Economics 101A
(Lecture 17)

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Outline

1. Cost Minimization: Example II
2. Cost Curves and Supply Function
3. One-step Profit Maximization
4. Introduction to Market Equilibrium

1 Cost Minimization: Example II

- Continue example above: $y = f(L, K) = AK^{\alpha}L^{\beta}$

- Define $B := w\left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{-\frac{\alpha}{\alpha+\beta}} + r\left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}$

- Cost-minimizing output choice:

$$\max py - B\left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}}$$

- First order condition:

$$p - \frac{1}{\alpha+\beta}\frac{B}{A}\left(\frac{y}{A}\right)^{\frac{1-(\alpha+\beta)}{\alpha+\beta}} = 0$$

- Second order condition:

$$-\frac{1}{\alpha+\beta}\frac{1-(\alpha+\beta)}{\alpha+\beta}\frac{B}{A^2}\left(\frac{y}{A}\right)^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}} < 0$$

- Solution:
  - $\alpha + \beta = 1$ (CRS):
    * S.o.c. equal to 0
    * Solution depends on p
    * For $p > \frac{1}{\alpha+\beta}\frac{B}{A}$, produce $y^* \to \infty$
    * For $p = \frac{1}{\alpha+\beta}\frac{B}{A}$, produce any $y^* \in [0, \infty)$
    * For $p < \frac{1}{\alpha+\beta}\frac{B}{A}$, produce $y^* = 0$

- $\alpha + \beta > 1$ (IRS):
  * S.o.c. positive
  * Solution of f.o.c. is a minimum!
  * Solution is $y^* \rightarrow \infty$.
  * Keep increasing production since higher production is associated iwth higher returns

- $\alpha + \beta < 1$ (DRS):
  * s.o.c. negative. OK!
  * Solution of f.o.c. is an interior optimum
  * This is the only "well-behaved" case under perfect competition
  * Here can define a supply function

2 Cost Curves

- Nicholson, Ch. 8, pp. 220–228; Ch. 9, pp. 256–259 [OLD: Ch. 12, pp. 307–312 and Ch. 13, pp. 342–346.]

- Marginal costs $MC = \partial c / \partial y \rightarrow$ Cost minimization

$$p = MC = \partial c\left(w, r, y\right) / \partial y$$

- Average costs $AC = c/y \rightarrow$ Does firm break even?

$$\begin{aligned} \pi &= py - c\left(w, r, y\right) > 0 \text{ iff} \\ \pi / y &= p - c\left(w, r, y\right) / y > 0 \text{ iff} \\ c\left(w, r, y\right) / y &= AC < p \end{aligned}$$

- **Supply function.** Portion of marginal cost MC above average costs.(price equals marginal cost)

- Assume only 1 input (expenditure minimization is trivial)

- **Case 1.** Production function. $y = L^{\alpha}$

  – Cost function? (cost of input is w):

  $$c(w, y) = wL^*(w, y) = wy^{1/\alpha}$$

  – Marginal cost?

  $$\frac{\partial c(w, y)}{\partial y} = \frac{1}{\alpha} wy^{(1-\alpha)/\alpha}$$

  – Average cost $c(w, y)/y$?

  $$\frac{c(w, y)}{y} = \frac{wy^{1/\alpha}}{y} = wy^{(1-\alpha)/\alpha}$$

- **Case 1a.** $\alpha > 1$. Plot production function, total cost, average and marginal. Supply function?

- **Case 1b.** $\alpha = 1$. Plot production function, total cost, average and marginal. Supply function?

- **Case 1c.** $\alpha < 1$. Plot production function, total cost, average and marginal. Supply function?

- **Case 2.** *Non-convex technology.* Plot production function, total cost, average and marginal. Supply function?

- **Case 3.** *Technology with setup cost.* Plot production function, total cost, average and marginal. Supply function?

2.1 Supply Function

- Supply function: $y^* = y^* (w, r, p)$

- What happens to y^* as p increases?

- Is the supply function upward sloping?

- Remember f.o.c:

$$p - c'_y (w, r, y) = 0$$

- Implicit function:

$$\frac{\partial y^*}{\partial p} = -\frac{1}{-c''_{y,y} (w, r, y)} > 0$$

as long as s.o.c. is satisfied.

- Yes! Supply function is upward sloping.

3 One-step Profit Maximization

- Nicholson, Ch. 9, pp. 265–270 [OLD: Ch. 13, pp. 346–350].

- One-step procedure: maximize profits

- Perfect competition. Price p is given
  - Firms are small relative to market
  - Firms do not affect market price p_M
  - Will firm produce at $p > p_M$?
  - Will firm produce at $p < p_M$?
  - $\Longrightarrow p = p_M$

- Revenue: $py = pf\left(L, K\right)$

- Cost: $wL + rK$

- Profit $pf\left(L, K\right) - wL - rK$

- Agent optimization:

$$\max_{L,K} pf(L,K) - wL - rK$$

- First order conditions:

$$pf'_L(L,K) - w = 0$$

and

$$pf'_K(L,K) - r = 0$$

- Second order conditions? $pf''_{L,L}(L,K) < 0$ and

$$\begin{aligned} |H| &= \begin{vmatrix} pf''_{L,L}(L,K) & pf''_{L,K}(L,K) \\ pf''_{L,K}(L,K) & pf''_{K,K}(L,K) \end{vmatrix} = \\ &= p^2 \left[f''_{L,L} f''_{K,K} - \left(f''_{L,K} \right)^2 \right] > 0 \end{aligned}$$

- Need $f''_{L,K}$ not too large for maximum

- Comparative statics with respect to to p, w, and r.

- What happens if w increases?

$$\frac{\partial L^*}{\partial w} = -\frac{\begin{vmatrix} -1 & pf''_{L,K}(L,K) \\ 0 & pf''_{K,K}(L,K) \end{vmatrix}}{\begin{vmatrix} pf''_{L,L}(L,K) & pf''_{L,K}(L,K) \\ pf''_{L,K}(L,K) & pf''_{K,K}(L,K) \end{vmatrix}} < 0$$

and

$$\frac{\partial L^*}{\partial r} =$$

- Sign of $\partial L^*/\partial r$ depends on $f''_{L,K}$.

4 Introduction to Market Equilibrium

- Nicholson, Ch. 10, pp. 279–295 [OLD: Ch. 14, pp. 368–382.

- Two ways to analyze firm behavior:
  - Two-Step Cost Minimization
  - One-Step Profit Maximization

- What did we learn?
  - Optimal demand for inputs L^*, K^* (see above)
  - Optimal quantity produced y^*

- **Supply function**. $y = y^*(p, w, r)$
  - From profit maximization:
  $$y = f\left(L^*(p, w, r), K^*(p, w, r)\right)$$
  - From cost minimization:
  $$MC \text{ curve above } AC$$
  - Supply function is increasing in p

- Market Equilibrium. Equate demand and supply.

- Aggregation?

- Industry supply function!

5 Next Lecture

- Aggregation
- Market Equilibrium
- Comparative Statics of Equilibrium
- Taxes and Subsidies
- Long-Run Equilibrium