

Economics 101A

(Lecture 9)

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Outline

1. Expenditure Minimization II
2. Slutsky Equation
3. Complements and substitutes
4. Do utility functions exist?

1 Expenditure minimization II

- Nicholson, Ch. 4, pp. 127-132 (109–113, 9th) + Ch. 5, pp. 151-154

- Solve problem **EMIN** (minimize expenditure):

$$\begin{aligned} \min p_1x_1 + p_2x_2 \\ \text{s.t. } u(x_1, x_2) \geq \bar{u} \end{aligned}$$

- Pick budget set which is tangent to indifference curve

- Optimum coincides with optimum of Utility Maximization!

- Formally:

$$h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u}))$$

- Expenditure function is expenditure at optimum
- $e(p_1, p_2, \bar{u}) = p_1 h_1(p_1, p_2, \bar{u}) + p_2 h_2(p_1, p_2, \bar{u})$
- $h_i(p_i)$ is *Hicksian or compensated demand* function
- Is h_i always decreasing in p_i ? Yes!
- Graphical proof: moving along a convex indifference curve
- (For non-convex indifferent curves, still true)

- Using first order conditions:

$$L(x_1, x_2, \lambda) = p_1x_1 + p_2x_2 - \lambda(u(x_1, x_2) - \bar{u})$$

$$\frac{\partial L}{\partial x_i} = p_i - \lambda u'_i(x_1, x_2) = 0$$

- Write as ratios:

$$\frac{u'_1(x_1, x_2)}{u'_2(x_1, x_2)} = \frac{p_1}{p_2}$$

- MRS = ratio of prices as in utility maximization!
- However: different constraint $\implies \lambda$ is different

- Example 1: Cobb-Douglas utility

$$\begin{aligned} \min & p_1 x_1 + p_2 x_2 \\ \text{s.t.} & x_1^\alpha x_2^{1-\alpha} \geq \bar{u} \end{aligned}$$

- Lagrangean =

- F.o.c.:

- Solution: $h_1^* =$, $h_2^* =$

- $\partial h_i^* / \partial p_i < 0$, $\partial h_i^* / \partial p_j > 0$, $j \neq i$

2 Slutsky Equation

- Nicholson, Ch. 5, pp. 155-158 (135–138, 9th)
- Now: go back to Utility Max. in case where p_2 increases to $p'_2 > p_2$
- What is $\partial x_2^*/\partial p_2$? Decompose effect:
 1. Substitution effect of an increase in p_i
 - $\partial h_2^*/\partial p_2$, that is change in EMIN point as p_2 decreases
 - Moving along an indifference curve
 - Certainly $\partial h_2^*/\partial p_2 < 0$

2. Income effect of an increase in p_i

- $\partial x_2^*/\partial M$, increase in consumption of good 2 due to increased income
- Shift out a budget line
- $\partial x_2^*/\partial M > 0$ for normal goods, $\partial x_2^*/\partial M < 0$ for inferior goods

- $h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u}))$
- How does the Hicksian demand change if price p_i changes?

$$\frac{dh_i}{dp_i} = \frac{\partial x_i^*(\mathbf{p}, e)}{\partial p_i} + \frac{\partial x_i^*(\mathbf{p}, e)}{\partial M} \frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_i}$$

- What is $\frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_i}$? Envelope theorem:

$$\begin{aligned} \frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_i} &= \frac{\partial}{\partial p_i} [p_1 h_1^* + p_2 h_2^* - \lambda(u(h_1^*, h_2^*, \bar{u}) - \bar{u})] \\ &= h_i^*(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p, \bar{u})) \end{aligned}$$

- Therefore

$$\frac{\partial h_i(\mathbf{p}, \bar{u})}{\partial p_i} = \frac{\partial x_i^*(\mathbf{p}, e)}{\partial p_i} + \frac{\partial x_i^*(\mathbf{p}, e)}{\partial M} x_1^*(p_1, p_2, e)$$

- Rewrite as

$$\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_i} = \frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} - x_1^*(p_1, p_2, M) \frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}$$

- Important result! Allows decomposition into substitution and income effect

- Two effects of change in price:

1. Substitution effect negative: $\frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} < 0$

2. Income effect: $-x_i^*(p_1, p_2, M) \frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}$

- negative if good i is normal $(\frac{\partial x_i^*(\mathbf{p}, M)}{\partial M} > 0)$

- positive if good i is inferior $(\frac{\partial x_i^*(\mathbf{p}, M)}{\partial M} < 0)$

- Overall, sign of $\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_i}$?

- negative if good i is normal

- it depends if good i is inferior

- Example 1 (ctd.): Cobb-Douglas. Apply Slutsky equation

- $x_i^* = \alpha M/p_i$

- $h_i^* =$

- Derivative of Hicksian demand with respect to price:

$$\frac{\partial h_i(\mathbf{p}, \bar{u})}{\partial p_i} =$$

- Rewrite h_i^* as function of m : $h_i(\mathbf{p}, v(\mathbf{p}, M))$

- Compute $v(\mathbf{p}, M) =$

- Substitution effect:

$$\frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} =$$

- Income effect:

$$-x_i^*(p_1, p_2, M) \frac{\partial x_i^*(\mathbf{p}, M)}{\partial M} =$$

- Sum them up to get

$$\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_i} =$$

- It works!

3 Complements and substitutes

- Nicholson, Ch. 6, pp. 182-187 (161–166, 9th)
- How about if price of another good changes?
- Generalize Slutsky equation

- Slutsky Equation:

$$\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_j} = \frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} - x_j^*(p_1, p_2, M) \frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}$$

- Substitution effect

$$\frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} > 0$$

for $n = 2$ (two goods). Ambiguous for $n > 2$.

- Income effect:

$$-x_j^*(p_1, p_2, M) \frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}$$

- negative if good i is normal
- positive if good i is inferior

- How do we define complements and substitutes?

- Def. 1. Goods i and j are **gross substitutes** at price \mathbf{p} and income M if

$$\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_j} > 0$$

- Def. 2. Goods i and j are **gross complements** at price \mathbf{p} and income M if

$$\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_j} < 0$$

- Example 1 (ctd.): $x_1^* = \alpha M/p_1$, $x_2^* = \beta M/p_2$.

- Gross complements or gross substitutes? Neither!

- Notice: $\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_j}$ is usually different from $\frac{\partial x_j^*(\mathbf{p}, M)}{\partial p_i}$

- Better definition.

- Def. 3. Goods i and j are **net substitutes** at price \mathbf{p} and income M if

$$\frac{\partial h_i^* (\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} = \frac{\partial h_j^* (\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} > 0$$

- Def. 4. Goods i and j are **net complements** at price \mathbf{p} and income M if

$$\frac{\partial h_i^* (\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} = \frac{\partial h_j^* (\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} < 0$$

- Example 1 (ctd.): $h_1^* = \bar{u} \left(\frac{\alpha p_2}{1-\alpha p_1} \right)^{1-\alpha}$

- Net complements or net substitutes? Net substitutes!

4 Do utility functions exist?

- Preferences and utilities are theoretical objects
- Many different ways to write them
- How do we tie them to the world?
- Use actual choices – revealed preferences approach

- Typical economists' approach. Compromise of:
 - realism
 - simplicity
- Assume a class of utility functions (CES, Cobb-Douglas...) with free parameters
- Estimate the parameters using the data

5 Next Lectures

- Applications:
 - Labor Supply
 - Intertemporal Choice
 - Economics of Altruism