Econ 101A – Final exam
Mo 19 May, 2008.

Stefano apologizes for not being at the exam today. His reason is called Thomas.
From Stefano: Good luck to you all, you are a great class!

Do not turn the page until instructed to.

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Good luck on solving the problems!

Problem 1. Utility Maximization and Barter. (68 points) Consider consumers with preferences $u(x_1, x_2) = \min(x_1, x_2)$ over the two good x_1 and x_2.

1. Draw the indifference curves for this utility function and provide an example of goods that could approximately fit this pattern of preferences. (5 points)

2. Consider a standard utility maximization problem. Each consumer solves the problem

$$\max_{x_1, x_2} \min (x_1, x_2)$$
$$s.t.\ p_1 x_1 + p_2 x_2 \leq M$$

   Solve for $x_1^* (p_1, p_2, M)$ and $x_2^* (p_1, p_2, M)$. Provide the steps for the solution. [Hint: Do not go for straight for the derivatives, help yourself with the figure in point 1] (10 points)

3. Compute how the optimal solutions for $x_1^* (p_1, p_2, M)$ and $x_2^* (p_1, p_2, M)$ varies as a function of p_1 and p_2. Provide economic intuition. (5 points)

4. Now solve a problem similar to the one in point 2, except that each consumer is endowed not with income M, but with initial endowments of the two goods ω_1 and ω_2. Hence, each consumer solves

$$\max_{x_1, x_2} \min (x_1, x_2)$$
$$s.t.\ p_1 x_1 + p_2 x_2 \leq p_1 \omega_1 + p \omega_2$$

   Solve for $x_1^* (p_1, p_2, \omega_1, \omega_2)$ and $x_2^* (p_1, p_2, \omega_1, \omega_2)$ (10 points)

5. Compute how the optimal solutions for $x_1^* (p_1, p_2, \omega_1, \omega_2)$ and $x_2^* (p_1, p_2, \omega_1, \omega_2)$ varies as a function of p_1 and p_2. Compare to your answers in point 3. Provide economic intuition. (10 points)

6. Now assume that there are two identical consumers, consumers A and B, each with preferences $u(x_1, x_2) = \min(x_1, x_2)$ and initial endowments $\left(\omega_1^A, \omega_2^A\right)$ and $\left(\omega_1^B, \omega_2^B\right)$. Assume first $\left(\omega_1^A, \omega_2^A\right) = (1, 3)$ and $\left(\omega_1^B, \omega_2^B\right) = (3, 1)$. Draw the Edgeworth box for this economy and determine the Individually Rational area, that is, the area of points that each individual prefers to the initial allocation. (5 points)

7. Still on the Edgeworth box, and still assuming $\left(\omega_1^A, \omega_2^A\right) = (1, 3)$ and $\left(\omega_1^B, \omega_2^B\right) = (3, 1)$, determine the Pareto Set, that is, the set of allocations $\left(x_1^A, x_2^A\right), \left(x_1^B, x_2^B\right)$ that are feasible and such that there is no other allocation $\left(\hat{x}_1^A, \hat{x}_2^A\right), \left(\hat{x}_1^B, \hat{x}_2^B\right)$ that *both* consumer would prefer to $\left(x_1^A, x_2^A\right), \left(x_1^B, x_2^B\right)$ [Notice that this does not restrict the analysis to points in the Individually Rational Area] Argue your logic on this one. (8 points)

8. Using the answers in points 6 and 7, determine the Contract Curve, that is, the set of points that are possible outcomes of barter starting for $\left(\omega_1^A, \omega_2^A\right) = (1, 3)$ and $\left(\omega_1^B, \omega_2^B\right) = (3, 1)$. (5 points)

9. For general $\left(\omega_1^A, \omega_2^A\right)$ and $\left(\omega_1^B, \omega_2^B\right)$ but assuming $\omega_1^A + \omega_1^B = \omega_2^A + \omega_2^B$ (that is, a square Edgeworth box), determine mathematically the expression for the Contract Curve in the barter case for two consumers for preferences $u(x_1, x_2) = \min(x_1, x_2)$. (10 points)

Problem 2. Supply and Demand. (87 points) When Yao Ming entered the NBA, his parents moved to Houston with him and opened up a restaurant called Yao's. (The food there is quite good!) Unfortunately, they were educated before Deng Xioaping came to power, so they need your help navigating the market economy. The cost of producing X meals is given by $c(X) = f + \frac{c}{2}X^2$. Annual demand for meals is linear and is given by $Q^D(p) = a - bp$, where p is the price charged in the market.

1. Solve for Yao's average cost and marginal cost functions. Plot them, assuming (just for the plot) $c = 1$ and $f = 1$. (5 points)

2. Plot the *short-run* supply function $X^*(p)$ for the values above. Write then the supply function analytically for generic values of c and f. Be precise! (8 points)

3. Now assume that there are many identical restaurants (firms) in the perfectly competitive market, and there is free entry. What do you know about the shape of the long run supply curve? Assume that the entry of one firm does not affect the cost of the other firms. (5 points)

4. Solve for the number of meals that Yao's restaurant serves. [To solve for this, note that with free entry, marginal cost must equate average costs] (8 points)

5. What happens to the number of meals when fixed costs, f, increase? What if the marginal cost f increases? What is the economic significance of these results? (5 points)

6. Using your result from part 3, derive the *long-run* supply price. [Remember the condition price=marginal cost] What happens to the price when fixed costs increase? What happens to the price when the cost parameter c increases? Does this make sense? (8 points)

7. Given the demand above, how many restaurants would you expect to find in Houston? (Your answer should be in terms of the parameters, so don't worry that the number of restaurants may be fractional.) (5 points)

8. Suppose for a second that Yao had the only restaurant in town. Before doing any calculations, how will the monopoly price compare to the competitive price? How will the monopoly quantity compare to the competitive quantity? (5 points)

9. Ok, now go ahead and solve for the monopoly price .(10 points)

10. Consider now the case of *long-run perfect competition* and suppose Yao could make a one time research investment in order to substantially lower his costs forever. In particular, he could spend R in order to make $c = 0$. Assume, however, that other restaurants can observe this innovation and copy him immediately. Would he ever make the research investment? Explain [Hint: You should not need calculations to answer this] (8 points)

11. [This problem is a bit harder, so only work on it if you have time.] Now suppose that Yao could obtain a patent, which would allow him to operate as a monopolist for a certain number of years. How many years would this patent have to last for Yao to find the research worthwhile? Assume that there is no discounting and $a/2b < \sqrt{2cf}$ for the other restaurants. (20 points)

Problem 3. Arbitration. (50 points) The Democratic National Council (DNC) is having a hard time finding a compromise between the Hillary Clinton and Barack Obama camp. The two camps disagree on how many delegates should go to the Michigan and Florida primaries. These delegations originally were disqualified for not respecting a deadline, but the primary election still took place and a majority of voters favored Hillary Clinton. (Though in reality Barack Obama was not even on the ballot in one state) Ex post, The Hillary camp wants these delegates fully counted despite the initial bar she had agreed to. The Obama camp wants to not count these delegates. The DNC wants to arrive at a number n of the delegates of the two States (This is a simplification of the facts that gets the main points) We will assume for simplicity that all delegates will vote for Hillary Clinton, so the Obama camp is trying to minimize n, while the Clinton camp is trying to maximize n.

The DNC is trying to broker a bipartisan solution between the two sides and they decided to seek the advice of outside consultants on how to determine n.

1. The DNC decides to hire an unnamed political consultant, whom we'll call Karl Rove. Karl gives the following suggestion: 'Let's try to find common ground and a fair solution. Clinton proposes a fair deal $\hat{n}_C$, Obama proposes a fair deal $\hat{n}_O$, and then the DNC determines the number of delegates as the mid-point: $n = (\hat{n}_C + \hat{n}_O)/2$.' Let's analyze whether this rule will work. Assume that this is a dynamic game where the order is determined by seniority. Clinton chooses first $\hat{n}_C$ and then Obama observes $\hat{n}_C$ and chooses $\hat{n}_O(\hat{n}_C)$ as a function of $\hat{n}_C$. Both sides choose a number between 0 and N (the total number of delegates in the two states), which can be fractional for simplicity. Since this is a dynamic game, we use backward induction as solve for the optimal $\hat{n}_O(\hat{n}_C)$. Remember that Obama wants to minimize n, while Clinton wants to maximize n, subject in both cases to $0 \leq n \leq N$. What does Obama maximize and what is the solution for $\hat{n}_O(\hat{n}_C)$? (8 points)

2. Go back now to the first period and determine $\hat{n}_C$. Explain how Hillary Clinton solves the problem. What is in equilibrium the number of delegates n^* determined by this procedure? Do the offers $\hat{n}_C$ and $\hat{n}_O$ strike you as 'fair deal' offers? (5 points)

3. Another political consultant says; 'It is ridiculous that Clinton moves first, that giver her a first mover advantage. They should choose simultaneously'. Solve for the Nash equilibrium of the game in which the Clinton and Obama camp choose simultaneously, and comment on the consultant's advice. (8 points)

4. Hillary Clinton proposes a change to the procedure. She wants to keep the same dynamic procedure suggested in point 1, but she suggests that the two camps ought to be able to choose any number between 0 and $2N$. In any case, she argues, 'We will make fair offers'. Go through steps 1 and 2 with the new constraint and solve for the subgame perfect equilibrium strategies? Does the solution n^* strike you as fair? (5 points)

5. Finally, the DNC, tired of the political consultants, hires an economist to fix this mess. The economist makes the following suggestion: 'You should get an arbitrator and have the two parties submit a final-offer arbitration of the type used in many labor markets, such as to determine the police employment contracts. The Clinton camp submits n_C^O and the Obama camp submits n_O^O simultaneously to the arbitrator. The arbitrator is bound to choosing one of the two offers, and will choose the offer n_C^i that minimizes $\left|n_C^i - n_{Arb}\right|$, that is, the absolute distance between n_C^i and n_{Arb}, where n_{Arb} is the number that the arbitrator considers fair. That is, if n_O^O is closer to n_{Arb} that n_C^O, the number of delegates chosen by the arbitrator will be n_O^O. Viceversa, the number chosen will be n_C^O. If the two offers are equally close, the arbitrator will toss a coin' Assume first that n_{Arb} is known to the two parties. We now solve for *all* the Nash equilibria of this game in a similar manner to what we did for the Bertrand game. Are there equilibria in which $n_C^O \geq n_O^O > n_{Arb}$ or $n_O^O \geq n_C^O > n_{Arb}$? If so, find them; if not, find a profitable deviation for at least one party. (8 points)

6. Continuing the analysis, are there any equilibria with $n_{Arb} > n_O^O \geq n_C^O$ or $n_{Arb} > n_C^O \geq n_O^O$? Are there any equilibria with $n_O^O > n_{Arb} > n_C^O$ or $n_C^O > n_{Arb} > n_O^O$? (8 points)

7. Are there any equilibria with $n_O^O = n_{Arb} \neq n_C^O$ or $n_C^O = n_{Arb} \neq n_O^O$? Is there an equilibrium with $n_C^O = n_O^O = n_{Arb}$? Summarize all the equilibria of this game. (8 points)

Problem 4. Coordination Games (38 points) Consider the following (simultaneous) coordination game. Two individuals are lost in New York and are trying to meet for an important business meeting. There are only two possible meeting places, the Rockefeller Center and Penn Station. If the two managers manage to meet, they obtain a payoff of 8 each, while if they miscoordinate they get a payoff of 0. In addition, anyone that goes to Rockefeller Center can visit the Center, which yields a payoff of 2. The payoff matrix hence is

1\2	Rockefeller Center	Penn Station
Rockefeller Center	$10, 10$	$2, 0$
Penn Station	$0, 2$	$8, 8$

Call r the probability that player 1 goes to Rockefeller Center, $1 - r$ the probability that player 1 goes to Rockefeller Center, R the probability that player 2 goes to Rockefeller Center, $1 - R$ the probability that player 2 goes to Rockefeller Center.

1. Write the definition of Nash Equilibrium. (5 points)
2. Using this definition, compute all the pure-strategy equilibria of the game, that is all the equilibria where each player chooses between Rockefeller and Penn, and does not consider probability distributions. (5 points)
3. Compute all the mixed strategy of the game (15 points)
4. Of all the mixed strategy equilibria, are there some that you find more plausible than others? Here use your intuition. (5 points)
5. Now we introduce the concept of risk-dominant equilibrium. A strategy is a risk-dominant equilibrium if each player chooses the action that maximizes expected payoffs, assuming that the other player randomizes equally among all the available actions (in this case, each player assumes that the other player goes to Rockefeller with 50 percent probability and to Penn with 50 percent probability). Intuitively, this equilibrium allows for risk in the sense that each player thinks that the other player will play randomly. What is the unique risk-dominant equilibrium in this game? [Hint: The calculations in question 3 may help you here] In what sense this helps with your concerns in point 4? (8 points)