Econ 101A
Midterm 2

Instructor: Stefano DellaVigna

November 10, 2009

Do not turn page unless instructed to.

Econ 101A – Midterm 2
Tu 10 November 2009.

You have approximately 1 hour and 20 minutes to answer the questions in the midterm. I will collect the exams at 11.00 sharp. Show your work, and good luck!

Problem 1. Cost Minimization. (40 points) Consider a company in a perfectly competitive industry with a familiar Cobb-Douglas production function, $y = f(L, K) = AL^{\alpha}K^{\beta}$. The company faces input costs $w > 0$ for labor and $r > 0$ for capital, and sells the final product for price $p > 0$.

1. Write down the one-step profit maximization. Remember, the company is choosing L and K to maximize profits, that is, revenue minus input costs. (5 points)
2. Derive the first-order conditions. (5 points)
3. Solve for $L^* = L^*(p, w, r, A)$ and $K^* = K^*(p, w, r, A)$. (10 points)
4. Comment on how L^* varies as each of the four parameter (p, w, r, A) varies. If the derivative is obvious, just point to the sign. Explain the intuition. (10 points)
5. Without solving for the second-order conditions, explain intuitively why the second order conditions are satisfied only if $\alpha + \beta < 1$. (10 points)

Problem 2. Extended Warranties (90 points + 10 extra) When purchasing electronics at the big-box store Okay Buy, consumers are given the opportunity to purchase a one-year warranty after they purchase any product. A one-year warranty is a full insurance policy against malfunction of the product: If it stops working within one year, the consumer gets a free replacement only if they purchased the warranty. The product is an MP3 player which sells at price \$100 and that has a probability of malfunction within one year of 1/10. Consumers have utility function $u(w_0) + v$, where $w_0 = w - 100$ is the wealth *after* purchasing the cell phone, at the time when she is faced with the decision of whether to buy a warranty, and $v > 0$ is the utility of owning a functioning player. Assume $u' > 0$ and $u'' < 0$.

Consumer really need to use this product for one whole year (that is, v is extremely large). If the MP3 player malfunctions and the consumer does not have a warranty, she will buy immediately a replacement which also costs \$100. We will assume that this replacement player will not fail.

1. If the consumer *does not* purchase the warranty, what is her expected utility? What if she *does* purchase the warranty (at price p)? (4 points)
2. Derive an expression for $\overline{p(w_0)}$, the maximum price that a consumer with wealth w_0 will be willing to pay for the warranty (Do not try to solve explicitly for $\overline{p(w_0)}$). (Hint: At the maximum price that the consumer is willing to pay, she will be indifferent between purchasing the insurance or not.) (5 points)
3. From this point on, assume that the utility function $u(w_0)$ is the exponential utility function $-\exp(-\rho w_0)$, with $\rho > 0$. Show that ρ is the absolute risk aversion coefficient. (3 points)
4. Plot $u(w_0) = -\exp(-\rho w_0)$ as a function of c. (Do not worry about the exact magnitudes, I am interested in the qualitative features of the function) Is it a problem that the utility $u(w_0)$ is always negative? Explain (8 points)
5. Show that $\overline{p(w_0)}$ implicitly defined in point 2 (and now assuming $u(w_0) = -\exp(-\rho w_0)$) does not depend on w_0 (If you get stuck here, move on to the next part of the problem). (10 points)
6. Given that $\bar{p}$ (the highest price that a consumer is willing to pay for a warranty) does not depend on w_0, derive for a representative consumer the demand function, that is, the demand for a warranty as a function of the price of the warranty p, that is, $D(p)$ (Remember that the demand D in this case can only be 0 or 1.) (5 points)

7. Assume now that there are N consumers of varying wealth levels who have purchased the MP3 player. Derive the aggregate demand curve for warranties $AD(p)$ analytically (that is, show the expression) and plot this on a graph with p on the vertical axis and quantity y on the horizontal axis. (10 points)

8. Now we will consider how Okay Buy chooses prices of its one-year warranty of MP3 players. Okay Buy acts as a monopolist in providing this warranty (If you buy your MP3 player there, you can't get your warranty elsewhere.). First, determine Okay Buy's expected cost function $E[C(y)]$ for providing y extended warranties. Assume that it costs c (with $c \leq 100$) to the company to replace one unit that fails. (5 points)

9. Write the profit maximization problem that will determine how Okay Buy sets p, the price of the warranty (Remember that $D(p)$ as the aggregate demand function). Solve for the profit-maximizing price p_M^* and quantity y_M^*. Do not use Lagrangeans, reason through the different cases for p. (Note: You can assume that consumers who are indifferent between buying or not buying the warranty will buy it.) (10 points)

10. Why do we know that profits will be positive? (5 points)

11. Relate this answer to whether Okay Buy offers a "fair" price for this warranty. Explain what the fair price is in this case (5 points)

12. (Extra credit, do the next two questions before) Now, derive the solution for the monopolist graphically. Plot the Demand curve $D(p)$ (you already did), the marginal revenue curve, and the marginal expected cost curve, and find p_M^* and y_M^* (10 points)

13. Now, the government comes in and introduces competition in the market for extended warranties. Okay Buy, like every other retailer, is forced to offer extended warranties by competitors. This introduces perfect competition in this market. Plot the aggregate marginal cost, which in this case is the derivative of the expected cost function $E[C(y)]$ derived above. Using the graph, derive the perfect competition price p_{PC}^* and quantity sold y_{PC}^* in the whole market. (10 points)

14. Compute the consumer surplus and producer surplus in the case of monopoly and in the case of perfect competition, and compare them. (10 points)