mathbf{p}, M)\right)}{\partial p_j} = \frac{\partial h_j^* \left(\mathbf{p}, v(\mathbf{p}, M)\right)}{\partial p_i} < 0$$

- Example 1 (ctd.): $h_1^* = \overline{u} \left(\frac{\alpha}{1-\alpha} \frac{p_2}{p_1} \right)^{1-\alpha}$

- Net complements or net substitutes? Net substitutes!

3 Do utility functions exist?

- Preferences and utilities are theoretical objects

- Many different ways to write them

- How do we tie them to the world?

- Use actual choices – revealed preferences approach

- Typical economists' approach. Compromise of:
  - realism
  - simplicity

- Assume a class of utility functions (CES, Cobb-Douglas...) with free parameters

- Estimate the parameters using the data

4 Labor Supply I

- Nicholson Ch. 16, pp. 477–484 [OLD: Ch. 22, pp. 606–613.]

- Labor supply decision: how much to work in a day.

- Goods: consumption good c, hours worked h

- Price of good p, hourly wage w

- Consumer spends $24 - h = l$ hours in units of leisure

- Utilify function: $u(c, l)$

- Budget constraint?

- Income of consumer: $M + wh = M + w(24 - l)$

- Budget constraint: $pc \leq M + w(24 - l)$ or

$$pc + wl \leq M + 24w$$

- Notice: leisure l is a consumption good with price w. Why?

- General category: **opportunity cost**

- Instead of enjoying one hour of TV, I could have worked one hour and gained wage w.

- You should value the marginal hour of TV w!

- Opportunity costs are very important!

- Example 2. CostCo has a warehouse in SoMa

- SoMa used to have low cost land, adequate for warehouses

- Price of land in SoMa triples in 10 years.

- Should firm relocate the warehouse?

- Did costs of staying in SoMa go up?

- No.

- Did the opportunity cost of staying in SoMa go up?

- Yes!

- Firm can sell at high price and purchase land in cheaper area.

- Let's go back to labor supply

- Maximization problem is

$$\max u(c, l)$$
$$s.t.\ pc + wl \leq M + 24w$$

- Standard problem (except for $24w$)

- First order conditions

- Assume utility function Cobb-Douglas:

$$u(c, l) = c^{\alpha} l^{1-\alpha}$$

- Solution is

$$
\begin{aligned}
c^* &= \alpha \frac{M + 24w}{p} \\
l^* &= (1-\alpha)\left(24 + \frac{M}{w}\right)
\end{aligned}
$$

- Both c and l are normal goods

- Unlike in standard Cobb-Douglas problems, c^* depends on price of other good w

- Why? Agents are endowed with M AND 24 hours of l in this economy

- Normally, agents are only endowed with M

5 Next Lectures

- More applications:
  - Intertemporal choice
  - Economics of Altruism