Econ 101A
Midterm 1

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Th 28 February 2008

Do not turn page unless instructed to.

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You have approximately 1 hour and 20 minutes to answer the questions in the midterm. Vikram will collect the exams at 11.00 sharp. Show your work, and good luck!

Problem 1. Three-Good Cobb-Douglas. (50 points) Seung likes three goods: x_1, x_2, and x_3. He is aware that in Econ 101A we only use two goods, but he is too attached to all of them to let go of one. He maximizes the utility function

$$u(x_1, x_2, x_3) = x_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3},$$

with $0 < \alpha_i < 1$ for $i = 1, 2, 3$. The consumption good x_i has price p_i (for $i = 1, 2, 3$) and the individual has total income M.

1. Compute the marginal utility of consumption with respect to good x_1, $\partial u(x_1, x_2, x_3)/\partial x_1$. (2 points)
2. What is the limit of the marginal utility for $x_1 \to 0$ and for $x_1 \to \infty$? Interpret the economic intuition behind this feature of this utility function. (5 points)
3. Write the budget constraint. (3 points)
4. Write the maximization problem of Seung. Seung wants to achieve the highest utility subject to the budget constraint. Write down the boundary constraints for x_1, x_2, x_3, and neglect them for now. (3 points)
5. Assuming that the budget constraint holds with equality, write down the Lagrangean and derive the first order conditions with respect to x_1, x_2, x_3, and λ. (5 points)
6. Solve for x_1^* as a function of the prices p_1, p_2, p_3, the income M, and the parameters α_1, α_2, and α_3. [Hint: combine the first and second first-order condition, then combine the first and third first-order condition, and finally plug in budget constraint] Similarly solve for x_2^* and x_3^*. (6 points)
7. True or false? Show your work: "Cobb-Douglas preferences have the feature that the share of money spent on each good does not depend on the income, or on prices" (6 points)
8. Are the boundary conditions for x_1, x_2, and x_3 satisfied? (2 points)
9. Is good x_1 a normal good (for all values of M and prices p_i)? Compute and answer. (4 points)
10. Plot the implied demand function for x_1, that is plot x_1 as a function of p_1. (Put p_1 on the y axis and x_1 on the x axis) (4 points)
11. Is good x_1 a Giffen good? Why did you know this already from the answer to question 9? (5 points)
12. Are goods x_1 and x_2 gross complements, gross substitutes, or neither? Define and answer. (5 points)

Problem 2. (26 points)

1. Angela has utilty function $u(x_1, x_2) = 2x_1 + 2x_2$.
   (a) Plot the indifference curves of Angela. What kind of goods do they represent? (4 points)
   (b) Using the plot you did, find the utility-maximing solution x_1^*, x_2^* for prices $p_1 = 1$, $p_2 = 2$ and income M. Argue the steps you make. (8 points)
2. Kim has utility function $u(x_1, x_2) = \min(x_1, 2x_2)$
   (a) Plot the indifference curves of Kim. What kind of goods do they represent? (4 points)
   (b) Are the preferences represented by this utility function monotonic? Define. (4 points)
   (c) Are they strictly monotonic? Define. (6 points)