Econ 101A – Final exam
F 12 December.

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Problem 1. Nash equilibrium in 2X2 games (24 points) Consider the following game:

1\2	Left	Right
Up	$0, 0$	$1, 1$
Down	$4, \alpha$	$-2, -2$

1. Find all the pure *and mixed* strategy Nash Equilibria of the above game for $\alpha = 4$. You may find it useful to call u the probability that player 1 plays Up, $1 - u$ the probability that player 1 plays Down, l the probability that Player 2 plays Left, and $1 - l$ the probability that Player 2 plays Right. (10 points)
2. How does your answer for the mixed stategy change if $\alpha = 2$? (6 points)
3. Notice that we reduced a payoff of player 2 from 4 to 2. Whose player equilibrium strategies change? Does this surprise you? Can you provide some explanation? (8 points)

Problem 2. Homeworks and Altruism. (25 points) Student i has $H > 0$ hours of time and can either work on own homework h_1, or on homework of student 2, h_2. We denote the number of hours spent by student i on homework j as h^i_j,with $h^i_j \geq 0$. The time constraint for student i therefore is

$$h^i_1 + h^i_2 \leq H$$

The happiness of student i, $i = 1, 2$, is given by

$$U_i \left(h^1_i, h^2_i\right) = \left(h^1_i + h^2_i\right)^{.5},$$

which captures the fact that the more hours are spent on homework h_i by the two students, the better the grade for i, and the happier student i. However, there are decreasing returns to spending time on the homework, captured by the exponent .5. Selfish student 2 maximizes $U_2 \left(h^1_2, h^2_2\right)$, while altruistic student 1 maximizes $U_1 \left(h^1_1, h^2_1\right) + \alpha U \left(h^1_2, h^2_2\right)$. Suppose that student 1 moves first, and student 2 moves second, after observing student 1's decision.

1. Solve utility maximization for student 2 subject to time constraint as a function of h^1_2, that is, of how much student 1 helps student 2. Find $h^{2*}_2 \left(h^1_2\right)$. (5 points)
2. Set up the utility maximization for student 1 subject to time constraint. Student 1 anticipates $h^{2*}_1 \left(h^1_2\right)$ and $h^{2*}_2 \left(h^1_2\right)$. Write the first order condition. (4 points)
3. Obtain the solution for h^{1*}_1 and h^{1*}_2 if $\alpha \leq 1$ (selfishness or limited altruism). Show that $h^{1*}_1 < H$ if $\alpha > 1$ (strong altruism) (10 points)
4. What is the solution for h^{1*}_1 and h^{1*}_2 if $\alpha < 0$ (spite)? (6 points)

Problem 3. Externalities in Production (38 points) In this exercise, we consider the problem of externalities generated by two firms. An externality occurs when a firm does not take into account in its maximization problem the effect of one of its decisions on other firms. The idea of this problem is that each of the firms produces a pollutant that increases the costs of production of the other firm. Assume that two firms compete á la Cournot. Firm i produces q_i units of the good at total cost cq_iq_{-i}, with $b > c > 0$. For example, the total production cost of firm 1 is cq_2q_1. This captures the fact that increased production by the competitor increases the production costs. For $c = 0$, we obtain a standard Cournot case with zero marginal cost of production. The inverse demand function is $P(Q) = a - bQ = a - b(q_1 + q_2)$.

1. Consider first the Cournot solution. Write down the profit function for firm i as a function of the production of the competitor, q_{-i}. Write down the first order condition of firm i and solve for q_i^* as a function of q_{-i}^*, that is, find the best response function for firm i, for $i = 1, 2$. (6 points)

2. Find the solution for the Cournot oligopoly by requiring that both best response functions hold. In other words, solve the system of two first order conditions for the two firms. Find the solutions for $q_1^* = q_2^*$ and for p^* (4 points)

3. In the plane (q_1, q_2) graph the best response function of both firms for $a = b = 1$ and $c = 0$. Indicate the Cournot Nash equilibrium in the graph. (6 points)

4. Now assume that the two firms merge together and become one monopolist. The monopolist maximizes the total profits from the two plants, that is $(a - bq_1 - bq_2)(q_1 + q_2) - 2cq_1q_2$. Write down the first order conditions with respect to q_1 and q_2. Compare these first order conditions to the ones of the Cournot case in point 1. There are two differences between the f.o.c.s. Where do they come from? What is the sense in which the monopolist internalizes externalities? (8 points)

5. Find the optimum for q_1^M and q_2^M, that is the quantity that the merged monopolist produces in each plant. (the monopolist will produce an equal quantity q_M in both plants) (4 points)

6. Compare the quantities produced in monopoly and duopoly. In particular, the ratio q^*/q^M should come out to be
$$\frac{q^*}{q_M} = \frac{4b + 2c}{3b + c}.$$
Show that this ratio is increasing in c. What is the intuition for the fact that the 'overproduction' of duopoly relative to monopoly is more accentuated when c is large? (10 points)

Problem 4. The economics of brass. (70 points) In this problem we will consider the production of a good, brass, that needs two inputs, copper and zinc. Unlike what we did in class, we are going to explicitely model the market for the inputs, that is, the prodution of copper and zinc. We are going to assume that there is a monopolist in each of the copper and zinc makerts.

1. Write down the chemical formula for a molecule of brass. Just kidding! That formula is Cu_3Zn_2 where Cu indicates one molecule of copper and Zn indicates one molecule of Zinc. Argue that therefore the production function for brass is $\min(c/3, z/2)$, where c is the quantity of copper and z is the quantity of zinc. (3 points)

2. Plot an isoquant for the production of brass. Remember, the axes are the quantities of inputs c and z. (4 points)

3. A brass producer wants to produce b units of brass. Assume that the cost of one unit of Copper is $p_c > 0$ and the cost of one unit of Zinc in $p_z > 0$. Solve for the cost-minimizing combination of inputs $c^*(p_c, p_z, b)$ and $z^*(p_c, p_z, b)$ that produce quantity b of output. Write down the cost function $C(p_c, p_z, b) = p_c c^*(p_c, p_z, b) + p_z z^*(p_c, p_z, b)$. (Hint: Be careful about taking derivatives, the drawing of the isoquant should help you in guiding you to the solution) (I can give you the answer for 10 points if you get stuck here) (10 points)

4. Assume that the brass industry is perfectly competitive. Derive the supply function of a brass firm $b^*(p_b, p_c, p_z)$. Graph the firm and industry supply function assuming N firms. Does it make a difference for the supply functions if there is just 1 or 10 or 100 firms? (5 points)

5. The market demand function for brass is $p_b(b) = a - b$, where b is the total quantity of brass produced in the economy. Plot demand and supply assuming $a = 10$, $p_c = 1$ and $p_z = 1$. Helping yourself with the graph, solve for the total market production of brass $b^*(p_c, p_z)$, as well as for the equilibrium price $p_b^*(p_c, p_z)$. (6 points)

6. Write down also the derived demand for the inputs $c^*(p_c, p_z)$ and $z^*(p_c, p_z)$ in equilibrium using the expression for demand for inputs you found in point 3. Argue that these are also the demand functions that the copper and zinc monopolists face. (4 points)

7. We now model the market for the inputs. Assume that the market for zinc is controlled by a monopolist, and that the market for copper is controlled by a (different) monopolist. The two monopolists decide on production simultaneously. In both cases, cost of production are zero. Write down the profit maximization problem for the copper producer as a function of the price of zinc p_z: $\max_{p_c} p_c c^*(p_c, p_z)$ (Hint: you are better off maximizing profits of the copper monopolist with respect to price p_c, rather than with respect to quantity c, although both give the same solution). Write down the first order conditions and solve for $p_c^*(p_z)$ and $c^*(p_z)$. Check the second order conditions (with respect to p_c) (7 points)

8. Similarly, write down and solve for the problem of the monopolist in the zinc industry. Find the solution for $p_z^*(p_c)$ and $z^*(p_c)$. (3 points)

9. Finally, now that we have solved for the company's production function as a function of the price set by the other monopolist, look for a Nash equilibrium for p_c^* and p_z^*. Derive also $z^*$and c^*. In which sense this game is like Cournot, despite the fact that the two firms are monopolists producing different goods? Compute the implied price of brass p_b^* (10 points)

10. The final results that you get for point 9 should imply $p_b^* = (2/3)\, a$. Suppose now that the copper and zinc monopolists, instead of competing with each other, merged and maximized the sum of their joint profits. I am not asking you to solve for this, I will give you the solution. In this case, the final price of brass will be $p_b^{*\prime} = a/2$. In correspondence of these two values of the price of brass, compute the quantity of brass b^* and $b^{*\prime}$ demanded in both cases using the demand function $p_b(b) = a - b$. Draw the solution graphically for $a = 10$ and compute the consumer surplus (I suggest measuring the area of the appropriate triangle in the graph). (8 points)

11. (Hard) Why is it that having one monopolist control the whole market of the inputs yields lower price and higher surplus in the final goods market? In other words, the situation with two competing monopolists in the input market yields higher price for brass and lower consumer welfare than the situation with just one firm that controls all the inputs. Why is this the case? Usually, it is bad for consumers if two firms merge and/or collude. (this is a famous counterexample) (10 points)