Econ 101A – Midterm 2
Th 10 April 2008.

You have approximately 1 hour and 20 minutes to answer the questions in the midterm. I will collect the exams at 11.00 sharp. Show your work, and good luck!

Problem 1. Manager with Expected Utility (25 points)**.** A manager decides how much effort e to put in managing a company. We assume $e \in [0, 1]$. The effort determines the probability of success, and hence the manager's pay. With probability e, a project succeeds and the manager gets paid $W > 0$. With probability $1 - e$, the project fails and the manager is fired (and hence is paid 0). The manager has initial wealth w and utility over consumption $U(c)$, with $U'(c) > 0$ for all c. The manager consumes all the income, including the initial wealth, after she is paid the salary (possibly zero). The cost of effort is $-e^2/2$.

1. Write the expected utility maximization of the manager with respect to e. (5 points)
2. Write the first order condition and derive the solution e^*. (5 points)
3. What is the effect of an increase in salary W on the optimal effort e^*? Interpret the intuition. (5 points)
4. What is the effect of an increase in the initial wealth w on the optimal effort e^*? Interpret the intuition, relating to what you know about attitude toward risk. (10 points)

Solution of Problem 1.

1. The manager maximizes
$$\max_e eU(w + W) + (1 - e) U(w) - e^2/2.$$
2. The first order condition is
$$U(w + W) - U(w) - e = 0$$
and hence $e^* = U(w + W) - U(w)$.
3. The effect of W is
$$\frac{\partial e^*}{\partial W} = U'(w + W) > 0.$$
4. The effect of w is
$$\frac{\partial e^*}{\partial w} = U'(w + W) - U'(w).$$
Whether this term is positive or negative depends on whether the marginal utility function $U'()$ is increasing or not, that is, whether $U()$ is convex or not. If $U()$ is convex (respectively, concave), that is, the agent is risk-seeking (risk-averse), then an increase in wealth will raise (lower) the effort. The intuition for the risk-aversion case is that an agent with diminishing marginal utility gets less and less incentive to work hard the wealthier s/he is because of the diminishing utility of wealth.

Problem 2. Profit Maximization with Taxes (96 points) We consider the market for widgets, which is characterized by the aggregate (inverse) demand function $p(X) = a - bX$, where X is the total quantity of widgets demanded in the market. The cost function of each company is $c(y) = cy^{\alpha}$, with $c > 0$.

1. Assume perfect competition (that is, the price p of the widget is given) and set up the profit maximization of each firm. (5 points)

2. Solve for the profit-maximizing level of production $y^*(p)$ (that is, the supply function) using the first-order condition. (5 points)

3. Check the second-order conditions. Under what values of the parameters are they satisfied? Interpret the economic significance of this parameter restriction. (5 points)

4. Now consider the conditions for the market equilibrium. For points 4-7, assume that the parameters are such that the second order conditions are satisfied. Assume that N firms produce and write the equation for the equilibrium price p^* that equates aggregate supply and demand. Do not attempt to solve explicitly for p^*. (5 points)

5. Now introduce taxation. Denote by p the price inclusive of tax that the consumer pays, and by $p - t$ the price net of tax that accrues to the producer. Rewrite the market equilibrium condition. (5 points) [Note: If you get stuck here, you can move on to point 8]

6. Use the implicit function theorem to compute $\partial p^*/\partial t$. (5 points)

7. Show that $0 < \partial p^*/\partial t < 1$. What does it mean economically? (5 points)

8. From now on, consider the case $\alpha = 1$. Assume for now no taxes ($t = 0$). What is the economic interpretation of this special case? (5 points)

9. Characterize mathematically the supply function $y^*(p)$ for an individual company, and plot it. (5 points)

10. Solve for the market equilibrium price p^* and total quantity X^* produced in the market, assuming no taxes. [Note: A figure may help you here] (5 points)

11. Compute the aggregate consumer surplus and the producer surplus. You can help yourself with a plot of market demand and supply. [Note: Do not worry here about the distinction between compensated and uncompensated demand] (5 points)

12. Solve now for the market equilibrium price p^* and total quantity X^* produced in the market assuming a tax t. How much of the tax do the companies pass through in prices (that is, what is $\partial p^*/\partial t$)? (5 points)

13. Compute the consumer surplus and the producer surplus for the case with a tax t. (5 points)

14. Compute the total surplus adding the consumer surplus, the producer surplus, and the revenue raised with the taxes. (5 points)

15. Using what you just found, argue that the deadweight loss from taxation is given by $t^2/2b$. Do you agree or disagree with the following statement? Provide a precise argument: 'Small taxes are not very distortionary, but large taxes can induce very large deadweight loss' (8 points)

16. Consider now the case of monopoly. Keep assuming $\alpha = 1$ and a tax t. [Note: As above, the price p denotes the price that consumers pay inclusive of taxes, and $p - t$ is the price that producers receive] Set-up the maximization problem and solve for the profit-maximizing price p^*_M and quantity X^*_M. (8 points)

17. How much of the tax does the firm pass through to the consumer (that is, what is $\partial p^*/\partial t$)? Compare to the case of perfect competition for $\alpha = 1$. (5 points)

18. Compute the producer surplus and the consumer surplus and compare to the case of perfect competition for $\alpha = 1$. (5 points)

Solution of Problem 2.

1. Each firm maximizes

$$\max_y py - c(y) = py - cy^\alpha$$

2. The first-order condition for y^* is

$$p - \alpha c y^{\alpha-1} = 0,$$

leading to the solution $y^* = (p/\alpha c)^{1/(\alpha-1)}$ (so long as α is different from zero)

3. The second-order condition for y^* is

$$-\alpha(\alpha-1) c y^{\alpha-2} < 0.$$

This condition is satisfied for $\alpha > 1$. This condition is the condition of decreasing returns to scale.

4. To solve for the equilibrium price, we must equate supply Y and demand X. First, remember that total production is $Y = Ny^* = N(p/\alpha c)^{1/(\alpha-1)}$. One way to do this is to substitute the solution for Y^* from the production decision into the demand function:

$$p - a + bN\left(\frac{p}{\alpha c}\right)^{1/(\alpha-1)} = 0$$

5. The market equilibrium with taxes is

$$p - a + bN\left(\frac{p-t}{\alpha c}\right)^{1/(\alpha-1)} = 0$$

6. We can compute

$$\begin{aligned}\frac{\partial p^*}{\partial t} &= -\frac{-\frac{bN}{\alpha-1}\left(\frac{1}{\alpha c}\right)^{1/(\alpha-1)}(p-t)^{-\frac{\alpha}{\alpha-1}}}{1+\frac{bN}{\alpha-1}\left(\frac{1}{\alpha c}\right)^{1/(\alpha-1)}(p-t)^{-\frac{\alpha}{\alpha-1}}} = \\ &= \frac{\frac{bN}{\alpha-1}\left(\frac{1}{\alpha c}\right)^{1/(\alpha-1)}(p-t)^{-\frac{\alpha}{\alpha-1}}}{1+\frac{bN}{\alpha-1}\left(\frac{1}{\alpha c}\right)^{1/(\alpha-1)}(p-t)^{-\frac{\alpha}{\alpha-1}}}\end{aligned}$$

7. Both numerator and denominator are positive (remember $\alpha > 1$), hence $\partial p^*/\partial t > 0$. Since the denominator equals the numerator plus a positive number (1), the ratio has to be smaller than 1, hence $\partial p^*/\partial t < 1$. This shows that the incidence of the tax is partially on the consumer and partially on the producer.

8. The case $\alpha = 1$ corresponds to the case of constant returns to scale with no fixed costs.

9. In this case the marginal cost function $c'(y)$ equals a constant c. Further, in order for the company to break even, the following condition has to be satisfied:

$$py - cy \geq 0,$$

which implies that the company produces only if $p \geq c$. Hence, the supply function is

$$y^*(p) = \begin{cases} \infty & \text{if} \quad p > c \\ \text{any } y \in [0,\infty) & \text{if} \quad p = c \\ 0 & \text{if} \quad p < c \end{cases}$$

10. Given that the supply function is horizontal, the price is determined by the marginal cost c: $p^* = c$. It follows that the quantity produced X^* is $c = a - bX^*$, or $X^* = (a-c)/b$.

11. The producer surplus is zero, since each firm produces at marginal cost and makes zero profits: $\pi = p^* y^* - cy^* = cy^* - cy^* = 0$. The consumer surplus is the area below the demand curve and above the price. This area is a triangle with base X^* and height $(a - p^*) = (a - c)$. Hence,

$$CS = \frac{(a-c)}{b}\frac{(a-c)}{2} = \frac{(a-c)^2}{2b}.$$

12. In the case with a tax t, the price that the producer is now willing to accept in order to break even is $c + t$. Hence, the price in equilibrium is $p^* = c + t$. The quantity produced X^* is $c + t = a - bX^*$, or $X^* = (a - c - t)/b$.

13. The producer surplus is still zero. The consumer surplus is now

$$CS = \frac{(a-c-t)}{b}\frac{(a-c-t)}{2} = \frac{(a-c-t)^2}{2b}.$$

14. The revenue raised with the taxes is $t * X^* = t(a-c-t)/b$. Hence, the total surplus is

$$\begin{aligned} TS &= \frac{(a-c-t)^2}{2b} + \frac{(a-c-t)t}{b} = \frac{(a-c-t)(a-c-t+2t)}{2b} = \\ &= \frac{(a-c-t)(a-c+t)}{2b} = \frac{(a-c)^2 - t^2}{2b} \end{aligned}$$

15. The deadweight loss of taxation is the difference between the total surplus before taxes are introduced, that is, $(a-c)^2/2b$, and the total surplus with taxes, in the above expression. The difference is $-t^2/2b$. This implies that the deadweight loss grows not linearly with the tax, but with the square power. Hence, while small taxes will induce a small distortion, large taxes can induce a large distortion.

16. With a tax t, a monopolist maximizes

$$\max_X (p(X) - t)X - cX = (a - bX - t)X - cX,$$

with first-order conditions

$$a - 2bX^* - t - c = 0,$$

which implies $X^* = (a-c-t)/2b$. The price is

$$p = a - bX^* = a - b\frac{a-c-t}{2b} = \frac{a+c+t}{2}.$$

17. The firm passes through only 1/2 of the tax ($\partial p^*/\partial t = 1/2$), which is half as much as in the case of perfect competition. This is because the monopolist finds it profit-maximizing to bear half of the burden. If it were to pass it on fully, it would lower demand 'too much'.

18. The producer surplus is larger than in the case of perfect competition (where it was zero):

$$\pi = (p^* - c - t)X^* = \left(\frac{a-t-c}{2}\right)\left(\frac{a-t-c}{2b}\right) = \frac{(a-t-c)^2}{4b}.$$

The consumer surplus is smaller than in perfect competition:

$$CS = \frac{a-t-c}{2b}\frac{a - \left(\frac{a+c+t}{2}\right)}{2} = \frac{a-t-c}{2b}\frac{a-c-t}{4} = \frac{(a-t-c)^2}{8b}.$$

The total surplus [not required] is

$$TS = \frac{(a-t-c)^2}{4b} + \frac{(a-t-c)^2}{8b} = \frac{3(a-t-c)^2}{8b}$$

Notice that this is smaller than in the case of perfect competition.