

Economics 101A

(Lecture 10)

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Outline

1. Do utility functions exist?
2. Application 1: Labor Supply
3. Application 2: Intertemporal choice

1 Do utility functions exist?

- Preferences and utilities are theoretical objects
- Many different ways to write them
- How do we tie them to the world?
- Use actual choices – revealed preferences approach

- Typical economists' approach. Compromise of:
 - realism
 - simplicity
- Assume a class of utility functions (CES, Cobb-Douglas...) with free parameters
- Estimate the parameters using the data

2 Labor Supply

- Nicholson Ch. 16, pp. 573-581 (477–484, 9th)
- Labor supply decision: how much to work in a day.
- Goods: consumption good c , hours worked h
- Price of good p , hourly wage w
- Consumer spends $24 - h = l$ hours in units of leisure
- Utility function: $u(c, l)$

- Budget constraint?
- Income of consumer: $M + wh = M + w(24 - l)$
- Budget constraint: $pc \leq M + w(24 - l)$ or

$$pc + wl \leq M + 24w$$
- Notice: leisure l is a consumption good with price w . Why?
- General category: **opportunity cost**
- Instead of enjoying one hour of TV, I could have worked one hour and gained wage w .
- You should value the marginal hour of TV w !

- Opportunity costs are very important!
- Example 2. CostCo has a warehouse in SoMa
- SoMa used to have low cost land, adequate for warehouses
- Price of land in SoMa triples in 10 years.
- Should firm relocate the warehouse?

- Did costs of staying in SoMa go up?

- No.

- Did the opportunity cost of staying in SoMa go up?

- Yes!

- Firm can sell at high price and purchase land in cheaper area.

- Let's go back to labor supply

- Maximization problem is

$$\begin{aligned} \max u(c, l) \\ \text{s.t. } pc + wl \leq M + 24w \end{aligned}$$

- Standard problem (except for $24w$)

- First order conditions

- Assume utility function Cobb-Douglas:

$$u(c, l) = c^\alpha l^{1-\alpha}$$

- Solution is

$$c^* = \alpha \frac{M + 24w}{p}$$

$$l^* = (1 - \alpha) \left(24 + \frac{M}{w} \right)$$

- Both c and l are normal goods
- Unlike in standard Cobb-Douglas problems, c^* depends on price of other good w
- Why? Agents are endowed with M AND 24 hours of l in this economy
- Normally, agents are only endowed with M

3 Intertemporal choice

- Nicholson Ch. 17, pp. 597-601 (502–506, 9th)
- So far, we assumed people live for one period only
- Now assume that people live for two periods:
 - $t = 0$ – people are young
 - $t = 1$ – people are old
- $t = 0$: income M_0 , consumption c_0 at price $p_0 = 1$
- $t = 1$: income $M_1 > M_0$, consumption c_1 at price $p_1 = 1$
- Credit market available: can lend or borrow at interest rate r

- Budget constraint in period 1?
- Sources of income:
 - M_1
 - $(M_0 - c_0) * (1 + r)$ (this can be negative)
- Budget constraint:

$$c_1 \leq M_1 + (M_0 - c_0) * (1 + r)$$

or

$$c_0 + \frac{1}{1 + r} c_1 \leq M_0 + \frac{1}{1 + r} M_1$$

- Utility function?

- Assume

$$u(c_0, c_1) = U(c_0) + \frac{1}{1 + \delta} U(c_1)$$

- $U' > 0, U'' < 0$

- δ is the discount rate

- Higher δ means higher impatience

- Elicitation of δ through hypothetical questions

- Person is indifferent between 1 hour of TV today and $1 + \delta$ hours of TV next period

- Maximization problem:

$$\begin{aligned} \max U(c_0) + \frac{1}{1 + \delta} U(c_1) \\ \text{s.t. } c_0 + \frac{1}{1 + r} c_1 \leq M_0 + \frac{1}{1 + r} M_1 \end{aligned}$$

- Lagrangean

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_0)}{U'(c_1)} = \frac{1 + r}{1 + \delta}$$

- Case $r = \delta$

- $c_0^* = c_1^*$?

- Substitute into budget constraint using $c_0^* = c_1^* = c^*$:

$$\frac{2+r}{1+r}c^* = \left[M_0 + \frac{1}{1+r}M_1 \right]$$

or

$$c^* = \frac{1+r}{2+r}M_0 + \frac{1}{2+r}M_1$$

- We solved problem virtually without any assumption on U !

- Notice: $M_0 < c^* < M_1$

- Case $r > \delta$

- $c_0^* = c_1^*$?

- Comparative statics with respect to income M_0

- Rewrite ratio of f.o.c.s as

$$U'(c_0) - \frac{1+r}{1+\delta}U'(c_1) = 0$$

- Substitute c_1 in using $c_1 = M_1 + (M_0 - c_0)(1+r)$ to get

$$U'(c_0) - \frac{1+r}{1+\delta}U'(M_1 + (M_0 - c_0)(1+r)) = 0$$

- Apply implicit function theorem:

$$\frac{\partial c_0^*(r, \mathbf{M})}{\partial M_0} = -\frac{-\frac{1+r}{1+\delta}U''(c_1)(1+r)}{U''(c_0) - \frac{1+r}{1+\delta}U''(c_1) * (-(1+r))}$$

- Denominator is always negative
- Numerator is positive
- $\partial c_0^*(r, \mathbf{M}) / \partial M_0 > 0$ — consumption at time 0 is a normal good.
- Can also show $\partial c_0^*(r, \mathbf{M}) / \partial M_1 > 0$

- Comparative statics with respect to interest rate r
- Apply implicit function theorem:

$$\frac{\partial c_0^*(r, \mathbf{M})}{\partial r} = \frac{-\frac{1}{1+\delta}U'(c_1)}{U''(c_0) - \frac{1+r}{1+\delta}U''(c_1) * (-(1+r))} \\ \frac{-\frac{1+r}{1+\delta}U''(c_1) * (M_0 - c_0)}{U''(c_0) - \frac{1+r}{1+\delta}U''(c_1) * (-(1+r))}$$

- Denominator is always negative
- Numerator: First term negative (substitution eff.)
- Numerator: Second term (income effect:)
 - positive if $M_0 > c_0$
 - negative if $M_0 < c_0$

4 Next Lectures

- Application 3: Altruism and Charitable Donations
- Introduction to Probability
- Risk Aversion
- Coefficient of risk aversion