

Econ 101A – Solutions to Final Exam
Th 15 December.

Please solve Problem 1, 2 and 3 in the first blue book and Problems 4 and 5 in the second Blue Book. Good luck!

Problem 1. Shorter problems. (50 points) Solve the following shorter problems.

1. Compute the pure-strategy and mixed strategy equilibria of the following coordination game. Call u the probability that player 1 plays Up, $1 - u$ the probability that player 1 plays Down, l the probability that Player 2 plays Left, and $1 - l$ the probability that Player 2 plays Right. (20 points)

1\2	Left	Right
Up	3, 2	1, 1
Down	1, 1	2, 3

2. For each of these cost functions, plot the marginal cost function and the supply function, *and* write out the supply function $S(p)$, with quantity as a function of price p (30 points):
 - (a) $C(q) = 2q$ (8 points)
 - (b) $C(q) = 2q^2 - q + 2$ (12 points)
 - (c) $C(q) = q^3 + 10q$ (10 points)

Solution to Problem 1.

1. The pure strategy Nash equilibria can be found in the matrix once we underline the best responses for each player:

1\2	Left	Right
Up	<u>3</u> , <u>2</u>	1, 1
Down	1, 1	<u>2</u> , <u>3</u>

The equilibria therefore are $(s_1^*, s_2^*) = (U, L)$ and $(s_1^*, s_2^*) = (D, R)$. To find the mixed strategy equilibria, we compute for each player the expected utility as a function of what the other player does. We start with player 1. Player 1 prefers Up to Down if

$$lu_1(U, L) + (1 - l)u_1(U, R) \geq lu_1(D, L) + (1 - l)u_1(D, R)$$

or

$$3l + (1 - l) \geq l + 2(1 - l)$$

or

$$l \geq 1/3.$$

Therefore, the Best Response correspondence for player 1 is

$$BR_1^*(l) = \begin{cases} u = 1 & \text{if } l > 1/3; \\ \text{any } u \in [0, 1] & \text{if } l = 1/3; \\ u = 0 & \text{if } l < 1/3. \end{cases}$$

We then compute the Best Response correspondence for player 2. Player 2 prefers Left to Right if

$$u * u_2(U, L) + (1 - u) u_2(D, L) \geq u * u_2(U, R) + (1 - u) u_2(D, R)$$

or

$$2u + (1 - u) \geq u + 3(1 - u)$$

or

$$u \geq 2/3.$$

Therefore, the Best Response correspondence for player 2 is

$$BR_2^*(u) = \begin{cases} l = 1 & \text{if } u > 2/3; \\ \text{any } l \in [0, 1] & \text{if } u = 2/3; \\ l = 0 & \text{if } u < 2/3. \end{cases}$$

Plotting the two Best Response correspondences, we see that the three points that are on the Best Response correspondences of both players are $(\sigma_1^*, \sigma_2^*) = (u = 1, l = 1)$, $(u = 0, l = 0)$, and $(u = 2/3, l = 1/3)$. The first two are the pure-strategy equilibria we had identified before, the other one is the additional equilibrium in mixed strategies.

2. We proceed case-by-case:

- (a) $C'(q) = C(q)/q = 2$. The marginal cost function is always (weakly) above the average cost function). Supply function:

$$S(p) = \begin{cases} q^* \rightarrow +\infty & \text{if } p > 2 \\ \text{any } q \in [0, \infty) & \text{if } p = 2 \\ q^* = 0 & \text{if } p < 2 \end{cases}$$

- (b) $C'(q) = 4q - 1$, $C(q)/q = 2q - 1 + 2/q$. Marginal cost is higher than average cost whenever $4q - 1 \geq 2q - 1 + 2/q$, or $2q^2 - 2 \geq 0$, or $q^2 \geq 1$, or $q \geq 1$. (we do not care about solutions with negative q) Plug in $q = 1$ in the marginal cost curve to find the lowest price level such that the marginal cost function lies above the average cost function: $p = 4 * (1) - 1$, or $p = 3$. We invert the marginal cost function $C'(q) = 4q - 1 = p$ to get $q = p/4 + 1/4$. The supply function therefore is

$$S(p) = \begin{cases} q^* = p/4 + 1/4 & \text{if } p \geq 3 \\ q^* = 0 & \text{if } p < 3 \end{cases}$$

3. $C'(q) = 3q^2 + 10$, $C(q)/q = q^2 + 10$. Marginal cost is higher than average cost whenever $3q^2 + 10 \geq q^2 + 10$, or $2q^2 \geq 0$, which is always true. We invert the marginal cost function $C'(q) = 3q^2 + 10 = p$ to get $q = \sqrt{\frac{p-10}{3}}$. Clearly, price has to be above 10 to justify a positive production q . (the marginal cost is never lower than 10, check this) The supply function therefore is

$$S(p) = \begin{cases} q^* = \sqrt{\frac{p-10}{3}} & \text{if } p \geq 10 \\ q^* = 0 & \text{if } p < 10. \end{cases}$$

Problem 2. Monopoly and Duopoly. (40 points). Initially there is one firm in a market for cars. The firm has a linear cost function: $C(q) = 2q$. The market inverse demand function is given by $P(Q) = 9 - Q$.

1. What price will the firm charge? What quantity of cars will the firm sell? (8 points)
2. How much profit will the firm make? (4 points)
3. Now, a second firm enters the market. The second firm has an identical cost function. What will the Cournot equilibrium output for each firm be? (8 points)
4. What is the Stackelberg equilibrium output for each firm if firm 2 enters second? (7 points)
5. How much profit will each firm make in the Cournot game? How much in Stackelberg? (5 points)
6. Which type of market do consumers prefer: monopoly, Cournot duopoly or Stackelberg duopoly? Why? (8 points)

Solution to Problem 2.

1. The first two questions are about the case of monopoly. A monopolistic firm maximizes profits:

$$\max_y (9 - q)q - 2q.$$

and the first order condition is

$$9 - 2q_M^* - 2 = 0.$$

It follows that

$$q_M^* = \frac{9 - 2}{2} = 7/2$$

and

$$p_M^* = 9 - q_M^* = 9 - 7/2 = 11/2.$$

2. As for the profits in monopoly, they equal

$$\pi_M^* = (9 - q_M^*)q_M^* - 2q_M^* = (9 - 7/2)7/2 - 7 = 77/4 - 7 = 49/4$$

3. We are now in a Cournot competition set-up. A duopolistic firm maximizes profits:

$$\max_{y_i} (9 - (q_i + q_{-i}))q_i - 2q_i. \tag{1}$$

The first order condition of problem (1) is

$$9 - 2q_i^* - q_{-i}^* - 2 = 0.$$

It follows that the first order conditions for the two firms are

$$9 - 2q_1^* - q_2^* - 2 = 0.$$

and

$$9 - 2q_2^* - q_1^* - 2 = 0.$$

From the first order condition, we get $q_2^* = 7 - 2q_1^*$. We substitute this expression into the second order condition to get $9 - 2(7 - 2q_1^*) - q_1^* - 2 = 0$ or $3q_1^* = 7$, so

$$q_1^* = \frac{7}{3}$$

and

$$q_2^* = 7 - 2q_1^* = 7 - 2\frac{7}{3} = \frac{7}{3}.$$

Not surprisingly, the quantities produced by firms 1 and 2 are equal. The total market output $Q^* = 2 * 7/3 = 14/3$.

4. In the Stackelberg case, firm 1 maximizes taking into account the reaction function of firm 2. The reaction function of firm 2 is $q_2^*(q_1) = (7 - q_1)/2$. Therefore, firm 1 maximizes

$$\max_{q_1} (9 - (q_1 + (7 - q_1)/2)) q_1 - 2q_1$$

which leads to the first order conditions

$$9 - 2q_1 - 7/2 + q_1 - 2 = 0$$

or

$$q_1^* = 7/2$$

and, using the reaction function of firm 2,

$$q_2^* = (7 - q_1^*)/2 = 7/4.$$

So the first mover produces twice as much. The total market output is $Q^* = 7/2 + 7/4 = 21/4$, larger than in Cournot.

5. In the Cournot case, the profit for either firm is

$$\pi_C = (9 - 14/3) 7/3 - 14/3 = 91/9 - 14/3 = 49/9.$$

Profits in Stackelberg differ. The profit of the leader is

$$\pi_S^1 = (9 - 21/4) 7/2 - 14/2 = 105/8 - 14/2 = 49/8 > \pi_C,$$

while the profit of the follower is

$$\pi_S^2 = (9 - 21/4) 7/4 - 14/4 = 105/16 - 14/4 = 49/16 < \pi_C$$

6. Consumers prefer the market with the most total output (and by consequence the lowest price), since this maximizes consumer surplus. Formally, consumers maximize

$$\int_0^Q (D(p) - p) dq' = \int_0^Q ((9 - q') - (9 - Q)) dq' = [Qq' - q'^2/2]_0^Q = Q^2 - Q^2/2 = Q^2/2,$$

which is increasing in total quantity produced. (this integration was not required) Therefore consumers prefer the Stackelberg duopoly, which has the highest total production.

Problem 3. Voting. (23 points) This paper provides a simple model of voting to illustrate the difficulties (and the strength) of an economic model of voting. Consider George, a committed Republican that is deciding whether to vote for Presidential elections. George's utility function is $U(P, v) = u(P) - cv$, where $u(P)$ equals U if Republicans win the election ($P = R$) and 0 if Democrats win the election ($P = D$). The variable $c \geq 0$ is the effort cost of going to vote, which George pays only if he votes ($v = 1$). If George does not vote ($v = 0$), George pays no voting cost. Finally, George believes that there is a probability p that his vote will decide the election, and probability $1 - p$ that his vote will not affect the elections. In addition, George believes that the average share of Republican voters is .5.

1. Compute the expected utility of George from voting ($v = 1$) and from non-voting ($v = 0$). (6 points)
2. Under what condition does George vote? Provide intuition (4 points)
3. Assume that the cost of voting is \$10 (an hour's wage) and the value of voting is U is \$1,000. What would this imply about the cutoff level of p such that George votes? Is it plausible that George will vote? (5 points)
4. Two empirical facts about votings are that (i) voter turnout is higher in closer elections; (ii) voter turnout is higher for more educated voters; (iii) voter turnout is higher for individuals with higher earnings; (iv) voter turnout is lower for younger people. Interpret these results in light of the model (8 points)

Solution to Problem 3.

1. If George does not votes, the probability that Republicans win is .5, and he does not pay the voting cost. His expected utility therefore is

$$EU(v = 0) = .5 * U + .5 * 0 = .5U.$$

If George votes, the probability that Republicans win is $.5 + p$ [this is not obvious from the text of the question, as someone pointed out, but this is the approach I clarified during the exam], and he pays the voting cost. His expected utility from voting therefore is

$$EU(v = 1) = (.5 + p) * U + (1 - .5 - p) * 0 - c = (.5 + p)U - c.$$

2. By the comparison of the two above utilities implies that George prefers to vote if

$$EU(v = 1) = (.5 + p)U - c \geq EU(v = 0) = .5U$$

or

$$p \geq c/U. \tag{2}$$

That is, George votes if the probability that e us a pivotal voter is larger than c/U , where c captures the cost of voting and U the benefit of voting. The larger the costs, and the lower the benefits for George of having Republicans in power (relative to Democrats), the higher the probability p needs to be to convince George to vote.

3. Using expression (2), we can see that the threshold level of p in this case would be $\bar{p} = 10/1000 = .01$. In fact, however, the probability that any given voter be pivotal in Presidential elections is closer to one over one million, that is, 10^{-6} , rather than 10^{-2} . Economic theory therefore cannot explain how people vote, unless the cost is very, very small and the benefits very large. Presumably, people think that voting is just the right thing to do even if it is not the economically rational thing to do, or they are altruistic and care about the wekfare of others, which will also increase the likelihood of voting (now I take into account the effect of voting on others). In other words, voting is a public good. We all want to live in a world where people vote, but individually we have incentives not to go.

4. In order: (i) In closer elections, p is higher (George is more likely to be pivotal), so it is more likely that condition (2) is met; (ii) More educated voters may have a higher willingness to pay for the party of their choosing, that is, a higher U (this is not obvious); (iii) To a first approximation, this runs counter to the prediction of the model, in that the value of time is higher for individuals with higher earnings, so c should be higher, leading to the opposite prediction. On the other hand, if higher earnings are also associated with higher value for a party (higher U), one can stretch the theory to fit this fact; (iv) Who knows! Younger people may have a more realistic perception that p is too small to justify voting, or they may perceive that the benefits of voting are lower, or they may have a higher cost of voting (they have not familiarized themselves with voting procedures)

Problem 4. Driving risk and insurance (38 points). Robert is an expected-utility maximizer that likes to drive fast, so his probability of an accident is $2/3$. Robert's preferences over wealth are $u(w) = w^5$. Suppose that Robert's initial wealth is \$100. If Robert has an accident, he incurs a \$51 loss.

1. What is Robert's expected utility? (5 points)
2. Now, assume there is one insurance company in existence with one policy available: Full insurance. That is to say, the insurance company charges a fixed premium and then pays Robert \$51 if he gets in an accident. If the insurance company is risk neutral, what is the premium π they need to charge to break even? (5 points)
3. Compute the expected utility for Robert if he purchases the insurance at premium π . Will Robert purchase the insurance? (Note: $(66)^5$ is approximately 8.1) (5 points)
4. Repeat the exercise for the case in which Robert has utility function $u(w) = w^2$. What is his utility if he does/does not purchase the insurance? Does he purchase the insurance? (Note: $(66)^2 = 4356$ and $(49)^2 = 2401$) (6 points)
5. Discuss the intuition for why Robert purchases the insurance in one case, but not in another. (5 points)
6. (Harder) Consider now the general case with $u(w) = w^\alpha$ with $\alpha > 0$. Remember that Jensen's inequality says $Ef(x) \geq f(Ex)$ if and only if f concave. Use Jensen's inequality to show analytically that, in fact, Robert purchases the insurance at premium π if and only if $\alpha < 1$ (and is indifferent for $\alpha = 1$). If you cannot do the maths, provide intuition on this. (12 points)

Solution to Problem 4.

1. The expected utility equals $2/3 \left[(100 - 51)^5 \right] + 1/3 \left[(100)^5 \right] = 2/3 * 7 + 1/3 * 10 = 24/3 = 8$.
2. The company's profits are given by the premium, minus the reimbursement for the loss in case the loss occurs: $\pi - 2/3 * 51$. Since profits must be zero, we obtain $\pi = 102/3 = \$34$.
3. Robert chooses whether or not to insure. If Robert takes the insurance, he has utility $(2/3 \left[(100 - 34 - 51 + 51)^5 \right] + 1/3 \left[(100 - 34)^5 \right]) = (100 - 34)^5 \approx 8.1$ Since the utility under insurance (8.1) is higher than without insurance (8), Robert purchases the insurance.
4. With the utility function $u(w) = w^2$, Robert without insurance has expected utility $2/3 \left[(100 - 51)^2 \right] + 1/3 \left[(100)^2 \right] = 2/3 * 2401 + 1/3 * 10,000 = 14,802/3 \approx 4,900$. With insurance, he has expected utility $2/3 \left[(100 - 34 - 51 + 51)^2 \right] + 1/3 \left[(100 - 34)^2 \right] = (100 - 34)^2 = 66^2 = 4,356$. Since $4,900 > 4,356$, Robert does not take up the insurance.
5. For concave utility function $u(w) = w^5$, Robert is risk-averse, so he purchases insurance whenever it is priced in actuarially fair manner. For convex utility function $u(w) = w^2$, Robert is risk-loving, so he does not purchase insurance that is priced in actuarially fair manner. The concavity/convexity of the utility function determines the attitude toward risk.
6. Jensen's inequality: $Ef(x) \leq f(Ex)$ if and only if f concave. In this case, the uncertainty is over whether the accident occurs or not. Consider $f(x)$ to be $u(x)$, where x is the post-accident wealth. Let x to be 100 with probability $1/3$ and 49 with probability $2/3$. Then, Jensen's inequality says

$$\frac{2}{3}u(49) + \frac{1}{3}u(100) \leq u\left(\frac{2}{3}49 + \frac{1}{3}100\right) = u\left(\frac{98 + 100}{3}\right) = u\left(\frac{198}{3}\right) = u(66) \text{ if and only if } u(\cdot) \text{ is concave.}$$

But we know that $\frac{2}{3}u(49) + \frac{1}{3}u(100)$ is the expected utility from the case of no-insurance purchase, and $u(66)$ is the expected utility in case Robert purchases the insurance. Therefore, Jensen's inequality says that Robert will purchase the insurance if and only if $u(\cdot)$ is concave. Since $u''(x) = \alpha(\alpha - 1)x^{\alpha-2}$, $u(x)$ is concave if and only if $\alpha - 1 < 0$, or $\alpha < 1$. This proves the desired point.

Problem 5. Bertrand Competition in discrete increments (64 points) (Note: This problem resembles one on last year's exam, but it's not the same, read carefully) Consider a variant of the Bertrand model of competition with two firms that we covered in class. The difference from the model in class is that prices are not a continuous variable, but rather a discrete variable. Prices vary in multiples of 1 cent. Firms can charge prices of 0, .01, .02, .03,... etc. The profits of firm i are

$$\pi_i(p_i, p_j) = \begin{cases} (p_i - c_i) D(p_i) & \text{if } p_i < p_j \\ (p_i - c_i) D(p_i) / 2 & \text{if } p_i = p_j. \\ 0 & \text{if } p_i > p_j. \end{cases}$$

Demand is extremely inelastic, that is, $D(p) = Q$ for all p . Both firms have the same marginal cost $c_1 = c_2 = c$, and the marginal cost c is a multiple of 1 cent. (The firm can charge $c - .01$, c , $c + .01$, $c + .02$, etc.) Consider first the case in which the two firms move simultaneously (as we did in class), and apply the Nash Equilibrium concept.

1. Write down the definition of Nash Equilibrium as it applies to this game, that is, with p_i as the strategy of player i and $\pi_i(p_i, p_j)$ as the function that player 1 maximizes. Provide both the formal definition and the intuition. Do not substitute in the expression for π_i . (8 points)
2. Show that $p_1^* = p_2^* = c$ (that is, marginal cost pricing) is a first Nash Equilibrium of this game. (8 points)
3. Show that $p_1^* = p_2^* = c + .01$ is a second Nash Equilibrium of this game. (8 points)
4. (Harder) Can you find another Nash Equilibrium (you need to prove that it is a Nash Equilibrium) [Hint: The peculiar feature of this setup is that the firm can only charge prices that are multiples of 1 cent] Why does it matter that demand is inelastic? (10 points)
5. Now, we change the setup in just one way. The game is now played sequentially, that is, firm 1 moves first, and firm 2 follows after observing the price choice of firm 1. We apply Subgame Perfection to solve this game, and therefore start from the last period, from the choice of player 2. Player 2's strategy will be a function of Player 1's price p_1 . Find the best response for player 2 as a function of p_1 , that is, find $p_2^*(p_1)$. (10 points)
6. Now let's continue with the backward induction and go back to player 1. Player 1 anticipates the best response of player 2 and chooses the price p_1 that will yield the highest profit. What is this price p_1^* ? To simplify the solution, assume that player 2 responds to a price of $c + .02$ by also setting price $c + .02$ (8 points)
7. Write down the subgame perfect equilibrium. How does it differ from the set of Nash Equilibria of the simultaneous game? (8 points)
8. Can you conjecture how the solution of the dynamic Bertrand game will differ if firms can set price continuously? (4 points)

Solution to Problem 5.

1. A set of price $p^* = (p_1^*, p_2^*)$ is a Nash Equilibrium if

$$\pi_i(p_i^*, p_j^*) \geq \pi_i(p_i, p_j^*) \text{ for all } p_i \text{ and for all players } i = 1, 2.$$

(We denote by p_j the payoff of the other player when considering player i).

2. We apply the definition at point 1 for player 1 first. It has to be the case that $\pi_1(c, c) \geq \pi_1(p, c)$ for all prices p . Since $\pi_1(c, c) = (c - c) D(c) / 2 = 0$, we need to show $0 \geq \pi_1(p, c)$ for all p . Consider a p higher than c . In this case, $\pi_1(p, c) = 0$, which satisfies the Nash Equilibrium definition. Consider now a p lower than c . In this case, $\pi_1(p, c) = (p - c) D(p) < 0$. Again, $0 < \pi_1(p, c)$. We have shown that it is optimal for player 1 to play $p_1 = c$. We can repeat the same proof for player 2 just substituting 2 for 1. Therefore $p_1^* = p_2^* = c$ is a Nash Equilibrium.

3. A second equilibrium is $p_1^* = p_2^* = c + .01$, that is, both firms charge one cent above marginal cost. We apply the definition of Nash equilibrium for player 1 first. It has to be the case that $\pi_1(c + .01, c + .01) \geq \pi_1(p, c + .01)$ for all prices p . Since $\pi_1(c + .01, c + .01) = (c + .01 - c)Q/2 = .005Q > 0$, we need to show $.005Q \geq \pi_1(p, c + .01)$ for all p . Consider a p higher than $c + .01$. In this case, $\pi_1(p, c) = 0 < .005Q$. This is not a profitable deviation. Consider now a p lower than $c + .01$. In this case, $\pi_1(p, c) = (p - c)D(p)$. This amount is 0 for $p = c$ and is negative for $p < c$. In both cases, the price yields lower profits than charging $c + .01$. We have shown that it is optimal for player 1 to play $p_1 = c + .01$. We can repeat the same proof for player 2 just substituting 2 for 1. Therefore $p_1^* = p_2^* = c + .01$ is a Nash Equilibrium.
4. Another equilibrium is $p_1^* = p_2^* = c + .02$, that is, both firms charge two cents above marginal cost. We apply the definition of Nash equilibrium for player 1 first. It has to be the case that $\pi_1(c + .02, c + .02) \geq \pi_1(p, c + .02)$ for all prices p . Since $\pi_1(c + .02, c + .02) = (c + .02 - c)Q/2 = .01Q > 0$, we need to show $.01Q \geq \pi_1(p, c + .02)$ for all p . Consider a p higher than $c + .02$. In this case, $\pi_1(p, c + .02) = 0 < .01Q$. This is not a profitable deviation. Consider now a p equal to, or lower than, c . In this case, $\pi_1(p, c + .02) = (p - c)Q$. This amount is 0 for $p = c$ and is negative for $p < c$. In both cases, the price yields lower profits than charging $c + .02$. It remains to show that charging $p = c + .01$ is not a profitable deviation. Notice that $\pi_1(c + .01, c + .02) = (c + .01 - c)Q = .01Q$, which is a payoff equal to the equilibrium payoff. Since ties do not destroy Nash equilibria, this is also not a profitable deviation. (Notice that here it is key that demand is inelastic. If demand were downward sloping, demand would be higher for $p = c + .01$ than at $p = c + .02$, which would destroy the equilibrium) We have shown that it is optimal for player 1 to play $p_1 = c + .02$ if player 2 plays $p_2 = c + .02$. We can repeat the same proof for player 2 just substituting 2 for 1. Therefore $p_1^* = p_2^* = c + .02$ is a Nash Equilibrium.
5. We distinguish several cases:
 - (a) Player 1 chose a price $p_1 > c + .02$. Then the best response is $p_2^*(p_1) = p_1 - .01$, that is, player 2 undercuts player 1 by exactly one cent. The profit for player 2 from choosing $p_2^* > p_1$ is 0. The profit from choosing $p_2^* = p_1$ is $(p_1 - c)Q/2$. The profit from choosing a price lower than p_1 is $(p_2 - c)Q$. Clearly, among the prices lower than p_1 , the most attractive is $p_1 - .01$, that is, undercutting by just one cent. The profit from undercutting by just one cent is $(p_1 - .01 - c)Q$, which is higher than $(p_1 - c)Q/2$ as long as $p_1 - c > .02$, as is the case here.
 - (b) Player 1 chooses price $p_1 = c + .02$. Then the best response is either $p_2^*(p_1) = c + .02$ or $p_2^*(p_1) = c + .01$, that is, player 2 undercuts player 1 by exactly one cent or sets the price equal. This part follows essentially the same argument as in point 4, and it follows from the fact that player 2 is indifferent between the two responses.
 - (c) Player 1 chooses price $p_1 = c + .01$. The best response here is $p_2^*(p_1) = p_1 = c + .01$. The profit from choosing $p_2^* > p_1$ is 0. The profit from choosing $p_2^* = p_1$ is $.005Q$. The profit from choosing a price lower than p_1 is $(p_2 - c)Q$, which is either 0 or negative.
 - (d) Player 1 chooses price $p_1 = c$. The best response here is either $p_2^*(p_1) = p_1 = c$ or any $p_2^*(p_1) > c$. The profit from choosing $p_2^* > p_1$ is 0. The profit from choosing $p_2^* = p_1$ is also 0. The profit from choosing a price lower than p_1 is negative.
 - (e) Player 1 chooses price $p_1 < c$. The best response here is any $p_2^*(p_1) > p_1$. The profit from choosing $p_2^* > p_1$ is 0. The profit from choosing $p_2^* = p_1$ or a price lower than p_1 is negative.
6. We consider it in steps. We show that the most profitable strategy for player 1 is to charge $p_1^* = c + .02$.
 - (a) Player 1 chose a price $p_1 > c + .02$. Then the best response of player 2 is $p_2^*(p_1) = p_1 - .01$, leading to player 1 being undercut and earning zero profits.
 - (b) Player 1 chooses price $p_1 = c + .02$. Then the best response of player 2 is either $p_2^*(p_1) = c + .02$ or $p_2^*(p_1) = c + .01$. We assume that the first is the case, leading to profits for player 1 of $.02Q/2 = .01Q$.
 - (c) Player 1 chooses price $p_1 = c + .01$. The best response here of player 2 is $p_2^*(p_1) = p_1 = c + .01$, leading to profit $.01Q/2 = .005Q$.

- (d) Player 1 chooses price $p_1 = c$. The best response here of player 2 is either $p_2^*(p_1) = p_1 = c$ or any $p_2^*(p_1) > p$, leading in any case to 0 profit.
- (e) Player 1 chooses price $p_1 < c$. The best response here of player 2 is any $p_2^*(p_1) > p$, and player 1 would make negative profits.

Among all these possibilities, player 1 earns the highest profits charging $p_1^* = c + .02$. Notice that this depends crucially from how we decide to solve the tie for player 1. Had we solved the tie the other way, player 1 would have then found it optimal to choose $p_1^* = c + .01$.

7. The subgame perfect equilibrium strategies are $p_1^* = c + .02$ for player 1 and for player 2

$$p_2^*(p_1) = \begin{cases} p_1 - .01 & \text{if } p_1 > c + .02 \\ c + .02 & \text{if } p_1 = c + .02 \\ c + .01 & \text{if } p_1 = c + .01 \\ \text{any } p_2 \text{ such that } p_2 > c & \text{if } p_1 = c \\ \text{any } p_2 \text{ such that } p_2 > p_1 & \text{if } p_1 < c. \end{cases}$$

Notice that the equilibrium strategy for player 2 is complicated, since we have to write the best response as a function of what player 1 did. (Strictly speaking, since there are multiple choices for $p_2^*(p_1)$ for the cases $p_1 = c$ and $p_1 < c$, there is an infinite number of equilibria). [Not required: The other set of equilibria where we break the indifference differently is $p_1^* = c + .01$ for player 1 and

$$p_2^*(p_1) = \begin{cases} p_1 - .01 & \text{if } p_1 > c + .02 \\ c + .01 & \text{if } p_1 = c + .02 \\ c + .01 & \text{if } p_1 = c + .01 \\ \text{any } p_2 \text{ such that } p_2 > c & \text{if } p_1 = c \\ \text{any } p_2 \text{ such that } p_2 > p_1 & \text{if } p_1 < c. \end{cases}$$

] The main difference is that in the dynamic game fewer equilibrium outcomes survive. So the outcome $p_1^* = p_2^* = c$, which is an equilibrium in the static game, is not an equilibrium in the dynamic one. In a way, in the dynamic game the first firm is able to enforce the better equilibrium for itself, the one with the price highest above c , that is, $p = c + .02$. This intuition is like in the Stackelberg case. Unlike in Stackelberg, being a first-mover does not bring advantages to firm 1, firm 1 and 2 obtain the same payoff in equilibrium in all the equilibria.

8. If prices could be set continuously in a dynamic Bertrand game, firms could not sustain equilibria like $p_1^* = p_2^* = c + .02$ or $p_1^* = p_2^* = c + .01$, because firm 2 would slightly undercut the offer of firm 1 if firm 1 were to offer any price $p > c$. So the only equilibrium outcome in the dynamic Bertrand game with continuous prices is $p_1^* = p_2^* = c$. So with continuous prices, having a dynamic vs. static game does not make a difference.