Economics 101A
(Lecture 22)

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Outline

1. Game Theory II
2. Oligopoly: Cournot
3. Oligopoly: Bertrand

1 Game Theory II

- Penalty kick in soccer (matching pennies)

Kicker \ Goalie	L	R
L	0, 1	1, 0
R	1, 0	0, 1

  - Kicker kicks left with probability k
  - Goalie kicks left with probability g
  - utility for kicker of playing L :

$$\begin{aligned} U_K(L,\sigma) &= gU_K(L,L) + (1-g)\,U_K(L,R) \\ &= (1-g) \end{aligned}$$

  - utility for kicker of playing R :

$$\begin{aligned} U_K(R,\sigma) &= gU_K(R,L) + (1-g)\,U_K(R,R) \\ &= g \end{aligned}$$

- Optimum?
  - $L \succ R$ if $1 - g > g$ or $g < 1/2$
  - $R \succ L$ if $1 - g < g$ or $g > 1/2$
  - $L \sim R$ if $1 - g = g$ or $g = 1/2$

- Plot best response for kicker

- Plot best response for goalie

- Nash Equilibrium is:
  - fixed point of best response correspondence
  - crossing of best response correspondences

2 Oligopoly: Cournot

- Nicholson, Ch. 14, pp. 524-530 (*better* than Ch. 14, pp. 418–419, 421–422, 9th)

- Back to oligopoly maximization problem

- Assume 2 firms, cost $c_i(y_i) = cy_i$, $i = 1, 2$

- Firms choose simultaneously quantity y_i

- Firm i maximizes:

$$\max_{y_i} p(y_i + y_{-i}) y_i - cy_i.$$

- First order condition with respect to y_i:

$$p'_Y\left(y_i^* + y_{-i}^*\right) y_i^* + p - c = 0,\ i = 1, 2.$$

- Nash equilibrium:
  - y_1 optimal given y_2;
  - y_2 optimal given y_1.
- Solve equations:

$$p'_Y(y_1^* + y_2^*)\,y_1^* + p - c = 0 \text{ and}$$

$$p'_Y(y_2^* + y_1^*)\,y_2^* + p - c = 0.$$

- Cournot -> Pricing above marginal cost

3 Oligopoly: Bertrand

- Cournot oligopoly: firms choose quantities

- Bertrand oligolpoly: firms first choose prices, and then produce quantity demanded by market

- Market demand function $Y(p)$

- 2 firms

- Profits:

$$\pi_i(p_i, p_{-i}) = \begin{cases} (p_i - c)\,Y(p_i) & \text{if} \quad p_i < p_{-i} \\ (p_i - c)\,Y(p_i)/2 & \text{if} \quad p_i = p_{-i} \\ 0 & \text{if} \quad p_i > p_{-i} \end{cases}$$

- First show that $p_1 = c = p_2$ is Nash Equilibrium

- Does any firm have a (strict) incentive to deviate?

- Check profits for Firm 1

- Symmetric argument for Firm 2

- Second, show that this equilibrium is unique.

- For each of the next 5 cases at least on firm has a profitable deviation

- Case 1. $p_1 > p_2 > c$

- Case 2. $p_1 = p_2 > c$

- Case 3. $p_1 > c \geq p_2$

- Case 4. $c > p_1 \geq p_2$

- Case 5. $p_1 = c > p_2$

- Only Case 6 remains: $p_1 = c = p_2$, which is Nash Equilibrium

- It is unique!

- Notice:

- To show that something is an equilibrium –> Show that there is *no* profitable deviation

- To show that something is *not* an equilibrium –> Show that there is *one* profitable deviation

- Surprising result of Bertrand Competition
- Marginal cost pricing
- Two firms are enough to guarantee perfect competition!
- Realistic? Price wars between PC makers

4 Next lecture

- Auctions
- Dynamic Games
- Stackelberg duopoly