Econ 101A – Midterm 2
Th 27 October 2004.

You have approximately 1 hour and 20 minutes to answer the questions in the midterm. We will collect the exams at 12.30 sharp. Show your work, and good luck!

Two plants different costs
Accounting with cost function
Fast-food Stands with license

Problem 1. Uncertainty (21 points). Consider the case of transportation accidents.

1. Mary is worried about car accidents next year. At the beginning of the year Mary has \$10,000 in wealth, with no additional earnings for the year. With probability 2/3 Mary has an accident and suffers a loss of \$7,500. (Mary is not hurt, just the car) With probability 1/3 there is no accident leading to zero loss. What is Mary's expected wealth? (3 points)
2. From now on, assume that Mary's utility function over wealth is $u(w) = w^{1/2}$, where w is the wealth left over after the accident. What is her expected utility? (3 points)
3. Mary can purchase an insurance with a premium \$5,100. This insurance will fully reimburse the damage (\$7,500) if an accident occurs and wil give no payment in case of no accident. What is Mary's expected wealth if she takes the insurance? Is the insurance premium fair? (5 points)
4. Will Mary take up the insurance? (Compute the expected utility and compare to expected utility in point 2) (3 points)
5. Angela is a friend of Mary. She hears Mary talk about her decision and she exclaims "*I* would not have purchased the insurance!" Given an example of a utility function such that Mary would *not* have purchased the insurance given a wealth of \$10,000 and the accident probabilities given above. (4 points)
6. Angela adds: "Mary, you are so risk-averse, relax!". Let's leave aside the 'relaxing' issue. Provide intuition on why risk-aversion translates into a concave utility function, like $u(w) = w^{1/2}$. (3 points)

Solution to Problem 1. Uncertainty.

1. Mary's expected wealth is $2/3 * (10,000 - 7,500) + 1/3 * (10,000 - 0) = 5,000$.
2. Mary's expected utility is $2/3 * u(10,000 - 7,500) + 1/3 * u(10,000 - 0) = 2/3 * (2,500)^{1/2} + 1/3 * (10,000)^{1/2} = 2/3 * 50 + 1/3 * 100 = 200/3 = 66.66$.
3. If Mary takes the insurance, her wealth is $\$10,000 - \$5,100 = \$4,900$ no matter whether there is an accident or not. The insurance is not fair since $\$5,100 > 2/3 * \$7,500 = \$5,000$.
4. Mary's expected utility if she takes the insurance is $2/3u(4,900) + 1/3 * u(4,900) = (4,900)^{1/2} = 70$. Since $70 > 66.66$, Mary will take the insurance.
5. For example, assume that Angela is risk neutral, for example $u(w) = w$. Then Angela chooses the action that maximizes expected wealth. In this case, Angela would not purchase the insurance given that the expected wealth of \$5,100 with no insurance is higher than the expected wealth of \$5,000 with no insurance. Any convex utility function would have done the trick as well.
6. A concave utility function implies that marginal utility of wealth is highest for low levels of wealth. When marginal utility of wealth is highest, Mary would really like to increase consumption. This leads Mary to avoids risks, since she would rather smooth consumption and get more consumption when times are bad and less consumption when times are good.

Problem 2. Production. (43 points) In this exercise, we consider a firm producing product y using two inputs, labor L and capital K. The production function is $y = f(L, K) = (L + K)^{\alpha}$. Assume that the wage of a worker is w and the cost of capital is r. Assume $L \geq 0$, $K \geq 0$, and $\alpha > 0$.

1. Draw a picture of the isoquants. What is the unusual feature of this production function? (5 point)
2. For which values of α does the function exhibit decreasing returns to scale (that is, $f(tL) < tf(L)$ for all $t > 1$ and all $L \geq 0$)? (3 points)
3. Consider now the first step of the cost minimization problem. The firm solves

$$\min wL + rK$$
$$s.t. f(L, K) \geq y$$

   for $y > 0$. What are the solutions for $L^*(w, r, y|\alpha)$ and $K^*(w, r, y|\alpha)$? (This notation stresses that the solution depends also on the parameter α. Hint: You are better off not using Lagrangeans. The pictures you drew in point 1 may be helpful) (9 points)
4. Write down the implied cost function $c(w, y|\alpha)$. (4 points)
5. Derive an expression for the average cost $c\left(w, y|\bar{L}, \alpha\right)/y$ and the marginal cost $c'_y\left(w, y|\bar{L}, \alpha\right)$ for $y > 0$ and $w < r$. Graph the average cost and marginal cost for $\alpha = .5$, $w = 1$, and $r = 2$. Graph the supply function for the same values of the parameters. [remember, y is on the horizontal axis]. (5 points)
6. Now that we graphically solved for the supply function, we also derive it formally for all $\alpha > 0$. Consider the second step of cost minimization

$$\max_y py - c\left(w, y|\bar{L}, \alpha\right).$$

   Write down the first order condition and the second order conditions. Solve for $y^*\left(w, p|\bar{L}, \alpha\right)$. (here do *not* assume $w < r$) For what values of α is the second order condition satisfied? (5 points)
7. From now on, assume $\alpha < 1$. Take the solution for $y^*\left(w, p|\bar{L}, \alpha\right)$ in point 6 and consider what happens to y^* as the wage w increases. Obtain the sign of $\partial y^*/\partial w$ for the cases $w < r$ and $w > r$. Provide intuition on this result. (5 points)
8. Consider now what happens to the supply function as price of output p increase. Obtain the sign of $\partial y^*/\partial p$ and provide intuition on the result. (3 points)
9. Does the company make, negative, or positive profits for $p > 0$? Provide an argument for your answer. (4 points)

Solution of Problem 2.

1. See Figure 1. In this particular firm, the inputs are perfect substitutes. The firm can perfectly substitute one unit of capital for one unit of labor and viceversa.
2. For this production function, $f(tL, tK) = (tL + tK)^{\alpha} = t^{\alpha}(L + K)^{\alpha} = t^{\alpha} f(L, K)$. Therefore, for $0 < \alpha < 1$ the firm will exhibit decreasing returns to scale since $t^{\alpha} < t$ for $t > 1$ and $\alpha < 1$.
3. From Point 1, we know that the firm is indifferent between a unit of labor and a unit of capital. Therefore, in its cost minimization problem the firm will just choose the input that is cheaper. That is, hte firm will use only L if $w < r$ and only K if $w > r$. For $w = r$, the firm is indifferent between any combination of labor and capital. One can see this graphically from the Figure in point 1. The isoquants are straight lines, so like in the case of expenditure minimization, the solution will be in the

corners. To fully solve the problem, consider first the case $w < r$. In this case the firm uses only labor and the cost minimization becomes

$$\begin{aligned}&\min wL\\ &s.t. L^{\alpha} \geq y.\end{aligned}$$

We already solved this in class. We know that the constraint will be binding and the firm will use just about enough labor to produce y. Therefore, $L^*(w, r, y|\alpha)$ is the solution of $L^*(w, r, y|\alpha)^{\alpha} = y$, or $L^*(w, r, y|\alpha) = y^{1/\alpha}$ and $K^*(w, r, y|\alpha) = 0$. Consider now the case $w > r$. Now the cost minimization becomes

$$\begin{aligned}&\min rK\\ &s.t. K^{\alpha} \geq y.\end{aligned}$$

Again, the firm will use just about enough labor to produce y. Therefore, $K^*(w, r, y|\alpha)$ is the solution of $K^*(w, r, y|\alpha)^{\alpha} = y$, or $K^*(w, r, y|\alpha) = y^{1/\alpha}$ and $L^*(w, r, y|\alpha) = 0$. Finally, for $r = w$, there are multiple solutions for $L^*(w, r, y|\alpha)$ and $K^*(w, r, y|\alpha)$, all satisfying $L^*(w, r, y|\alpha) + K^*(w, r, y|\alpha) = y^{1/\alpha}$.

4. The cost function $c(w, y|\alpha)$ is $wL^*(w, r, y|\alpha) + rK^*(w, r, y|\alpha)$, which equals

$$c(w, y|\alpha) = \begin{cases} ry^{1/\alpha} & \text{if} \quad w > r \\ wy^{1/\alpha} = ry^{1/\alpha} & \text{if} \quad w = r \\ wy^{1/\alpha} & \text{if} \quad w < r \end{cases}$$

or, in a more compact format, $c(w, y|\alpha) = \min(w, r) * y^{1/\alpha}$.

5. Under the assumption $w < r$, the cost function is $wy^{1/\alpha}$. The average cost function $c\left(w, y|\bar{L}, \alpha\right)/y$ is $wy^{1/\alpha}/y = wy^{(1-\alpha)/\alpha}$. The marginal cost function $c_y'\left(w, y|\bar{L}, \alpha\right)$ is $w\frac{1}{\alpha}y^{(1-\alpha)/\alpha}$. For $\alpha = 1/2$, the average cost function is wy and the marginal cost function is $2wy$. Both are increasing, and the marginal cost function lies always above the average cost function. This implies that the supply function is just the marginal cost function.

6. The first order condition is

$$p - c_y'\left(w, y|\bar{L}, \alpha\right) = 0$$

or

$$p - \frac{1}{\alpha}\min(w, r)y^{(1-\alpha)/\alpha} = 0.$$

This implies

$$y^*\left(w, y|\bar{L}, \alpha\right) = p^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha}{\min(w, r)}\right)^{\frac{\alpha}{1-\alpha}} \tag{1}$$

The second order condition is

$$-c_{y,y}''\left(w, y|\bar{L}, \alpha\right) < 0$$

or

$$-\min(w, r)\frac{(1-\alpha)}{\alpha^2}y^{(1-2\alpha)/\alpha} < 0.$$

This condition is satisfied for $\alpha < 1$.

7. Expression (1) for y^* implies $\partial y^*/\partial w < 0$ for $w < r$, since in this case $y^* = p^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}$. However, $\partial y^*/\partial w = 0$ for $w > r$, since in this case $y^* = p^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}$, and w does not show up in this expression. Increases in the cost of labor only matter in the supply function as long as labor is used, that is, as long as it is the cheapest input ($w < r$). If labor is not the cheapest input, instead, small increases in labor costs do not matter since no labor is employed.

8. Expression (1) for y^* implies $\partial y^*/p = \frac{\alpha}{1-\alpha}p^{\frac{\alpha}{1-\alpha}-1}\left(\frac{\alpha}{\min(w,r)}\right)^{\frac{\alpha}{1-\alpha}} > 0$. As the price of the final good increases, the company finds it profitable to supply more output to the market. The higher final price makes it worthwhile to produce even if increasing production raises the marginal cost.

9. Given that the marginal cost function always lies above the average cost function (see point 5) the firm is making positive profits for all levels of p, as long as $p > 0$.

Problem 3. (Exercise with less guidance than usual) (18 points) Consider now the decision making of a governor that has limited funds to spend and wants to minimize the accidents on freeways and railways. Each freeway accident occurs with probability p_F and a railway accident occurs with probability p_R. Either accident generates a social loss of L. If there is no loss, the social utility is 0. The expected social utility therefore is $-p_F L - p_R L$, with $L > 0$. The governor maximizes social utility by spending the State Budget M on improving streets (M_F) and railways (M_R), with $M_F + M_R \leq M$. In particular, the probabilities of accident depend on the funds spent as follows: $p_F(M_F) = \exp(-M_F)$ and $p_R(M_R) = \exp(-M_R)$

1. Graph $p_F(M_F) = \exp(-M_F)$. Comment briefly on how increased expenditure affects the probability of an accident. (3 points)
2. Solve for the optimal levels of spending M_F^* and M_R^*, as well as for p_F^* and p_R^*. Comment on the solution you found. Will the governor reduce the probability of accidents to zero if the budget M is very large? (15 points)

Solution to Problem 3.

1. See Figure. The increased expenditure reduces the probability of an accident and the effectiveness of spending \$1 is lower the more is spent already.
2. The governor maximizes

$$\max_{M_F, M_R} -\exp(-M_F) L - \exp(-M_R) L$$
$$s.t. M_F + M_R \leq M$$

The constraint will be satisfied with equality since the function to be maximized is increasing in both M_F and M_R. We can therefore substitute $M_F = M - M_R$ in the objective function to get

$$\max_{M_R} -\exp(-M + M_R) L - \exp(-M_R) L.$$

The first order conditions are

$$-\exp(-M + M_R) L + \exp(-M_R) L = 0$$

or

$$\exp(-M + M_R) = \exp(-M_R)$$

or

$$-M + M_R = M_R$$

or $M_R^* = M/2 = M_L^*$. The optimum accident probabilities are $p_F^* = p_R^* = \exp(-M/2)$. The higher the funds, the lower the accident probabilities. Since the effectiveness of expenditure is the same and the cost of accidents the same across railway and freeway, in equilibrium the governor shares the funds equally across railway and freeway.

The second order condition is satisfied since

$$-\exp(-M + M_R) F - 4\exp(-2M_R) R < 0.$$