

# Economics 101A

## (Lecture 10)

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## Outline

1. Complements and substitutes
2. Do utility functions exist?
3. Application 1: Labor Supply
4. Application 2: Intertemporal choice

# 1 Complements and substitutes

- Nicholson, Ch. 6, pp. 161–166 [OLD: 152–158].
- How about if price of another good changes?
- Generalize Slutsky equation

- Slutsky Equation:

$$\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_j} = \frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} - x_j^*(p_1, p_2, M) \frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}$$

- Substitution effect

$$\frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} > 0$$

for  $n = 2$  (two goods). Ambiguous for  $n > 2$ .

- Income effect:

$$-x_j^*(p_1, p_2, M) \frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}$$

- negative if good  $i$  is normal
- positive if good  $i$  is inferior

- How do we define complements and substitutes?

- Def. 1. Goods  $i$  and  $j$  are **gross substitutes** at price  $\mathbf{p}$  and income  $M$  if

$$\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_j} > 0$$

- Def. 2. Goods  $i$  and  $j$  are **gross complements** at price  $\mathbf{p}$  and income  $M$  if

$$\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_j} < 0$$

- Example 1 (ctd.):  $x_1^* = \alpha M/p_1$ ,  $x_2^* = \beta M/p_2$ .
- Gross complements or gross substitutes? Neither!
- Notice:  $\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_j}$  is usually different from  $\frac{\partial x_j^*(\mathbf{p}, M)}{\partial p_i}$

- Better definition.

- Def. 3. Goods  $i$  and  $j$  are **net substitutes** at price  $\mathbf{p}$  and income  $M$  if

$$\frac{\partial h_i^*(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} = \frac{\partial h_j^*(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} > 0$$

- Def. 4. Goods  $i$  and  $j$  are **net complements** at price  $\mathbf{p}$  and income  $M$  if

$$\frac{\partial h_i^*(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} = \frac{\partial h_j^*(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} < 0$$

- Example 1 (ctd.):  $h_1^* = \bar{u} \left( \frac{\alpha p_2}{1-\alpha p_1} \right)^{1-\alpha}$

- Net complements or net substitutes? Net substitutes!

## 2 Do utility functions exist?

- Preferences and utilities are theoretical objects
- Many different ways to write them
- How do we tie them to the world?
- Use actual choices – revealed preferences approach

- Typical economists' approach. Compromise of:
  - realism
  - simplicity
- Assume a class of utility functions (CES, Cobb-Douglas...) with free parameters
- Estimate the parameters using the data

### 3 Labor Supply

- Nicholson Ch. 16, pp. 477–484 [OLD: Ch. 22, pp. 606–613.]
- Labor supply decision: how much to work in a day.
- Goods: consumption good  $c$ , hours worked  $h$
- Price of good  $p$ , hourly wage  $w$
- Consumer spends  $24 - h = l$  hours in units of leisure
- Utility function:  $u(c, l)$

- Budget constraint?
- Income of consumer:  $M + wh = M + w(24 - l)$
- Budget constraint:  $pc \leq M + w(24 - l)$  or
 
$$pc + wl \leq M + 24w$$
- Notice: leisure  $l$  is a consumption good with price  $w$ . Why?
- General category: **opportunity cost**
- Instead of enjoying one hour of TV, I could have worked one hour and gained wage  $w$ .
- You should value the marginal hour of TV  $w$ !

- Opportunity costs are very important!
- Example 2. CostCo has a warehouse in SoMa
- SoMa used to have low cost land, adequate for warehouses
- Price of land in SoMa triples in 10 years.
- Should firm relocate the warehouse?

- Did costs of staying in SoMa go up?
  
- No.
  
- Did the opportunity cost of staying in SoMa go up?
  
  
- Yes!
  
  
- Firm can sell at high price and purchase land in cheaper area.

- Let's go back to labor supply

- Maximization problem is

$$\begin{aligned} \max u(c, l) \\ \text{s.t. } pc + wl \leq M + 24w \end{aligned}$$

- Standard problem (except for  $24w$ )

- First order conditions

- Assume utility function Cobb-Douglas:

$$u(c, l) = c^\alpha l^{1-\alpha}$$

- Solution is

$$c^* = \alpha \frac{M + 24w}{p}$$

$$l^* = (1 - \alpha) \left( 24 + \frac{M}{w} \right)$$

- Both  $c$  and  $l$  are normal goods
- Unlike in standard Cobb-Douglas problems,  $c^*$  depends on price of other good  $w$
- Why? Agents are endowed with  $M$  AND 24 hours of  $l$  in this economy
- Normally, agents are only endowed with  $M$

## 4 Intertemporal choice

- Nicholson Ch. 17, pp. 502–506 [OLD: Ch. 23, pp. 628–632.]
- So far, we assumed people live for one period only
- Now assume that people live for two periods:
  - $t = 0$  – people are young
  - $t = 1$  – people are old
- $t = 0$ : income  $M_0$ , consumption  $c_0$  at price  $p_0 = 1$
- $t = 1$ : income  $M_1 > M_0$ , consumption  $c_1$  at price  $p_1 = 1$

- Credit market available: can lend or borrow at interest rate  $r$

- Budget constraint in period 1?
- Sources of income:
  - $M_1$
  - $(M_0 - c_0) * (1 + r)$  (this can be negative)
- Budget constraint:

$$c_1 \leq M_1 + (M_0 - c_0) * (1 + r)$$

or

$$c_0 + \frac{1}{1 + r} c_1 \leq M_0 + \frac{1}{1 + r} M_1$$

- Utility function?

- Assume

$$u(c_0, c_1) = U(c_0) + \frac{1}{1 + \delta} U(c_1)$$

- $U' > 0, U'' < 0$

- $\delta$  is the discount rate

- Higher  $\delta$  means higher impatience

- Elicitation of  $\delta$  through hypothetical questions

- Person is indifferent between 1 hour of TV today and  $1 + \delta$  hours of TV next period

- Maximization problem:

$$\begin{aligned} \max U(c_0) + \frac{1}{1 + \delta} U(c_1) \\ s.t. c_0 + \frac{1}{1 + r} c_1 \leq M_0 + \frac{1}{1 + r} M_1 \end{aligned}$$

- Lagrangean

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_0)}{U'(c_1)} = \frac{1 + r}{1 + \delta}$$

- Case  $r = \delta$

- $c_0^* = c_1^*$ ?

- Substitute into budget constraint using  $c_0^* = c_1^* = c^*$ :

$$\frac{2+r}{1+r}c^* = \left[ M_0 + \frac{1}{1+r}M_1 \right]$$

or

$$c^* = \frac{1+r}{2+r}M_0 + \frac{1}{2+r}M_1$$

- We solved problem virtually without any assumption on  $U$ !

- Notice:  $M_0 < c^* < M_1$

- Case  $r > \delta$

- $c_0^* = c_1^*$ ?

- Comparative statics with respect to income  $M_0$

- Rewrite ratio of f.o.c.s as

$$U'(c_0) - \frac{1+r}{1+\delta}U'(c_1) = 0$$

- Substitute  $c_1$  in using  $c_1 = M_1 + (M_0 - c_0)(1+r)$  to get

$$U'(c_0) - \frac{1+r}{1+\delta}U'(M_1 + (M_0 - c_0)(1+r)) = 0$$

- Apply implicit function theorem:

$$\frac{\partial c_0^*(r, \mathbf{M})}{\partial M_0} = -\frac{-\frac{1+r}{1+\delta}U''(c_1)(1+r)}{U''(c_0) - \frac{1+r}{1+\delta}U''(c_1) * (-(1+r))}$$

- Denominator is always negative
- Numerator is positive
- $\partial c_0^*(r, \mathbf{M}) / \partial M_0 > 0$  — consumption at time 0 is a normal good.
- Can also show  $\partial c_0^*(r, \mathbf{M}) / \partial M_1 > 0$

- Comparative statics with respect to interest rate  $r$
- Apply implicit function theorem:

$$\frac{\partial c_0^*(r, \mathbf{M})}{\partial r} = \frac{-\frac{1}{1+\delta}U'(c_1)}{U''(c_0) - \frac{1+r}{1+\delta}U''(c_1) * (-(1+r))} \\ \frac{-\frac{1+r}{1+\delta}U''(c_1) * (M_0 - c_0)}{U''(c_0) - \frac{1+r}{1+\delta}U''(c_1) * (-(1+r))}$$

- Denominator is always negative
- Numerator: First term negative (substitution eff.)
- Numerator: Second term (income effect:)
  - positive if  $M_0 > c_0$
  - negative if  $M_0 < c_0$ .

# 5 Next Lectures

- More applications:
  - Finish Intertemporal choice
  - Economics of Altruism