Economics 101A
(Lecture 14)

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Outline

1. Investment in Risky Asset II
2. Measures of Risk Aversion
3. Time Consistency
4. Time Inconsistency

1 Investment in Risk Asset II

- Individual has:
  - wealth w
  - utility function u, with $u' > 0$

- Two possible investments:
  - Asset B (bond) yields return 1 for each dollar
  - Asset S (stock) yields uncertain return $(1 + r)$:
    * $r = r_+ > 0$ with probability p
    * $r = r_- < 0$ with probability $1 - p$
    * $Er = pr_+ + (1 - p)\, r_- > 0$

- Share of wealth invested in stock S $= \alpha$

- Individual maximization:

$$\max_{\alpha} (1-p)\, u\left(w\left[(1-\alpha)+\alpha\left(1+r_{-}\right)\right]\right)+$$
$$+pu\left(w\left[(1-\alpha)+\alpha\left(1+r_{+}\right)\right]\right)$$
$$s.t. 0 \leq \alpha \leq 1$$

- Case of risk neutrality: $u(x) = a + bx$, $b > 0$

- Assume $a = 0$ (no loss of generality)

- Maximization becomes

$$\max_{\alpha} b\,(1-p)\left(w\left[1+\alpha r_{-}\right]\right)+bp\left(w\left[1+\alpha r_{+}\right]\right)$$

or

$$\max_{\alpha} bw+\alpha bw\left[(1-p)\, r_{-}+pr_{+}\right]$$

- Sign of term in square brackets? Positive!

- Set $\alpha^* = 1$

- Case of risk aversion: $u'' < 0$

- Assume $0 \leq \alpha^* \leq 1$, check later

- First order conditions:

$$\begin{aligned} 0 &= (1-p)(wr_-)\,u'(w[1+\alpha r_-]) + \\ &\quad +p(wr_+)\,u'(w[1+\alpha r_+]) \end{aligned}$$

- Can $\alpha^* = 0$ be solution?

- Solution is $\alpha^* > 0$ (positive investment in stock)

- Exercise: Check s.o.c.

2 Measures of Risk Aversion

- Nicholson, Ch. 18, pp. 541–545 [OLD: Ch. 8, pp. 207–210].

- How risk averse is an individual?

- Two measures:

  – Absolute Risk Aversion r_A:

$$r_A = -\frac{u''(x)}{u'(x)}$$

  – Relative Risk Aversion r_R:

$$r_R = -\frac{u''(x)}{u'(x)}x$$

- Examples in the Problem Set

3 Time consistency

- Intertemporal choice

- Three periods, $t = 0$, $t = 1$, and $t = 2$

- At each period i, agents:
  - have income $M_i' = M_i$+savings/debts from previous period
  - choose consumption c_i;
  - can save/borrow $M_i' - c_i$
  - no borrowing in last period: at $t = 2$ $M_2' = c_2$

- Utility function at $t = 0$

$$u(c_0, c_1, c_2) = U(c_0) + \frac{1}{1+\delta} EU(c_1) + \frac{1}{(1+\delta)^2} EU(c_2)$$

- Utility function at $t = 1$

$$u(c_1, c_2) = U(c_1) + \frac{1}{1+\delta} EU(c_2)$$

- Utility function at $t = 2$

$$u(c_2) = U(c_2)$$

- $U' > 0$, $U'' < 0$

- Question: Do preferences of agent in period 0 agree with preferences of agent in period 1?

- **Period 1**.

- Budget constraint at $t = 1$:

$$c_1 + \frac{1}{1+r}c_2 \leq M_1' + \frac{1}{1+r}M_2$$

- Maximization problem:

$$\max U(c_1) + \frac{1}{1+\delta}EU(c_2)$$
$$s.t.\ c_1 + \frac{1}{1+r}c_2 \leq M_1' + \frac{1}{1+r}M_2$$

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_1)}{EU'(c_2)} = \frac{1+r}{1+\delta}$$

- Back to **period 0**.

- Agent at time 0 can commit to consumption at time 1 as function of uncertain income M_1.

- Anticipated budget constraint at $t = 1$:

$$c_1 + \frac{1}{1+r}c_2 \leq M_1' + \frac{1}{1+r}M_2$$

- Maximization problem:

$$\max U(c_0) + \frac{1}{1+\delta}U(c_1) + \frac{1}{(1+\delta)^2}EU(c_2)$$
$$s.t.\ c_1 + \frac{1}{1+r}c_2 \leq M_1' + \frac{1}{1+r}M_2$$

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_1)}{EU'(c_2)} = \frac{1+r}{1+\delta}$$

- The two conditions coincide!

- **Time consistency.** Plans for future coincide with future actions.

- To see why, rewrite utility function $u(c_0, c_1, c_2)$:

$$
\begin{aligned}
& U(c_0) + \frac{1}{1+\delta} U(c_1) + \frac{1}{(1+\delta)^2} EU(c_2) \\
= \; & U(c_0) + \frac{1}{1+\delta} \left[U(c_1) + \frac{1}{1+\delta} EU(c_2) \right]
\end{aligned}
$$

- Expression in brackets coincides with utility at $t = 1$

- Is time consistency right?

  - addictive products (alcohol, drugs);

  - good actions (exercising, helping friends);

  - immediate gratification (shopping, credit card borrowing)

4 Time Inconsistency

- Alternative specification (Akerlof, 1991; Laibson, 1997; O'Donoghue and Rabin, 1999)

- Utility at time t is $u\left(c_t, c_{t+1}, c_{t+2}\right)$:

$$u(c_t) + \frac{\beta}{1+\delta} u(c_{t+1}) + \frac{\beta}{(1+\delta)^2} u(c_{t+2}) + ...$$

- Discount factor is

$$1, \frac{\beta}{1+\delta}, \frac{\beta}{(1+\delta)^2}, \frac{\beta}{(1+\delta)^3}, ...$$

  instead of

$$1, \frac{1}{1+\delta}, \frac{1}{(1+\delta)^2}, \frac{1}{(1+\delta)^3}, ...$$

- What is the difference?

- *Immediate gratification:* $\beta < 1$

- Back to our problem: **Period 1**.

- Maximization problem:

$$\max U(c_1) + \frac{\beta}{1+\delta} EU(c_2)$$
$$s.t.\ c_1 + \frac{1}{1+r} c_2 \leq M_1' + \frac{1}{1+r} M_2$$

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_1^*)}{EU'(c_2^*)} = \beta \frac{1+r}{1+\delta}$$

- Now, **period 0** with commitment.

- Maximization problem:

$$\max U(c_0) + \frac{\beta}{1+\delta} U(c_1) + \frac{\beta}{(1+\delta)^2} EU(c_2)$$
$$s.t.\ c_1 + \frac{1}{1+r} c_2 \leq M_1' + \frac{1}{1+r} M_2$$

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_1^{*,c})}{EU'(c_2^{*,c})} = \frac{1+r}{1+\delta}$$

- The two conditions differ!

- Time inconsistency: $c_1^{*,c} < c_1^*$ and $c_2^{*,c} > c_2^*$

- The agent allows him/herself too much immediate consumption and saves too little

- Ok, we agree. but should we study this as economists?

- YES!

  - One trillion dollars in credit card debt;

  - Most debt is in teaser rates;

  - Two thirds of Americans are overweight or obese;

  - $10bn health-club industry

- Is this testable?

  - In the laboratory?

  - In the field?

5 Next Lecture

- An Example: Health club Attendance

- Cost Minimization

- Solve an Example

- Cases in which s.o.c. are not satisfied

- Start Profit Maximization