Economics 101A
(Lecture 11)

Stefano DellaVigna

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Outline

1. Application 3: Altruism and charitable donations II
2. Introduction to probability
3. Expected Utility
4. Insurance

1 Altruism and Charitable Donations II

- Wendy maximizes

$$\max_{c_M, D} u(c_W) + \alpha u\left(M_M + D\right)$$
$$s.t.\ c_W \leq M_W - D$$

- Rewrite as:

$$\max_{D} u(M_W - D) + \alpha u\left(M_M + D\right)$$

- First order condition:

$$-u'(M_W - D^*) + \alpha u'\left(M_M + D^*\right) = 0$$

- Second order conditions:

$$u''(M_W - D^*) + \alpha u''\left(M_M + D^*\right) < 0$$

- Assume $\alpha = 1$.
  - Solution?
  - $u'(M_W - D) = u'(M_M + D^*)$
  - $M_W - D^* = M_M + D^*$ or $D^* = (M_W - M_M)/2$
  - Transfer money so as to equate incomes!
  - Careful: $D < 0$ (negative donation!) if $M_M > M_W$

- Corrected maximization:

$$\max_D u(M_W - D) + \alpha u(M_M + D)$$
$$s.t. D \geq 0$$

- Solution ($\alpha = 1$):

$$D^* = \begin{cases} (M_W - M_M)/2 & \text{if } M_W - M_M > 0 \\ 0 & \text{otherwise} \end{cases}$$

- Assume interior solution. ($D^* > 0$)

- Comparative statics 1 (altruism):

$$\frac{\partial D^*}{\partial \alpha} = -\frac{u'(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0$$

- Comparative statics 2 (income of donor):

$$\frac{\partial D^*}{\partial M_W} = -\frac{-u''(M_W + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0$$

- Comparative statics 3 (income of recipient):

$$\frac{\partial D^*}{\partial M_M} = -\frac{\alpha u''(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} < 0$$

- A quick look at the evidence

- From Andreoni (2002)

2 Introduction to Probability

- So far deterministic world:
  - income given, known M
  - interest rate known r
- But some variables are unknown at time of decision:
  - future income M_1?
  - future interest rate r_1?
- Generalize framework to allow for uncertainty
  - Events that are truly unpredictable (weather)
  - Event that are very hard to predict (future income)

- Probability is the language of uncertainty

- Example:

  - Income M_1 at $t = 1$ depends on state of the economy

  - Recession ($M_1 = 20$), Slow growth ($M_2 = 25$), Boom ($M_3 = 30$)

  - Three probabilities: p_1, p_2, p_3

  - $p_1 = P(M_1) = P(\text{recession})$

- Properties:

  - $0 \leq p_i \leq 1$

  - $p_1 + p_2 + p_3 = 1$

- Mean income: $EM = \sum_{i=1}^{3} p_i M_i$

- If $(p_1, p_2, p_3) = (1/3, 1/3, 1/3)$,

$$EM = \frac{1}{3}20 + \frac{1}{3}25 + \frac{1}{3}30 = \frac{75}{3} = 25$$

- Variance of income: $V(M) = \sum_{i=1}^{3} p_i (M_i - EM)^2$

- If $(p_1, p_2, p_3) = (1/3, 1/3, 1/3)$,

$$\begin{aligned} V(M) &= \frac{1}{3}(20-25)^2 + \frac{1}{3}(25-25)^2 + \frac{1}{3}(30-25)^2 \\ &= \frac{1}{3}5^2 + \frac{1}{3}5^2 = 2/3 * 25 \end{aligned}$$

- Mean and variance if $(p_1, p_2, p_3) = (1/4, 1/2, 1/4)$?

3 Expected Utility

- Nicholson, Ch. 7, pp. 202-209 (Ch. 18, pp. 533–541, 9th)

- Consumer at time 0 asks: what is utility in time 1?

- At $t = 1$ consumer maximizes

$$\max U(c^1)$$
$$s.t.\ c_i^1 \leq M_i^1 + (1 + r)(M^0 - c^0)$$

  with $i = 1, 2, 3$.

- What is utility at optimum at $t = 1$ if $U' > 0$?

- Assume for now $M^0 - c^0 = 0$

- Utility $U\left(M_i^1\right)$

- This is uncertain, depends on which i is realized!

- How do we evaluate future uncertain utility?

- **Expected utility**

$$EU = \sum_{i=1}^{3} p_i U\left(M_i^1\right)$$

- In example:

$$EU = 1/3U(20) + 1/3U(25) + 1/3U(30)$$

- Compare with $U(EC) = U(25)$.

- Agents prefer riskless outcome EM to uncertain outcome M if

$$\begin{aligned} 1/3U(20) + 1/3U(25) + 1/3U(30) &< U(25) \text{ or} \\ 1/3U(20) + 1/3U(30) &< 2/3U(25) \text{ or} \\ 1/2U(20) + 1/2U(30) &< U(25) \end{aligned}$$

- Picture

- Depends on sign of U'', on concavity/convexity

- Three cases:

  - $U''(x) = 0$ for all x. (linearity of U)

    * $U(x) = a + bx$

    * $1/2U(20) + 1/2U(30) = U(25)$

  - $U''(x) < 0$ for all x. (concavity of U)

    * $1/2U(20) + 1/2U(30) < U(25)$

  - $U''(x) > 0$ for all x. (convexity of U)

    * $1/2U(20) + 1/2U(30) > U(25)$

- If $U''(x) = 0$ (linearity), consumer is indifferent to uncertainty

- If $U''(x) < 0$ (concavity), consumer dislikes uncertainty

- If $U''(x) > 0$ (convexity), consumer likes uncertainty

- Do consumers like uncertainty?

- Do *you* like uncertainty?

- **Theorem. (Jensen's inequality)** If a function $f(x)$ is concave, the following inequality holds:

$$f(Ex) \geq Ef(x)$$

where E indicates expectation. If f is strictly concave, we obtain

$$f(Ex) > Ef(x)$$

- Apply to utility function U.

- Individuals dislike uncertainty:

$$U(Ex) \geq EU(x)$$

- Jensen's inequality then implies U concave ($U'' \leq 0$)

- Relate to diminishing marginal utility of income

4 Insurance

- Nicholson, Ch. 7, pp. 216–221 (18, pp. 545–551, 9th) Notice: different treatment than in class

- Individual has:
  - wealth w
  - utility function u, with $u' > 0$, $u'' < 0$

- Probability p of accident with loss L

- Insurance offers coverage:
  - premium $\$q$ for each $\$1$ paid in case of accident
  - units of coverage purchased α

- Individual maximization:

$$\max_{\alpha} (1-p)\, u\, (w - q\alpha) + pu\, (w - q\alpha - L + \alpha)$$
$$s.t. \alpha \geq 0$$

- Assume $\alpha^* \geq 0$, check later

- First order conditions:

$$\begin{aligned} 0 &= -q\,(1-p)\,u'\,(w - q\alpha) \\ &\quad + (1-q)\,pu'\,(w - q\alpha - L + \alpha) \end{aligned}$$

or

$$\frac{u'\,(w - q\alpha)}{u'\,(w - q\alpha - L + \alpha)} = \frac{1-q}{q}\frac{p}{1-p}.$$

- Assume first $q = p$ (insurance is fair)

- Solution for $\alpha^* =$?

- $\alpha^* > 0$, so we are ok!

- What if $q > p$ (insurance needs to cover operating costs)?

- Insurance will be only partial (if at all): $\alpha^* < L$

- Exercise: Check second order conditions!

5 Next Lectures

- Risk aversion

- Applications:

  - Portfolio choice

  - Consumption choice II