Economics 101A
(Lecture 8)

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Outline

1. Indirect Utility Function II
2. Comparative Statics (Introduction)
3. Income Changes
4. Price Changes
5. Expenditure Minimization

1 Indirect utility function

- Nicholson, Ch. 4, pp. 124-127 (106–108, 9th)

- Define the indirect utility $v(\mathbf{p}, M) \equiv u(\mathbf{x}^*(\mathbf{p}, M))$, with $\mathbf{p}$ vector of prices and $\mathbf{x}^*$ vector of optimal solutions.

- $v(\mathbf{p}, M)$ is the utility at the optimimum for prices $\mathbf{p}$ and income M

- Some comparative statics: $\partial v(\mathbf{p}, M)/\partial M =$?

- Hint: Use Envelope Theorem on Lagrangean function

- What is the sign of λ?

- $\lambda = u'_{x_i}/p > 0$

- $\partial v(\mathbf{p}, M)/\partial p_i =$?

- Properties:

  – Indirect utility is always increasing in income M

  – Indirect utility is always decreasing in the price p_i

2 Comparative Statics (introduction)

- Nicholson, Ch. 5, pp. 141-151 (121–131, 9th)

- Utility maximization yields $x_i^* = x_i^*(p_1, p_2, M)$

- Quantity consumed as a function of income and price

- What happens to quantity consumed x_i^* as prices or income varies?

- Simple case: Equal increase in prices and income.

- $M' = tM$, $p_1' = tp_1$, $p_2' = tp_2$.

- Compare $x^*(tM, tp_1, tp_2)$ and $x^*(M, p_1, p_2)$.

- What happens?

- Write budget line: $tp_1x_1 + tp_2x_2 = tM$

- Demand is homogeneous of degree 0 in $\mathbf{p}$ and M:

  $x^*(tM, tp_1, tp_2) = t^0 x^*(M, p_1, p_2) = x^*(M, p_1, p_2)$.

- Consider Cobb-Douglas Case:

$$x_1^* = \frac{\alpha}{\alpha + \beta} M/p_1, x_2^* = \frac{\beta}{\alpha + \beta} M/p_2$$

- What is $\partial x_1^* / \partial M$?

- What is $\partial x_1^* / \partial p_1$?

- What is $\partial x_1^* / \partial p_2$?

- General results?

3 Income changes

- Income increases from M to to $M' > M$.

- Budget line ($p_1x_1 + p_2x_2 = M$) shifts out:
$$x_2 = \frac{M'}{p_2} - x_1\frac{p_1}{p_2}$$

- New optimum?

- Engel curve: $x_i^*(M)$: demand for good i as function of income M holding fixed prices p_1, p_2

- Does x_i^* increase with M?
  - Yes. Good i is *normal*
  - No. Good i is *inferior*

4 Price changes

- Price of good i increases from p_i to to $p'_i > p_i$

- For example, decrease in price of good 2, $p'_2 < p_2$

- Budget line tilts:

$$x_2 = \frac{M}{p'_2} - x_1 \frac{p_1}{p'_2}$$

- New optimum?

- Demand curve: $x_i^*(p_i)$: demand for good i as function of own price holding fixed p_j and M

- Odd convention of economists: plot price p_i on vertical axis and quantity x_i on horizontal axis. Better get used to it!

- Does x_i^* decrease with p_i?
  - Yes. Most cases
  - No. Good i is *Giffen*
  - Ex.: Potatoes in Ireland
  - Do not confuse with Veblen effect for luxury goods or informational asimmetries: these effects are real, but not included in current model

5 Expenditure minimization

- Nicholson, Ch. 4, pp. 127-132 (109–113, 9th)

- Solve problem **EMIN** (minimize expenditure):

$$\begin{aligned} &\min p_1 x_1 + p_2 x_2 \\ &s.t.\ u(x_1, x_2) \geq \bar{u} \end{aligned}$$

- Choose bundle that attains utility $\bar{u}$ with minimal expenditure

- Ex.: You are choosing combination CDs/restaurant to make a friend happy

- If utility u strictly increasing in x_i, can maximize s.t. equality

- Denote by $h_i(p_1, p_2, \bar{u})$ solution to EMIN problem

- $h_i(p_1, p_2, \bar{u})$ is *Hicksian or compensated demand*

- Graphically:
  - Fix indifference curve at level $\bar{u}$
  - Consider budget sets with different M
  - Pick budget set which is tangent to indifference curve

- Optimum coincides with optimum of Utility Maximization!

- Formally:

$$h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u}))$$

- Expenditure function is expenditure at optimum

- $e(p_1, p_2, \bar{u}) = p_1 h_1(p_1, p_2, \bar{u}) + p_2 h_2\,(p_1, p_2, \bar{u})$

- $h_i(p_i)$ is *Hicksian or compensated demand* function

- Is h_i always decreasing in p_i? Yes!

- Graphical proof: moving along a convex indifference curve

- (For non-convex indifferent curves, still true)

- Using first order conditions:

$$L(x_1, x_2, \lambda) = p_1 x_1 + p_2 x_2 - \lambda \left(u(x_1, x_2) - \bar{u} \right)$$

$$\frac{\partial L}{\partial x_i} = p_i - \lambda u_i'(x_1, x_2) = 0$$

- Write as ratios:

$$\frac{u_1'(x_1, x_2)}{u_2'(x_1, x_2)} = \frac{p_1}{p_2}$$

- $MRS =$ ratio of prices as in utility maximization!

- However: different constraint $\Longrightarrow \lambda$ is different

- Example 1: Cobb-Douglas utility

$$\min p_1 x_1 + p_2 x_2$$
$$s.t.\ x_1^{\alpha} x_2^{1-\alpha} \geq \bar{u}$$

- Lagrangean $=$

- F.o.c.:

- Solution: $h_1^* =$ $\qquad\qquad\qquad\qquad$ $, h_2^* =$

- $\partial h_i^* / \partial p_i < 0$, $\partial h_i^* / \partial p_j > 0$, $j \neq i$

6 Next Lectures

- Slutsky Equation

- Complements and Substitutes

- Then moving on to applications:
  - Labor Supply
  - Intertemporal choice
  - Economics of Altruism