Econ 101A – Midterm 1
Tu 30 September 2003.

You have approximately 1 hour and 20 minutes to answer the questions in the midterm. I will collect the exams at 12.30 sharp. Show your work, and good luck!

Problem 1. Utility maximization. (52 points) In this exercise, we consider a standard maximization problem with an unusual utility function. The utility function is

$$u(x, y) = \sqrt{x} + \sqrt{y}.$$

The price of good x is p_x and the price of good y is p_y. We denote income by M, as usual, with $M > 0$. This function is well-defined for $x > 0$ and for $y > 0$. From now on, assume $x > 0$ and $y > 0$ unless otherwise stated.

1. Compute $\partial u/\partial x$ and $\partial^2 u/\partial^2 x$. Is the utility function increasing in x? Is the utility function concave in x? (3 points)
2. The consumer maximizes utility subject to a budget constraint. Write down the maximization problem of the consumer with respect to x and y. Explain *briefly* why the budget constraint is satisfied with equality. (Hint: you can use the answer in point 1) (5 points)
3. Write down the Lagrangean function. (2 points)
4. Write down the first order conditions for this problem with respect to x, y, and λ. (4 points)
5. Solve explicitely for x^* and y^* as a function of p_x, p_y, and M. (8 points)
6. Do the solutions for x^* and y^* satisfy the positivity constraint, that is, $x^* > 0$ and $y^* > 0$?.(2 points)
7. Are these points maxima of the problem above? Check that the determinant of the bordered Hessian is positive at x^*, y^*, and λ^*. (8 points)
8. Use the expression for x^* that you obtained in point 6. Differentiate it with respect to M, that is, compute $\partial x^*/\partial M$. Is this good a normal good? (4 points)
9. We are now interested in the sign of $\partial x^*/\partial p_x$. That is, we would like to know if the demand function is downward sloping. (at higher prices, a lower quantity of the good is demanded). Argue, using the answer that you gave in point 9 and something you learnt in class, that we know the sign of $\partial x^*/\partial p_x$. What is this sign? (8 points)
10. Can you guess the solutions for x^* and y^* for the following maximization problem? Develop your argument. (8 point)

$$\max \left(\sqrt{x} + \sqrt{y}\right)^2$$
$$s.t. p_x x + p_y y \leq M$$

Short problems. (26 points) In this part, you are required to solve the problems below. Provide the steps in the derivation of your answer.

Short problem 1. (16 points) Consider the utility function

$$U(x) = \begin{cases} x & \text{if} \quad 0 \leq x \leq 10 \\ (20 - x) & \text{if} \quad 10 < x \leq 20 \end{cases}$$

defined for $0 \leq x \leq 20$.

1. Plot the utility function as a function of x. (1 point)
2. Is $U(10) > U(5)$? Is $U(20) > U(5)$? Are the preferences represented by this utility function monotonic (that is, if $y \geq x$, then $y \succeq x$)? (5 points)
3. (Harder) Write down the set of preferences over the numbers between 0 and 20 that this utility function represents. Try to be precise, but you can help yourself using words. [Hint: for each x, $y \in [0, 20]$, when is $x \succeq y$?] (10 points)

Short problem 2. (10 points)

1. Consider the implicit function $g(x, y) = y - 1 - \ln(x * y) = 0$ for $x > 0$, $y > 0$.
   (a) Use the implicit function theorem to write down $\partial y / \partial x$.(6 points)
   (b) Is this derivative well-defined for $(x = 1, y = 1)$? (4 points)