Economics 101A
(Lecture 12)

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Outline

1. Insurance II

2. Investment in Risky Asset

3. Risk Aversion and Lottery

4. Measures of Risk Aversion

5. Mid-Term Feedback

1 Insurance II

- Individual maximization:

$$\max_{\alpha} \left(1 - p\right) u \left(w - q\alpha\right) + pu \left(w - q\alpha - L + \alpha\right)$$
$$s.t. \alpha \geq 0$$

- Assume $\alpha^* \geq 0$, check later

- First order conditions:

$$
\begin{aligned}
0 = \; & -q \left(1 - p\right) u' \left(w - q\alpha\right) \\
& + \left(1 - q\right) pu' \left(w - q\alpha - L + \alpha\right)
\end{aligned}
$$

or

$$\frac{u' \left(w - q\alpha\right)}{u' \left(w - q\alpha - L + \alpha\right)} = \frac{1 - q}{q} \frac{p}{1 - p}.$$

- Assume first $q = p$ (insurance is fair)

- Solution for $\alpha^* = ?$

- $\alpha^* > 0$, so we are ok!

- What if $q > p$ (insurance needs to cover operating costs)?

- Insurance will be only partial (if at all): $\alpha^* < L$

- Exercise: Check second order conditions!

2   Investment in Risk Asset

- Individual has:

  - wealth w

  - utility function u, with $u' > 0$

- Two possible investments:

  - Asset B (bond) yields return 1 for each dollar

  - Asset S (stock) yields uncertain return $(1 + r)$:

    * $r = r_+ > 0$ with probability p

    * $r = r_- < 0$ with probability $1 - p$

    * $Er = pr_+ + (1 - p)\, r_- > 0$

- Share of wealth invested in stock S $= \alpha$

- Individual maximization:

$$\max_{\alpha} (1-p)\, u\left(w\left[(1-\alpha) + \alpha\left(1+r_-\right)\right]\right) +$$
$$+pu\left(w\left[(1-\alpha) + \alpha\left(1+r_+\right)\right]\right)$$
$$s.t. 0 \leq \alpha \leq 1$$

- Case of risk neutrality: $u(x) = a + bx$, $b > 0$

- Assume $a = 0$ (no loss of generality)

- Maximization becomes

$$\max_{\alpha} b\left(1-p\right)\left(w\left[1 + \alpha r_-\right]\right) + bp\left(w\left[1 + \alpha r_+\right]\right)$$

or

$$\max_{\alpha} bw + \alpha bw\left[(1-p)\, r_- + pr_+\right]$$

- Sign of term in square brackets? Positive!

- Set $\alpha^* = 1$

- Case of risk aversion: $u'' < 0$

- Assume $0 \leq \alpha^* \leq 1$, check later

- First order conditions:

$$
\begin{aligned}
0 \; = \; & (1-p)\,(wr_-)\,u'\,(w\,[1+\alpha r_-]) + \\
& + p\,(wr_+)\,u'\,(w\,[1+\alpha r_+])
\end{aligned}
$$

- Can $\alpha^* = 0$ be solution?

- Solution is $\alpha^* > 0$ (positive investment in stock)

- Exercise: Check s.o.c.

3   Risk aversion and Lottery

- Risk aversion:

  - individuals dislike uncertainty

  - u concave, $u'' < 0$

- Implications?

  - purchase of insurance (possible accident)

  - investment in risky asset (risky investment)

  - choice over time (future income uncertain)

- Experiment — Are you risk-averse?

4 Measures of Risk Aversion

- Nicholson, Ch. 7, pp. 209-213 (Ch. 18, pp. 541–545, 9th)

- How risk averse is an individual?

- Two measures:

  - Absolute Risk Aversion r_A:

  $$r_A = -\frac{u''(x)}{u'(x)}$$

  - Relative Risk Aversion r_R:

  $$r_R = -\frac{u''(x)}{u'(x)}x$$

- Examples in the Problem Set

5 Mid-Term Feedback

- Thanks for the feedback!

6 Next lecture and beyond

- Time Inconsistency

- Production Function