Economics 101A
(Lecture 15)

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October 20, 2005

Outline

1. Time Inconsistency II
2. Health Club Attendance
3. Production: Introduction
4. Production Function
5. Returns to Scale
6. Two-step Cost Minimization

1 Time Inconsistency II

- Alternative specification (Akerlof, 1991; Laibson, 1997; O'Donoghue and Rabin, 1999)

- Utility at time t is $u\left(c_t, c_{t+1}, c_{t+2}\right)$:

$$u(c_t) + \frac{\beta}{1+\delta} u(c_{t+1}) + \frac{\beta}{(1+\delta)^2} u(c_{t+2}) + ...$$

- Discount factor is

$$1, \frac{\beta}{1+\delta}, \frac{\beta}{(1+\delta)^2}, \frac{\beta}{(1+\delta)^3}, ...$$

  instead of

$$1, \frac{1}{1+\delta}, \frac{1}{(1+\delta)^2}, \frac{1}{(1+\delta)^3}, ...$$

- What is the difference?

- *Immediate gratification:* $\beta < 1$

- Back to our problem: **Period 1**.

- Maximization problem:

$$\max U(c_1) + \frac{\beta}{1+\delta} EU(c_2)$$
$$s.t.\ c_1 + \frac{1}{1+r} c_2 \leq M_1' + \frac{1}{1+r} M_2$$

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_1^*)}{EU'(c_2^*)} = \beta \frac{1+r}{1+\delta}$$

- Now, **period 0** with commitment.

- Maximization problem:

$$\max U(c_0) + \frac{\beta}{1+\delta} U(c_1) + \frac{\beta}{(1+\delta)^2} EU(c_2)$$
$$s.t.\ c_1 + \frac{1}{1+r} c_2 \leq M_1' + \frac{1}{1+r} M_2$$

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_1^{*,c})}{EU'(c_2^{*,c})} = \frac{1+r}{1+\delta}$$

- The two conditions differ!

- Time inconsistency: $c_1^{*,c} < c_1^*$ and $c_2^{*,c} > c_2^*$

- The agent allows him/herself too much immediate consumption and saves too little

- Ok, we agree. but should we study this as economists?

- YES!
  - One trillion dollars in credit card debt;
  - Most debt is in teaser rates;
  - Two thirds of Americans are overwight or obese;
  - $10bn health-club industry

- Is this testable?
  - In the laboratory?
  - In the field?

2 Health Club Attendance

- Health club industry study (DellaVigna and Malmendier, 2002)

- 3 health clubs

- Data on attendance from swiping cards

- Choice of contracts:

  - Monthly contract with average price of \$75

  - 10-visit pass for \$100

- Consider users that choose monthly contract. Attendance?

- Attend on average 4.8 times per *month*

- Pay on average over $17

- Average delay of 2.2 months ($185) between last attendance and contract termination

- Over membership, user could have saved $700 by paying per visit

- Health club attendance:
  - immediate cost c
  - delayed benefit b

- At sign-up (attend tomorrow):

$$NB^t = -\frac{\beta}{1+\delta}c + \frac{\beta}{(1+\delta)^2}b$$

- Plan to attend if $NB^t > 0$

$$c < \frac{1}{(1+\delta)}b$$

- Once moment to attend comes:

$$NB = -c + \frac{\beta}{(1+\delta)}b$$

- Attend if $NB > 0$

$$c < \frac{\beta}{(1+\delta)}b$$

- Interpretations?

- Users are buying a commitment device

- User underestimate their future self-control problems:
  - They overestimate future attendance
  - They delay cancellation

3 Production: Introduction

- Second half of the economy. **Production**

- Example. Ford and the Minivan (Petrin, 2002):
  - Ford had idea: "Mini/Max" (early '70s)
  - Did Ford produce it?
  - No!
  - Ford was worried of cannibalizing station wagon sector
  - Chrysler introduces Dodge Caravan (1984)
  - Chrysler: $1.5bn profits (by 1987)!

- Why need separate treatment?

- Perhaps firms maximize utility...

- ...we can be more precise:
  - Competition
  - Institutional structure

4 Production Function

- Nicholson, Ch. 7, pp. 183–190; 195–200 [OLD: Ch. 11, pp. 268–275; 280–285]

- Production function: $y = f(\mathbf{z})$. Function $f : R_+^n \to R_+$

- Inputs $\mathbf{z} = (z_1, z_2, ..., z_n)$: labor, capital, land, human capital

- Output y: Minivan, Intel Pentium III, mangoes (Philippines)

- Properties of f:

  - no free lunches: $f(0) = 0$

  - positive marginal productivity: $f_i'(\mathbf{z}) > 0$

  - decreasing marginal productivity: $f_{i,i}''(\mathbf{z}) < 0$

- Isoquants $Q(y) = \{\mathbf{x} | f(\mathbf{x}) = y\}$

- Set of inputs $\mathbf{z}$ required to produce quantity y

- Special case. Two inputs:
  - $z_1 = L$ (labor)
  - $z_2 = K$ (capital)

- Isoquant: $f(L, K) - y = 0$

- Slope of isoquant $dK/dL = MRTS$

- Convex production function if convex isoquants

- Reasonable: combine two technologies and do better!

- Mathematically, $d^2K/d^2L =$

5 Returns to Scale

- Nicholson, Ch. 7, pp. 190–193 [OLD: Ch. 11, pp. 275–278]

- Effect of increase in labor: f'_L

- Increase of all inputs: $f(t\mathbf{z})$ with t scalar, $t > 1$

- How much does input increase?

  – Decreasing returns to scale: for all $\mathbf{z}$ and $t > 1$,

$$f(t\mathbf{z}) < tf(\mathbf{z})$$

  – Constant returns to scale: for all $\mathbf{z}$ and $t > 1$,

$$f(t\mathbf{z}) = tf(\mathbf{z})$$

– Increasing returns to scale: for all $\mathbf{z}$ and $t > 1$,

$$f\left(t\mathbf{z}\right) > tf\left(\mathbf{z}\right)$$

- Example: $y = f(K, L) = AK^{\alpha}L^{\beta}$

- Marginal product of labor: $f_L' =$

- Decreasing marginal product of labor: $f_L'' =$

- $MRTS =$

- Convex isoquant?

- Returns to scale: $f(tK, tL) = A(tK)^{\alpha}(tL)^{\beta} = t^{\alpha+\beta}AK^{\alpha}L^{\beta} = t^{\alpha+\beta}f(K, L)$

6 Two-step Cost minimization

- Nicholson, pp. 212–220 [OLD, Ch. 12 , pp. 298–307]

- Objective of firm: Produce output that generates maximal profit.

- Decompose problem in two:
  - Given production level y, choose cost-minimizing combinations of inputs
  - Choose optimal level of y.

- *First step.* Cost-Minimizing choice of inputs

- Two-input case: Labor, Capital

- Input prices:
  - Wage w is price of L
  - Interest rate r is rental price of capital K

- Expenditure on inputs: $wL + rK$

- Firm objective function:

$$\min wL + rK$$
$$s.t. f(L, K) \geq y$$

- Compare with expenditure minimization for consumers

- First order conditions:

$$w - \lambda f'_L = 0$$

and

$$r - \lambda f'_K = 0$$

- Rewrite as

$$\frac{f'_L\left(L^*, K^*\right)}{f'_K\left(L^*, K^*\right)} = \frac{w}{r}$$

- MRTS (slope of isoquant) equals ratio of input prices

- Graphical interpretation

- Derived demand for inputs:
  - $L = L^* (w, r, y)$
  - $K = K^* (w, r, y)$
- Value function at optimum is **cost function:**

$$c (w, r, y) = wL^* (r, w, y) + rK^* (r, w, y)$$

- *Second step.* Given cost function, choose optimal quantity of y as well

- Price of output is p.

- Firm's objective:

$$\max py - c(w, r, y)$$

- First order condition:

$$p - c'_y(w, r, y) = 0$$

- Price equals marginal cost – very important!

7 Next Lecture

- Continue Cost Minimization
- Solve an Example
- Cases in which s.o.c. are not satisfied
- Start Profit Maximization