Econ 101A – Final exam
Th 16 December.

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Please solve Problem 1 and 2 in the first blue book and Problems 3 and 4 in the second Blue Book. Good luck!

Problem 1. Car production (34 points). Consider a market for cars with just one firm. The firm has a linear cost function $C(q) = 2q$. The market inverse demand function is $P(Q) = 9 - Q$, where Q is the total quantity produced. Since initially there is just one firm, $q = Q$.

1. Set up the maximization problem for the monopolist and determine the optimal price and quantity of cars produced (6 points)

2. How much profit does the firm make? (4 points)

3. Consider now the case of a second firm entering the market. The two firms choose quantities simultaneously, that is, they compete á la Cournot. Set up the maximization problem. Determine the optimal price and quantity of cars produced. (6 points)

4. Compare the quantities and prices produced to the monopoly case. Provide intuition on the result. (4 points)

5. Compare total profits in Cournot and in monopoly (3 points).

6. Draw a graph with price on the y axis and quantity on the x axis. Locate the Cournot and monopoly outcomes. Compute the consumer surplus for the Cournot and the monopoly cases. Which market do consumers prefer? Provide intuition for the answer (7 points)

7. On the graph, identify the deadweight loss of going from Cournot to monopoly. (4 points)

Problem 2. Car Driving. (52 points) Two agents ($i = 1, 2$) are deciding how fast to drive and how much to consume. Each individual chooses speed x_i and get utility $u(x_i)$ from the choice of speed, with $u'(x) > 0$ and $u''(x) < 0$ or all x. (that is, each agent like faster speed because it allows her to get to more places in less time; in addition, there are diminishing gains to higher speed). The cost of driving faster is that it increases the probability of an accident. The probability of an accident for agent i is $\pi(x_i) + \pi(x_j)$, with $\pi'(x) > 0$ and $\pi''(x) > 0$. [Notation: x_j denotes the speed chosen by the other driver] The faster any of the agents drives, the higher the probability of accident for both. Furthermore, the probability of accident is convex in driving speed. The cost of an accident is c. The quantity of consumption is y_i, with price normalized to 1. The overall utility $v(x_i, y_i)$ of agent i is

$$v(x_i, y_i) = u(x_i) + y_i,$$

where y_i is the amount consumed of good i. The budget constraint is

$$(\pi(x_i) + \pi(x_j))\, c + y_i = M_i,$$

where M_i is the income of agent i.

1. Write out the maximization problem. Obtain the first-order condition for agent i with respect to x_i^* and write the expression for y_i^* [You are better off substituting the constraint into the utility function] (5 points)

2. Check the second order conditions. (4 points)

3. Use the implicit function theorem to obtain an expression for $\partial x_i^*/\partial c$ (speed of driving and cost of accident) [You can hold x_j^* constant while doing this] What is the sign? Provide intuition (5 points)

4. Use the implicit function theorem to obtain an expression for $\partial x_i^*/\partial M$? (speed of driving and income) Provide intuition, in particular on the specific assumptions driving this result. (8 points)

5. Now assume that the decisions on speed (x_1, x_2) and consumption (y_1, y_2) are taken by a central planner. The central planner maximizes the sum of the utilities of the two agents subject to the two budget constraints. Write the maximization problem. [Recommended substitution of the budget constraints into the objective function] (3 points)

6. Obtain the first order conditions of the problem of the planner. Do the solutions for x_1^P (planner problem) differ from x_1^* (individual problem)? In which direction? Provide intuition and try to characterize the general problem surfacing here. (10 points)

7. Go back now to the individual optimization problem. Assume now that agent i pays a fine t for each accident that involves her (no matter who caused it). (for example, the insurance premium increases in subsequent years) Solve the new problem for the individual. (6 points)

8. What is the level of fine t such that the solution to the individual problem coincides with the social optimum? Comment on the magnitude you find (7 points)

9. Does this problem suggest also a justification for speed limits? (4 points)

Problem 3. Altruism and dictator games. (41 points) In an experiment called the dictator game, a dictator (Player D) decides how to share \$10 dollars with the recipient (Player R). Label g (for gift) the transfer from the dictator to the recipient, with $0 \leq g \leq 10$. The monetary payoff π_D for the dictator is $(10 - g)$ and the monetary payoff π_R for the recipient is g. The typical outcome of this game is that 50 percent of subjects chooses $g = \$5$, and 50 percent chooses $g = \$0$. We now consider different models to see if they can rationalize this behavior.

1. Consider the case of dictator A, a selfish dictator. His utility function is $u_D = \pi_D$. What g does a selfish dictator choose in this case to maximize utility? (4 points)

2. Consider the case of dictator B that has the following utility function: $u_D = (1-\rho)\pi_D + \rho\pi_R$. Provide an interpretation for this utility function and for parameter ρ, with $0 \leq \rho < 1$. How do you interpret the special cases $\rho = 0$ and $\rho = .5$? (6 points)

3. How do you interpret the case $\rho < 0$? (3 points)

4. Keep assuming $u_D = (1 - \rho)\pi_D + \rho\pi_R$, and $\rho < 1$. Solve for the optimal gift $g^*(\rho)$. (The notation reminds you that g^* is a function of ρ.) (7 points)

5. How well can dictators of type A or B explain observed play in the dictator game? (3 points)

6. Consider now the case of dictator C, with utility function

$$u_D = \begin{cases} (1-\rho)\pi_D + \rho\pi_R & \text{if} \quad \pi_D \geq \pi_R \\ (1-\sigma)\pi_D + \sigma\pi_R & \text{if} \quad \pi_D < \pi_R. \end{cases}$$

   Assume $\rho \in [0, 1]$, $\sigma < \rho$. Provide an interpretation of this utility function. What psychological intuition does it capture? (6 points)

7. Solve for the optimal g^* for the case $.5 < \rho < 1$ and $\sigma < .5$. (8 points)

8. Provide intuition for the solution. Does the behavior of dictator C help to explain the data? (4 points)

Problem 4. Bertrand Competition in discrete increments (46 points) Consider a variant of the Bertrand model of competition with two firms that we covered in class. The difference from the model in class is that prices are not a continuous variable, but rather a discrete variable. Prices vary in multiples of 1 cent. Firms can charge prices of 0, .01, .02, .03,... etc. The profits of firm i are

$$\pi_i(p_i, p_j) = \begin{cases} (p_i - c_i)\, D(p_i) & \text{if} \quad p_i < p_j \\ (p_i - c_i)\, D(p_i)/2 & \text{if} \quad p_i = p_j. \\ 0 & \text{if} \quad p_i > p_j. \end{cases}$$

The demand function $D(p)$ is strictly decreasing in p, that is, $D'(p) < 0$. Assume first that both firms have the same marginal cost $c_1 = c_2 = c$, and that the marginal cost c is a multiple of 1 cent. (The firm can charge $c - .01$, c, $c + .01$, $c + .02$, etc.)

1. Write down the definition of Nash Equilibrium as it applies to this game, that is, with p_i as the strategy of player i and $\pi_i(p_i, p_j)$ as the function that player 1 maximizes. Provide both the formal definition and the intuition. Do not substitute in the expression for π_i. (7 points)

2. Show that $p_1^* = p_2^* = c$ (that is, marginal cost pricing) is a Nash Equilibrium. (5 points)

3. Unlike in the case in which prices are continuous, this is not the only Nash Equilibrium. Find one other Nash Equilibrium. (you need to prove that it is a Nash Equilibrium) [Hint: The peculiar feature of this setup is that the firm can only charge prices that are multiples of 1 cent] (8 points)

4. (Harder, and long) Characterize all the (pure strategy) Nash Equilibria of the game. Show that there are no other Nash Equilibria. (18 points)

5. Now consider the case of two firms with different marginal costs, both multiples of 1 cent. Firm 1 has marginal cost c_1, with $c_1 < c_2$, the marginal cost of firm 2. For simplicity, assume $c_2 - c_1 > .01$, that is, the difference in marginal costs is more than one cent. Is $p_1^* = c_2 - .01$, $p_2^* = c_2$ an equilibrium? Explain intuitively as well. (8 points)