Economics 101A
(Lecture 16)

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Outline

1. Cost Minimization: Example II
2. Cost Curves and Supply Function
3. One-step Profit Maximization

1 Cost Minimization: Example II

- Continue example above: $y = f(L, K) = AK^{\alpha}L^{\beta}$
- Cost minimization:

$$\begin{array}{l} \min wL + rK \\ s.t. AK^{\alpha}L^{\beta} = y \end{array}$$

- Solutions:
  - Optimal amount of labor:

$$L^*(r, w, y) = \left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{-\frac{\alpha}{\alpha+\beta}}$$

  - Optimal amount of capital:

$$\begin{aligned} K^*(r, w, y) &= \frac{w}{r}\frac{\alpha}{\beta}\left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{-\frac{\alpha}{\alpha+\beta}} = \\ &= \left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} \end{aligned}$$

- Check various comparative statics:

  - $\partial L^*/\partial A < 0$ (technological progress and unemployment)

  - $\partial L^*/\partial y > 0$ (more workers needed to produce more output)

  - $\partial L^*/\partial w < 0$, $\partial L^*/\partial r > 0$ (substitute away from more expensive inputs)

- Parallel comparative statics for K^*

- Cost function

$$
\begin{aligned}
c(w,r,y) &= wL^*(r,w,y) + rK^*(r,w,y) = \\
&= \left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}} \begin{bmatrix} w\left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{-\frac{\alpha}{\alpha+\beta}} + \\ +r\left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} \end{bmatrix}
\end{aligned}
$$

- Define $B := w\left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{-\frac{\alpha}{\alpha+\beta}} + r\left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}$

- Cost-minimizing output choice:

$$
\max py - B\left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}}
$$

- First order condition:

$$p - \frac{1}{\alpha+\beta}\frac{B}{A}\left(\frac{y}{A}\right)^{\frac{1-(\alpha+\beta)}{\alpha+\beta}} = 0$$

- Second order condition:

$$-\frac{1}{\alpha+\beta}\frac{1-(\alpha+\beta)}{\alpha+\beta}\frac{B}{A^2}\left(\frac{y}{A}\right)^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}}$$

- When is the second order condition satisfied?

- Solution:
  - $\alpha + \beta = 1$ (CRS):
    - S.o.c. equal to 0
    - Solution depends on p
    - For $p > \frac{1}{\alpha+\beta}\frac{B}{A}$, produce $y^* \to \infty$
    - For $p = \frac{1}{\alpha+\beta}\frac{B}{A}$, produce any $y^* \in [0, \infty)$
    - For $p < \frac{1}{\alpha+\beta}\frac{B}{A}$, produce $y^* = 0$

- $\alpha + \beta > 1$ (IRS):
  * S.o.c. positive
  * Solution of f.o.c. is a minimum!
  * Solution is $y^* \to \infty$.
  * Keep increasing production since higher production is associated iwth higher returns

- $\alpha + \beta < 1$ (DRS):
  - s.o.c. negative. OK!
  - Solution of f.o.c. is an interior optimum
  - This is the only "well-behaved" case under perfect competition
  - Here can define a supply function

2 Cost Curves

- Nicholson, Ch. 10, pp. 330-338; Ch. 11, pp. 365-369 (Ch. 8, pp. 220–228; Ch. 9, pp. 256–259, 9th)

- Marginal costs $MC = \partial c/\partial y \rightarrow$ Cost minimization
$$p = MC = \partial c\left(w, r, y\right)/\partial y$$

- Average costs $AC = c/y \rightarrow$ Does firm break even?
$$\begin{aligned}\pi &= py - c\left(w, r, y\right) > 0 \text{ iff} \\ \pi/y &= p - c\left(w, r, y\right)/y > 0 \text{ iff} \\ c\left(w, r, y\right)/y &= AC < p\end{aligned}$$

- **Supply function.** Portion of marginal cost MC above average costs.(price equals marginal cost)

- Assume only 1 input (expenditure minimization is trivial)
- **Case 1.** Production function. $y = L^{\alpha}$
  - Cost function? (cost of input is w):
$$c(w, y) = wL^{*}(w, y) = wy^{1/\alpha}$$
  - Marginal cost?
$$\frac{\partial c(w, y)}{\partial y} = \frac{1}{\alpha} wy^{(1-\alpha)/\alpha}$$
  - Average cost $c(w, y)/y$?
$$\frac{c(w, y)}{y} = \frac{wy^{1/\alpha}}{y} = wy^{(1-\alpha)/\alpha}$$

- **Case 1a.** $\alpha > 1$. Plot production function, total cost, average and marginal. Supply function?

- **Case 1b.** $\alpha = 1$. Plot production function, total cost, average and marginal. Supply function?

- **Case 1c.** $\alpha < 1$. Plot production function, total cost, average and marginal. Supply function?

- **Case 2.** *Non-convex technology.* Plot production function, total cost, average and marginal. Supply function?

- **Case 3.** *Technology with setup cost.* Plot production function, total cost, average and marginal. Supply function?

2.1 Supply Function

- Supply function: $y^* = y^* (w, r, p)$

- What happens to y^* as p increases?

- Is the supply function upward sloping?

- Remember f.o.c:

$$p - c'_y (w, r, y) = 0$$

- Implicit function:

$$\frac{\partial y^*}{\partial p} = -\frac{1}{-c''_{y,y} (w, r, y)} > 0$$

as long as s.o.c. is satisfied.

- Yes! Supply function is upward sloping.

3 One-step Profit Maximization

- Nicholson, Ch. 11, pp. 374-380 (Ch. 9, pp. 265–270, 9th)

- One-step procedure: maximize profits

- Perfect competition. Price p is given
  - Firms are small relative to market
  - Firms do not affect market price p_M
  - Will firm produce at $p > p_M$?
  - Will firm produce at $p < p_M$?
  - $\Longrightarrow p = p_M$

- Revenue: $py = pf\left(L, K\right)$

- Cost: $wL + rK$

- Profit $pf\left(L, K\right) - wL - rK$

- Agent optimization:

$$\max_{L,K} pf(L,K) - wL - rK$$

- First order conditions:

$$pf'_L(L,K) - w = 0$$

and

$$pf'_K(L,K) - r = 0$$

- Second order conditions? $pf''_{L,L}(L,K) < 0$ and

$$\begin{aligned} |H| &= \begin{vmatrix} pf''_{L,L}(L,K) & pf''_{L,K}(L,K) \\ pf''_{L,K}(L,K) & pf''_{K,K}(L,K) \end{vmatrix} = \\ &= p^2 \left[f''_{L,L} f''_{K,K} - \left(f''_{L,K} \right)^2 \right] > 0 \end{aligned}$$

- Need $f''_{L,K}$ not too large for maximum

- Comparative statics with respect to to p, w, and r.

- What happens if w increases?

$$\frac{\partial L^*}{\partial w} = -\frac{\begin{vmatrix} -1 & pf''_{L,K}(L,K) \\ 0 & pf''_{K,K}(L,K) \end{vmatrix}}{\begin{vmatrix} pf''_{L,L}(L,K) & pf''_{L,K}(L,K) \\ pf''_{L,K}(L,K) & pf''_{K,K}(L,K) \end{vmatrix}} < 0$$

and

$$\frac{\partial L^*}{\partial r} =$$

- Sign of $\partial L^*/\partial r$ depends on $f''_{L,K}$.

4 Next Lecture

- Aggregation
- Market Equilibrium
- Comparative Statics of Equilibrium