Handout for Piecemeal-Preferences Seminars At Two Great State Universities

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Highly simplified setting: Life arrives at us as a series of decision opportunities,

$$\{f_{11}, ..., f_{G1}; f_{12}, ..., f_{G2}; ...; f_{1M}, ..., f_{GM}\},$$

where each f_{ij} is a probability distribution over possible choice sets the person will face, $L_{ij} \subseteq \triangle(R^K)$, of probability distributions over K-dimensional vectors of consumption, $(c_{1ij}, ..., c_{Kij}) \in R^K$. Primary and simplest example: $K = 1 \Longrightarrow$ so each element of L_{ij} is a lottery over $.

Realizations of $\{f_{ij}\}$ and $\{L_{ij}\}$ all statistically independent of everything else, and $\{f_{ij}\}_{i=1,...,G}$ and $\{L_{ij}\}_{i=1,...,G}$ are i.i.d. for all j. [We can allow some non-independence by interpreting some of the dimensions as state-contingent.]

Realizations of $\{f_{ij}\}$, choices $l_{ij} \in L_{ij}$, and realizations of uncertainty in l_{ij} together determine grand outcome $o \in \triangle(R^K)$ putting weight on all realizations $(\sum_{ij} c_{1ij}, \sum_{ij} c_{2ij}, ..., \sum_{ij} c_{Kij}) \in R^K$.

I'll consider preferences $u(o)$ over grand outcomes $o \in \triangle(R^K)$ — very much allowing for non-EU preferences and (notation notwithstanding) non-utility preferences.

Each L_{ij} in the support of each f_{ij} contains a default choice, $l^*_{ij} \in L_{ij}$, that is implemented if not over-ridden.

Piecemeal preferences: A mapping $\rho : L_{ij} \rightarrow \triangle(L_{ij})$ such that for all $L_{ij} = L_{i'j'}$, $\rho(L_{ij}) = \rho(L_{i'j'})$.

Definition: Piecemeal preferences ρ are *constrained optimal* (COPP) if there do not exist piecemeal preferences ρ' such that (abusing notation) $u(\rho') > u(\rho)$.

Definition: Piecemeal preferences ρ are *myopic* (MYPP) if for all L_{ij}, person chooses $l_{ij} =$ argmax $_{l_{ij} \in L_{ij}}$ $u(l_{ij})$.

For any two distributions $f, g \in \triangle(R^K)$, let $\mu_f, \mu_g \in R^K$ be their means, and let $f^n, g^n \in \triangle(R^K)$ be n independent plays of the gambles f and g.

Definition: $u : \triangle(R^K) \to R$ is *limit average complete*, quasi-convex, and monotonic (LAC) if for all closed, convex, finte $Q \subseteq R^K$ there exists complete, monotonic, quasi-convex (or whatever) $v : Q \to R$ such that, for all $f, g \in \triangle(R^K)$ with $\mu_f, \mu_g \in Q$, there exists $\overline{n}$ such that for all $n > \overline{n}$, $u(f^n) > u(g^n)\ iff\ v(\mu_f) > v(\mu_g)$.

For all $L \subseteq \triangle(R^K)$, for all $\widehat{\alpha} \in \triangle^K$, for all $\epsilon > 0$, let $Z(L, \widehat{\alpha}, \epsilon) \subseteq \triangle(L)$ be the set of (possibly stochastic) choices from L that Max $E\{\sum_{k=1}^{K} \alpha_k c_k\}$ for some $\alpha \in \triangle^K$. Then say that ρ is $\alpha^*, \epsilon - LEV$ (ρ is *Linear Expected Value*) for $\alpha^* \in \triangle(R^K), \epsilon > 0$ if for all L_{ij} with positive probability in environment $\rho(Lij) \in Z(L, \alpha^*, \epsilon)$.

For environment f, $M > 0$, and preferences u, let $\rho^{u,f,M}_{COPP}$ be the corresponding COPP. (I am writing and notating as if this is unique, but I don't think this matters at all for the results.)

First Fundamental Theorem of COPP: For all LAC u, for all f (with bounded support in R^K), there exists $\alpha^* \in \triangle(R^K)$ such that for all $\epsilon > 0$, there exists $\overline{M}$ such that for all $M > \overline{M}$, $\rho^{u,f,M}_{COPP}$ is α^*, ϵ-LEV.

Second Fundamental Theorem of COPP: I think something like this is truish, but not clear how to formalize in a conceptually clear way: In limit as $M \to \infty$, $\rho^{u,f,M}_{COPP}$ becomes close to first-best optimal.