Competition and Price Variation when Consumers are Loss Averse*

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Abstract We introduce consumer loss aversion into the Salop (1979) model of price competition. Firms face uncertain costs of production, and after observing their own cost realizations simultaneously set prices. A consumer derives "gain-loss utility" from comparing the purchase price and her satisfaction with the acquired product to her recent expectations regarding the same variables, and dislikes losses more than she likes same-sized gains. Consumers' sensitivity to losses in money makes competition more intense at higher relative to lower market prices, reducing or eliminating price variation in a number of senses consistent with observed pricing regularities. For any joint cost distribution, an equilibrium in which all firms always charge the same, "focal," price for differentiated products exists if and only if no two possible cost realizations differ by more than a given constant. When firms face common stochastic costs, in any symmetric equilibrium the markup is strictly decreasing in cost, and the price may be constant over parts or all of the range of possible costs. Even if firms have asymmetric idiosyncratic shocks, if those have overlapping supports and sufficiently dense distributions, any equilibrium is focal. Because a change in consumers' price sensitivity affects competition more when margins are high, the above tendencies are stronger in less competitive industries. Finally, because the loss in product satisfaction she would suffer makes a consumer difficult to attract from a competitor, loss aversion decreases competition and increases prices.

Keywords: Reference-dependent utility, focal price, wage rigidity, kinked demand curve, countercyclical markups, (seemingly) collusive behavior.

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1 Introduction

This paper investigates how consumer loss aversion affects oligopolistic competition. Our results provide a novel perspective on a number of widely studied regularities related to reduced price variation between products and over time. First, we identify circumstances under which non-collusive firms subject to industry-wide or asymmetric idiosyncratic shocks charge an identical and sticky, "focal," price for differentiated products.[1] This challenges the blanket interpretation of focal prices as a collusive device. Second, our theory can explain why a retailer selling a number of differentiated products often sets the same, "uniform," price for them.[2] Third, our model predicts that prices are less volatile in more concentrated (or less competitive) industries.[3] Fourth, it yields a smaller pass-through of cost changes than the corresponding standard model, which, combined with evidence that marginal costs move procyclically, can be viewed as predicting countercyclical markups.[4] Finally, our theory can be reinterpreted as a model of an imperfectly competitive labor market, giving rise to results on wage stickiness analogous to those above.

Our results derive from profit-maximizing firms' reaction to consumers who—beyond the instrumental value of money—dislike paying a price that exceeds their expectation of the purchase price. Because consumers' sensitivity to such losses makes demand more responsive at higher than at lower parts of the market price distribution, competition is more intense at higher prices, decreasing or eliminating price variation. And because a change in the responsiveness of demand affects competition more when margins are high, this tendency is stronger in more concentrated industries.

Section 2 presents our model, which builds on the Salop (1979) model of imperfect competition. In Salop's model, a consumer's "taste" for a product is drawn uniformly from the circumference of a circle, where in addition n products are located equidistant from each other. A consumer's

[1] For evidence of sticky prices, see Kashyap (1995), Slade (1999), and Chevalier, Kashyap and Rossi (2000). The pricing of cigarette companies in Hungary (analyzed by Heidhues and Kőszegi (2005)) is an example of focal pricing. Another is provided by Kashyap (1995, page 263) who states that "[...] Orvis price-setters told me that they were matching Bean's price moves for Hudson Bay blankets."

[2] For evidence, see McMillan (2004) and Einav and Orbach (2005).

[3] For evidence, see for example Mills (1927), Means (1935), Wilder, Willimans and Singh (1977), Carlton (1986, 1989) and Geroski (1992).

[4] For a review of evidence that marginal costs move procyclically while markups are countercyclical, see Rotemberg and Woodford (1999).

utility from or "satisfaction" with a product is decreasing in the product's distance from her ideal variety. In addition, she suffers additive disutility from paying the product's price.

Using the framework developed by Kőszegi and Rabin (forthcoming), we append these classical preferences to account for loss aversion. A consumer compares outcomes in money and product satisfaction to relevant "reference points" in each dimension, with losses being more painful than equal-sized gains are pleasant.[5] Crucially, we posit that a consumer's reference point is her lagged expectations (i.e. full probabilistic beliefs) about the outcomes she is going to get. For example, if she had been expecting to spend \$14.99 on a Britney Spears CD—her favorite music—she experiences a sensation of loss if she pays \$18.98 instead. But she also experiences a loss if she buys a Madness CD for \$14.99.[6] And if she expected to pay either \$14.99 or \$19.99, paying \$18.98 generates a mixture of two feelings, a loss of \$3.99 and a gain of \$1.01, with the weight on the loss increasing in the probability with which she expected to pay \$14.99. Expectations are determined endogenously, in a *personal equilibrium*, by the requirement that the stochastic outcome implied by optimal behavior conditional on expectations be identical to expectations.

Consumers are in the market to purchase a single product. We assume that the n products are sold by n different firms, but our results on how the products are priced would be identical if some firms had multiple non-neighboring products—so that predictions of focal pricing below also correspond to uniform pricing of goods sold by the same firm. The firms face uncertain costs of production, and, after privately observing their cost realizations, simultaneously set prices. We define a *market equilibrium* as a situation where all firms maximize profits given other firms' behavior and consumer expectations, and consumers play a personal equilibrium correctly anticipating the distribution of prices. A *focal-price equilibrium* is a market equilibrium where all firms always charge the same *focal price* p^*.

[5] Loss aversion is a well-established facet of individual behavior. In laboratory selling and purchasing decisions, subjects ask for a higher price when selling a good than they are willing to pay when offered the opportunity to buy the same good, even though the two roles are randomly allocated between subjects. This probably occurs because subjects construe giving up the object they have just acquired as a loss, and are very sensitive to this loss (Kahneman, Knetsch, and Thaler 1990, 1991). In monetary gambles, loss aversion is reflected in people's reluctance to accept small favorable lotteries, presumably because they are more afraid of the possibility of loss than they are thrilled about a somewhat larger gain (Kahneman and Tversky 1979, Rabin 2000).

[6] Actual prices are taken on September 4th, 2005, from www.amazon.com. \$14.99 is the retail price of both CD's (and numerous others), while \$18.98 is a typical list price.

Since characterizing the full set of market equilibria for arbitrary cost distributions seems very difficult, we consider several more specific questions. After analyzing consumer behavior in Section 3, we begin in Section 4 by establishing conditions under which a focal-price equilibrium exists. Suppose a consumer had expected to pay p^* with probability one. Because she would then assess buying at a price greater than p^* as a loss in money, and buying at a price lower than p^* merely as a gain in money, demand is much more responsive to unilateral price increases from p^* than to unilateral price decreases from p^*. Due to this kink in residual demand, for a range of cost levels p^* is the optimal price to charge. Thus, for any distribution of costs such that any two cost realizations of any two firms are within a given distance, a focal-price equilibrium exists. In particular, this is true when one firm has higher costs than another in all states of the world.

We next establish two properties of focal-price equilibria. First, a focal-price equilibrium is more likely to exist in more concentrated industries. In more concentrated industries, the markup and hence the value of a marginal consumer is higher, so that the asymmetric demand responsiveness from above yields a greater range of costs for which p^* is the optimal price to charge.

Second, we prove that loss aversion increases prices: a focal price is higher than the entire support of possible equilibrium prices without loss aversion. A consumer attracted to a firm from a competitor receives a price cut in exchange for buying a product further from her ideal than she expected. Since she is more sensitive to the latter loss than to the former gain, she is difficult to attract, decreasing competition between firms.

In Section 5, we turn to considering how firms respond to "industry-wide" cost shocks. We assume that firms have common stochastic marginal costs, and characterize all market equilibria with symmetric pricing strategies. Our main result is that markups are strictly decreasing in cost, and under conditions we identify precisely, the price is focal in any symmetric equilibrium. Using the empirical observation that costs are strongly procyclical, this means that markups are countercyclical. Intuitively, if a consumer expected stochastic prices, the sense of loss from comparing the realized price to lower possible ones would make her demand more responsive at higher than at lower prices in the anticipated distribution. Hence, competition is fiercer at higher prices, decreasing markups. In addition, it may be that competition at higher prices is tougher to an extent that is inconsistent with firms raising their price in response to cost increases at all. Over such a region of costs, only a sticky price is possible. Because a change in demand responsiveness affects competition more when markups are higher, an increase in industry concentration raises both the tendency for prices to be sticky and the countercyclicality of markups when prices are not sticky.

Finally, in Section 6, we consider idiosyncratic cost shocks, and derive conditions under which any market equilibrium is a focal-price equilibrium even when firms have different cost distributions. We establish our result in two parts. On the one hand, we provide conditions under which a firm sets a deterministic price in any equilibrium. The intuition is closely related to the previous paragraph. If the consumer expected a firm's prices to be stochastic, then—at least to the extent that she expected to purchase this firm's product—her demand would be more responsive at higher than at lower prices within that distribution. If the firm's costs do not vary much, in contradiction to equilibrium it wants to deviate either by decreasing high prices or by increasing low prices. On the other hand, we show that if there is no firm all of whose possible costs are strictly higher than all possible costs of another firm, then firms never set different deterministic prices in equilibrium. Suppose firm H always charges the high price p_H and firm L always charges the low price $p_L < p_H$, and consider the producers' incentives when firm H has cost no greater than firm L. In this situation, firm H has a lower demand and a higher markup than firm L, which makes it more profitable for firm H to attract new demand. Furthermore, because paying p_H is assessed as a loss relative to paying p_L, firm H faces at least as responsive demand as firm L. This means that in contradiction to market equilibrium, either firm H will want to lower its price or firm L will want to raise its price.

Our results—derived in a noncooperative static game—provide a counterpoint to the widespread informal view that reduced price variation in concentrated industries is the result of collusive behavior.[7] Athey, Bagwell, and Sanchirico (2004) formalize the collusion view in a repeated price-setting game, showing that there is a focal-price equilibrium enforced by the threat of price war in case of a price change. While we believe this explanation is part of the story, our results provide an

[7] This view is expressed, for instance, in Carlton (1989, pages 914-915) and Knittel and Stango (2003, pages 1704-1705). In addition, focal prices and reduced price variability seemed to have raised suspicions of collusion in some antitrust cases, such as the recent SONY/BMG merger case in Europe.

intuitive and in some ways more complete alternative explanation for focal pricing. The attempt to maintain a collusive scheme explains why each firm should set a *fixed* price, but it does not explain why they should set the *same* price. Indeed, in collusion models this result is commonly *assumed* in some form through imposing Bertrand competition or symmetry in firms' situations or strategies. Similarly, if each colluding oligopolist sells multiple products, there is no reason for the cartel to set the same price for all these products. Furthermore, in a repeated-game setting there are typically equilibria that are more efficient than focal pricing (Athey and Bagwell 2001, Aoyagi 2002). In contrast, in many circumstances our model predicts focal and uniform pricing as the unique possible outcome and does so without imposing symmetry between the firms. To emphasize this point, we argue in Section 7 that our results are robust to a number of modifications of our model, including demand asymmetries and shocks, heterogeneity in consumer preferences, and endogenous determination of the number of firms.

2 Setup

Our model is based on the Salop (1979) model of differentiated-products competition, and is behaviorally equivalent to it when there is no loss aversion ($\lambda = 1$ below). To introduce consumer loss aversion, we build on the methods of Kőszegi and Rabin (forthcoming) and Heidhues and Kőszegi (2004). By basing the reference-dependent "gain-loss utility" on classical "intrinsic utility" and fully endogenizing the reference point, this approach allows little arbitrariness in modeling choices.

2.1 Consumer Behavior

We begin by describing intrinsic utility as taken from the Salop (1979) model. There is a mass one of consumers with differing tastes indexed by $\chi \in [0, 1]$, where χ is uniformly distributed on a circle with perimeter one. There are $n \geq 2$ products denoted $y_1, \ldots, y_n$ on the same circle equidistant from each other, where we normalize $y_1 = 0$, $y_2 = 1/n$, and so on. A consumer can buy at most one product, and to avoid unenlightening extra notation, we assume her utility from not consuming is negative infinity, so that she always does buy a product.[8] For any two locations χ and y, let $d(\chi, y)$ be their distance on the circle. The intrinsic utility of consumer χ from buying product y at price p is $v - t \cdot d(\chi, y) - p$, where $k_1 = v - t \cdot d(\chi, y)$ is her intrinsic utility from or "satisfaction" with the good and $k_2 = -p$ is her intrinsic disutility from paying its price. Like previous authors, we interpret χ as the consumer's "ideal variety," which yields intrinsic benefit v. If a person consumes a product y different from her ideal, she suffers disutility equal to $t \cdot d(\chi, y)$, where t is a measure of the intrinsic differentiation between varieties of the product.

Based on the above intrinsic utility, we formulate consumers' reference-dependent utility function. For a riskless consumption outcome $k = (k_1, k_2)$ and riskless reference point $r = (r_1, r_2)$, total utility $u(k|r)$ is composed of two additive terms: intrinsic utility introduced above, and reference-dependent "gain-loss utility" equal to $\mu(k_1 - r_1) + \mu(k_2 - r_2)$. To capture loss aversion, we assume that μ is two-piece linear with a slope of 1 for gains and a slope of $\lambda > 1$ for losses.

This specification incorporates two key assumptions. First, the consumer evaluates gains and losses in the two dimensions, satisfaction and money, separately. For example, if she consumes a good that costs more but is closer to her taste than expected, this is assessed as a loss in money and a gain in satisfaction (instead of, for example, a single gain or loss depending on total intrinsic utility relative to expectations). This seems to reflect the underlying psychology for most applications, and is consistent with much experimental evidence commonly interpreted in terms of loss aversion.[9] Second, the consumer's sense of gain or loss is directly related to the intrinsic value of the changes in question—it is more painful to lose something we value (e.g. $100) than to lose something we do not (e.g. a paper clip).

Since we assume below that the reference point is expectations, we extend the above utility

[8] Our results would be identical if consumers had an option of not buying, but v below was sufficiently high (or costs and product differentiation sufficiently low) so that no consumer would take advantage of this option in equilibrium.

If some consumers did not consume in equilibrium, some firms would effectively be monopolists in the market for their product. Since our interest in this paper is in the effect of loss aversion in competitive environments, we rule out such possibilities. Our earlier paper (Heidhues and Kőszegi 2004) analyzes monopolistic pricing with loss-averse consumers.

[9] Specifically, it is key to explaining the endowment effect and other observed regularities in riskless trades. If gains and losses were defined over the value of an entire transaction, loss aversion would have no implications for such trades.

function to allow for the reference point to be a probability measure Γ over $\mathbb{R}^2$:

$$U(k|\Gamma) = \int_r u(k|r)d\Gamma(r). \tag{1}$$

This formulation captures the idea that in evaluating k, the consumer compares it to each possibility in the reference lottery. For example, if she had been expecting to pay either \$15 or \$20 for her favorite CD, paying \$17 feels like a loss of \$2 relative the possibility of paying \$15, and like a gain of \$3 relative to the possibility of paying \$20. In addition, the higher the probability with which she expected to pay \$15, the more important is the loss in the overall experience.

Having specified the utility function, we turn to modeling behavior. Suppose that the consumer has a prior $F \in \Delta(\mathbb{R}^n_+)$ on the non-negative price vectors she might face, and also has a prior on χ. Her decision of which good to buy is made after observing the realized χ and the realized price vector, and is described by the strategy $\sigma : [0,1] \times \mathbb{R}^n_+ \to \{1, \ldots, n\}$. The premise of our model is that the consumer's preferences, and therefore also the strategy σ, depend on lagged rational expectations about outcomes. Specifically, the consumer's reference point is the distribution $\Gamma_{\sigma,F}$ induced by σ, F, and her prior over χ, over vectors (k_1, k_2) of product satisfaction and expenditure. To deal with the resulting interdependence between behavior (σ) and expectations ($\Gamma_{\sigma,F}$), we use the personal-equilibrium concept from Kőszegi and Rabin (forthcoming), which requires the behavior generating expectations to be optimal given expectations:

Definition 1. σ is a personal equilibrium for the price distribution F if

$$\sigma(\chi, p) \ \in \ \text{argmax}_{i\in\{1,\ldots,n\}} \ U\left(v - t \cdot d(\chi, y_i)\ ,\ p_i \ | \ \Gamma_{\sigma,F}\right) \text{ for all } \chi \in [0,1] \text{ and } p \in \mathbb{R}^n.$$

2.2 Market Equilibrium

The timing of our full market model is illustrated in Figure 1. Consumers first form the expectations regarding consumption outcomes that later determine their reference point. Next, firms observe their cost realizations and simultaneously set prices, aiming to maximize expected profits.[10] Then, consumers observe their ideal varieties and the realized market prices, and purchase a good.

[10] We make this assumption to capture our impression that firms display reference-dependent preferences far less than consumers do, and to isolate the effect of consumer loss aversion on market outcomes.

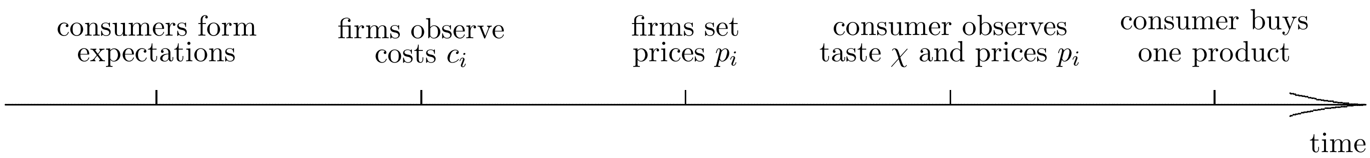

Figure 1: Timing of Full Market Model

We assume that consumers' prior on χ is identical to the population distribution, $U[0,1]$. Since it gives rise to the same distribution of consumption outcomes, an equivalent model is one in which consumers know their ideal variety, but are uncertain about the positioning of the firms' products.[11] A situation where consumers have a very good idea about their ideal variety as well as the products offered corresponds to a narrow or even degenerate prior distribution on χ, and yields a different model. In Section 7, we argue that our results in Sections 4 and 5 carry over to this case unchanged, and that reasonable specifications also yield our results in Section 6.

For most of the paper, we assume that the n available products are produced by n different firms, with firm i producing good i, y_i. In Section 7, we argue that as long as each product is owned by exactly one firm and no firm owns neighboring products, our results on how products are priced extend unchanged to situations where some or all firms produce multiple products.

Firms' costs are jointly distributed according to Θ on the set $\prod_{i=1}^{n}[\underline{c}_i, \overline{c}_i]$, where $[\underline{c}_i, \overline{c}_i]$ is the smallest closed interval containing the support of firm i's cost distribution. Let $\underline{c} = \min\{\underline{c}_i\}$ and $\overline{c} = \max\{\overline{c}_i\}$. Denote firm i's pricing function by $P_i : [\underline{c}_i, \overline{c}_i] \to \mathbb{R}_+$.[12] Let $P = (P_1, \ldots, P_n)$ be the vector of pricing strategies, F_P the market price distribution induced by P and Θ, and $P_{-i}(c_{-i}) = (P_j(c_j))_{j \neq i}$ the price vector of firms other than i.

Definition 2. The strategy profile $\{P, \sigma\}$ is a market equilibrium if

1. σ is a personal equilibrium for the price distribution F_P.

[11] More precisely, our model is identical to one in which each consumer knows her ideal variety, and firm 1's location is drawn from a uniform distribution on the circle, with the n firms still equidistant from each other.

[12] Without loss of generality, we restrict attention to equilibria in strategies that are pure *conditional on* the realized cost. In the settings we consider in this paper, no firm would randomize in equilibrium.

2. For each i and $c_i \in [\underline{c}_i, \bar{c}_i]$,

$$P_i(c_i) \in \text{argmax}_{p_i \in \mathbb{R}_+} \ (p_i - c_i) \cdot \text{Prob}\left[\sigma(p_i, P_{-i}(c_{-i}), \chi) = i \ \mid \ c_i\right].$$

Our definition of market equilibrium extends Bayesian Nash equilibrium to allow for reference effects in consumer behavior. A market equilibrium needs to satisfy two conditions. First, consumers play a personal equilibrium given the correctly forecasted price distribution. For notational simplicity, our definition imposes that there is a single representative consumer, or all consumers play the same personal equilibrium. We argue in Section 7 that this does not affect our results. Second, each firm at each cost realization plays a best response to other firms' pricing strategies, taking consumers' expectations (and hence reference point) as given.[13]

One of our goals in this paper is to investigate circumstances under which all firms charge the same price irrespective of their cost positions. We introduce a term for such a situation.

Definition 3. A market equilibrium is a focal-price equilibrium if there is a price p^* that all firms charge with probability one. In a focal-price equilibrium, p^* is the focal price.

To address what we believe is mostly a technical issue that arises in our model as well as the standard Salop model with cost asymmetry, we introduce a restricted class of market equilibria.

Definition 4. A market equilibrium is an interior equilibrium if with probability 1, all firms sell to a positive measure of consumers on each side of their location.

A priori, it is possible that a firm prices so low relative to its neighbor that it attracts all consumers between them. In the standard Salop model, at such a price level there is a discontinuity in the firm's demand, as it suddenly captures *all* consumers on the other side of its neighbor. In our model, both convex kinks and jumps in demand are possible. To avoid the difficulties of dealing with such situations in equilibrium, we follow previous models with cost uncertainty (e.g. Aghion and Schankerman 2004) and focus on (what we call) interior equilibria.

[13] This means that at the stage when firms' prices are chosen, these prices do not influence consumer expectations. Firms might be able to influence consumer expectations through public commitments to prices, advertising, or other marketing activities. Analyzing these motives is beyond the scope of this paper.

2.3 Consumption Examples

We provide a few examples of how our model fits various shopping scenarios.

Uncertainty about Taste: A consumer is looking to buy a new car. She knows what cars are available—she knows the products offered—but does not know which one leads to the most Fahrvergnügen. Once she test-drives the cars offered, she finds out each car's price (p_i) and how well it suits her (χ), and decides which one to buy. A similar shopping pattern applies to clothes, shoes, TV's, and many other goods consumers test to see whether they like it.

Unpredictability of Taste: There is a given set of beer brands available at entertainment establishments a person may visit. But she is uncertain as to which brew she will be in the mood (χ) for on a particular night. Once the evening comes, she learns her mood, the prices of the beers, and decides what to drink. The same pattern applies to many types of food and entertainment consumption, where a person's momentary taste is often unpredictable.

Uncertainty about products: A homeowner decides to buy a new fridge. She knows exactly what combination of features she would ideally like to have (she knows χ). But she does not know what is being offered in the marketplace. Upon visiting the store, she finds out what is available (she learns the products' locations and prices), and decides which fridge to buy.

In this example, the consumer knows her ideal variety, χ, but does not know the products offered in the market. In our formal model and the previous two examples, the converse is the case. But as we have emphasized above, the two variants of the theory are in fact equivalent.

Uncertainty about distance: While driving in her new car to visit an out-of-town friend who is waiting for her with refrigerated, ice-cold beer, a driver is looking to get gas. Though she knows the chains of gas stations out there, she is uncertain as to how conveniently they will be located. As in the previous example, the driver is uncertain about the distance between her ideal (most convenient) gas station and the actual gas stations.

2.4 A Reinterpretation to Labor Markets

While our formal theory is about pricing decisions, it has a reinterpretation to an important application, wage setting. A wage paid by a firm to an individual is for both parties of opposite sign as a price paid by the individual to the firm, and labor productivity is for a firm of opposite sign as a cost. With the signs of the cost and price variables flipped to become productivity and wage variables, therefore, our model becomes one of an imperfectly competitive labor market. In this interpretation, employees differ in what kind of job they would ideally like. These differences can derive, for instance, from their commuting preferences or family situation, or from nonpecuniary aspects of work. Employees always take exactly one job, and suffer disutility if it is far from their ideal one. Firms, which have positions at different points of employees' taste spectrum, are subject to labor-productivity shocks, and after observing their own productivity simultaneously offer wages. In Appendix A, we define this wage-setting game and its correspondence with a price-setting game formally, and observe that there is a one-to-one correspondence between market equilibria in the two games where the wage-setting function is obtained as a constant plus the negative of the price-setting function. Hence, although all of our results and discussion are in the language of price setting, appropriately restated they also apply to wage setting.

3 Consumer Behavior

In this section, we analyze how price changes affect the demand of loss-averse consumers with fixed expectations. The expression we derive will be crucial for our analysis of firm behavior. Suppose that—as is always the case for interior equilibria—for price vectors in a neighborhood of a realized price vector there is an indifferent consumer between firms 1 and 2 who partitions the interval between the firms into consumers buying good 1 and consumers buying good 2. Holding p_2 fixed, we solve for how the location of this indifferent consumer, and hence part of firm 1's demand, changes with p_1.

Suppose a consumer expects the distribution of her purchase price and the distribution of her acquired product's distance from ideal to be $F(\cdot)$ and $G(\cdot)$, respectively, and her realized taste is

$\chi \in [0, 1/n]$. Her utility from buying good 1 at price p_1 is then

$$\begin{aligned} u_1 &= v - \chi t - p_1 \\ &- \lambda \int_0^{p_1} (p_1 - p)\ dF(p) + \int_{p_1}^{\infty} (p - p_1)\ dF(p) \\ &- \lambda t \int_0^{\chi} (\chi - s)\ dG(s) + t \int_{\chi}^{1/n} (s - \chi)\ dG(s). \end{aligned} \tag{2}$$

The first line is the intrinsic utility from consuming good 1. The second line represents the consumer's gain-loss utility in money. To the extent that she expected to pay lower prices than p_1, paying p_1 feels like a loss. This is captured in the first term. But to the extent that she expected to buy at higher prices, paying p_1 feels like a gain. The third line is the consumer's gain-loss utility in product satisfaction. As with money, in as much as she expected to get a product closer to her taste, buying good 1 feels like a loss. But in as much as she expected to buy a less optimal product, she experiences a gain.

Similarly, the consumer's utility from buying good 2 at price p_2 is

$$\begin{aligned} u_2 &= v - ((1/n) - \chi)t - p_2 \\ &- \lambda \int_0^{p_2} (p_2 - p)\ dF(p) + \int_{p_2}^{\infty} (p - p_2)\ dF(p) \\ &- \lambda t \int_0^{(1/n)-\chi} (((1/n) - \chi) - s)\ dG(s) + t \int_{(1/n)-\chi}^{1/n} (s - ((1/n) - \chi))\ dG(s). \end{aligned} \tag{3}$$

Denote the indifferent consumer—for whom $u_1 = u_2$—by $x^+ \in [0, 1/n]$, and right and left limits by $\downarrow$ and $\uparrow$, respectively. The number x^+ is also firm 1's demand on the interval $[0, 1/n]$. Equations (3) and (4) are differentiable with respect to χ, and right and left differentiable with respect to p_1. Hence:

$$\left.\frac{dx^+}{dp_1}\right|_{\downarrow} = \frac{\left.\frac{\partial u_1}{\partial p_1}\right|_{\downarrow} - \left.\frac{\partial u_2}{\partial p_1}\right|_{\downarrow}}{\frac{\partial u_1}{\partial \chi} - \frac{\partial u_2}{\partial \chi}} = -\frac{1}{2t} \cdot \left[\frac{2 + (\lambda - 1)F(p_1)}{2 + \frac{\lambda - 1}{2}[G(x^+) + G(1/n - x^+)]}\right], \tag{4}$$

and the response to a price decrease is given by the expression where $F_{\uparrow}(p_1)$ replaces $F(p_1)$ above.

The price responsiveness of firm 1's demand derives partly from the sensitivity of the consumer's utility to changes in the purchase price. Paying a price p is experienced as a loss relative to lower prices in the expected price distribution, and as a gain relative to higher prices in that distribution. An increase (decrease) in p is therefore counted as a loss (avoided loss) to the extent that the consumer expected lower prices, and as a foregone gain (gain) to the extent that she expected higher prices. Due to this "comparison effect," demand responsiveness to price increases (decreases) is a continuously increasing function of the probability with which the consumer expected to pay lower prices, $F(p)$ ($F_{\uparrow}(p)$).

The comparison effect has two important implications we will use repeatedly in the paper. First, the residual demand curve is kinked at p if and only if the purchase-price distribution has an atom at p. Second, demand responsiveness is higher at higher prices in the purchase-price distribution.

More subtle than the effect of price itself is the effect of product satisfaction on the price responsiveness of demand. The consumers who switch to product 2 when firm 1 raises its price are approximately indifferent between a product at a distance x^+ from ideal and a product at a distance $1/n - x^+$ from ideal, and the responsiveness of demand is increasing in the number of these consumers. If consumers buying a product at x^+ or $1/n - x^+$ from optimal are very sensitive to a product's distance from ideal, small changes in the ideal variety induce large changes from the utility of choosing either product, so that few of them will be close to indifferent. And similarly to utility in the money dimension, a consumer's sensitivity to changes in the distance from ideal near x^+ ($1/n - x^+$) is increasing in $G(x^+)$ ($G(1/n - x^+)$), the probability with which she expected to buy a product closer to her taste. This gives rise to the denominator in Equation (4).

Firm 1's total demand is composed of its demand between firms 1 and 2 (analyzed above) and its demand between firms 1 and n. If $1 - x^-$ is the indifferent consumer between firms 1 and n, the right derivative of firm 1's total demand, $D(p_1, p_{-1})$, with respect to p_1 is

$$D_{1\downarrow}(p_1, p_{-1}) = -\frac{1}{2t} \cdot \left[\frac{2 + (\lambda - 1)F(p_1)}{2 + \frac{\lambda-1}{2}[G(x^+) + G(1/n - x^+)]} \right] - \frac{1}{2t} \cdot \left[\frac{2 + (\lambda - 1)F(p_1)}{2 + \frac{\lambda-1}{2}[G(x^-) + G(1/n - x^-)]} \right], \tag{5}$$

with $D_{1\uparrow}(p_1, p_{-1})$ being given by an expression in which $F_{\uparrow}(p_1)$ replaces $F(p_1)$ above. Whenever firms 2 and n set the same price p, we will denote the indifferent consumer on each side by $x =$

$x^+ = x^-$, and firm 1's demand as a function of its price by $D(\cdot, p)$.

4 Focal-Price Equilibria

We begin by establishing a necessary and sufficient condition for the existence of focal-price equilibria. A focal-price equilibrium can exist despite stochastic costs, and even when it is commonly known that firms have *different* (stochastic or deterministic) costs.

To derive the conditions under which a focal-price equilibrium exists, we solve for the cost levels c_1 for which firm 1 does not want to deviate from a focal price of p^*. In a market equilibrium, the consumer correctly anticipates that all prices will be p^*, so that she expects to spend p^* with probability 1 and to buy the product closest to her taste. This means that the distribution $G(\cdot)$ of the acquired product's distance from ideal is the uniform distribution on $[0, 1/(2n)]$.

Given these considerations, Equation (5) implies that $D_{1\downarrow}(p^*, p^*) = -1/t$. Using that $D(p^*, p^*) = 1/n$, so long as $(p^* - c_1)/t \geq 1/n$ firm 1 cannot benefit from raising its price locally. Similarly, since $D_{1\uparrow}(p^*, p^*) = -2/(t(1+\lambda))$, so long as $2(p^* - c_1)/(t(1+\lambda)) \leq 1/n$ firm 1 cannot benefit from lowering its price locally. Combining and rearranging these conditions, charging p^* is locally optimal if and only if

$$p^* - \frac{t}{n} \cdot \frac{1+\lambda}{2} \leq c_1 \leq p^* - \frac{t}{n}. \tag{6}$$

In the appendix, we show that when local deviations are unprofitable, non-local deviations are also unprofitable. Therefore:

Proposition 1. *A focal-price equilibrium exists if and only if*

$$\overline{c} - \underline{c} \leq \frac{\lambda - 1}{2} \cdot \frac{t}{n}.$$

Figure 2 depicts the residual demand curve firm 1 faces in our model and the standard Salop model when all competitors charge price p^*. While the demand curve in the standard model is linear, in ours it is kinked at p^*. Intuitively, a price decrease expands demand less than a price increase reduces demand because consumers are not as attracted by a gain in money as they dislike a loss in money. Since demand is kinked, there is a downward jump in marginal revenue at p^*, so that p^* is a (locally) optimal price for a range of marginal-cost levels.[14]

Proposition 1 has a number of important comparative-statics implications for when a focal-price equilibrium exists. A focal-price equilibrium is more likely to exist when consumer loss aversion (λ) is greater. The greater is λ, the greater is the difference between a consumer's sensitivity to price increases from p^* and price decreases from p^*. Hence, the greater is the downward jump in marginal revenue, and the greater is the range of cost levels for which p^* is the optimal price.

More excitingly, a focal-price equilibrium is more likely to exist when market power as measured by product differentiation relative to the number of firms (t/n) is greater. To develop an intuition for this result, suppose a firm has cost $\overline{c}$ and is just indifferent to raising its price from p^*. Then, due to a kink in demand, it strictly prefers not to decrease its price, and for a range of cost decreases it will have the same preference. We argue that this range is increasing in the markup $p^* - \overline{c}$ that makes the firm indifferent to price increases, so that it is larger in less competitive industries. With a higher markup, the value of a marginal consumer is higher, so the responsiveness of demand is more important in determining the firm's incentives to change its price. Hence, the lower responsiveness of demand to price decreases makes the firm very reluctant to cut its price, and it will not want to do so for a greater range of cost decreases.

In addition to identifying conditions under which a focal-price equilibrium exists, Inequality (6) determines what the focal price level can be. The following proposition states this level and compares it to prices in a model without loss aversion.

Proposition 2. *There is a focal-price equilibrium with focal price $p*$ if and only if*

$$\overline{c} + \frac{t}{n} \leq p^* \leq \underline{c} + \frac{t}{n} \cdot \frac{1+\lambda}{2}.$$

In the corresponding Salop model without loss aversion, the support of a firm's interior-Bayesian-Nash-equilibrium prices is bounded above by $\overline{c} + \frac{t}{n}$, and this bound can only be attained if the firm has realized cost $\overline{c}$.

[14] Figure 2 also shows that the demand curve is convex for price decreases from p^*. Hence, profits are not necessarily globally concave, so the fact that p^* is locally optimal does not guarantee that it is globally optimal. The proof of the proposition implies that the demand curve is not so convex that non-local price cuts are profitable.

Proposition 2 says that in a focal-price equilibrium, consumer loss aversion leads to increased prices: even at the *lowest* possible cost, a firm charges a higher price than it would in the standard model at the *highest* possible cost. For part of the intuition, consider a firm that tries to increase demand by lowering its price below those of other firms. The firm attracts some consumers whose tastes are closer to the neighboring firms' products and are compensated for this by the lower price, and who therefore consume a good that both costs and matches their taste less than expected. Since consumers are more sensitive to the loss in satisfaction than to the gain in money, lowering one's price attracts fewer of them than without loss aversion.

Now contrast the above with the firm's incentive to raise its price, which leads some of its consumers to choose other products. These consumers must either pay more than expected to obtain their favorite product, or make do with a less satisfactory product than they expected was possible. Either choice involves a loss, so the firm loses the same number of consumers as without loss aversion.[15] Since loss aversion decreases a firm's incentive to lower its price and leaves a firm's incentive to raise its price unchanged, it increases equilibrium prices.

Proposition 2 implies that if there is a focal-price equilibrium, there are generically multiple ones, with the set of possible focal prices being a closed interval. If consumers' expectation of the price increases from p to $p' > p$, the difference between paying p' and p turns from a loss to a foregone gain. Because this makes demand less responsive, firms are more willing to increase prices, within limits exactly matching the increased expectations.[16]

Beyond a theoretical possibility, our model predicts that focal-price equilibria can exist for calibrationally non-trivial amounts of cost variation. Assuming $\lambda = 3$, which corresponds to the conventional assumption of two-to-one loss aversion in observable choices, a focal-price equilibrium exists for cost variation $\overline{c} - \underline{c}$ up to t/n. Since by Proposition 2 the equilibrium markup lies in the interval $[t/n, 2t/n]$, the allowed cost variation is at least half the size of the markup.

[15] In fact, this is why in Figure 2, on the left of p^* the demand curve is tangent to the corresponding standard one.

[16] Of course, firms prefer higher equilibrium prices to lower ones, and therefore have a strong incentive to manage consumers' price expectations. Certain types of advertising and price-leadership behavior—which are outside our model—may partly serve this purpose.

5 Industry-Wide Cost Shocks

We turn from studying the existence and properties of focal-price equilibria to investigating market equilibria more generally. In this section we fully characterize symmetric equilibria when firms always have identical marginal costs, a case we view as a model of oligopolists' reponse to industry-wide cost shocks. We show that markups decline with cost in any market equilibrium, and that there may be regions where the price is sticky (unchanging in cost). Furthermore, markups decline faster with cost, and prices tend to be more sticky, in more concentrated industries.

Throughout this section, we assume that in all states of the world firms have the same marginal cost, and this marginal cost is continuously distributed according to Θ on $[\underline{c}, \bar{c}]$. We analyze the symmetric market equilibria of the resulting game, first establishing two basic properties:

Lemma 1. *Suppose firms have identical, continuously distributed marginal costs. In a symmetric market equilibrium, price is a continuous and non-decreasing function of marginal cost.*

To understand the lemma, take costs c and c' and corresponding prices p and $p' > p$, and suppose that residual demand is differentiable at both p and p'. Inframarginal demand is the same at the two prices. In addition, due to the comparison effect, consumers are more responsive to price changes at higher prices within the price distribution, where these changes are assessed more as changes in losses rather than as changes in gains. Hence, demand responsiveness is weakly greater at p' than at p. In order for firms' first-order conditions to be satisfied at both costs, therefore, c' must be greater than c and not be arbitrarily close to it.[17]

As a step toward a full analysis, we posit that for a cost c, $P(c)$ is not an atom of the market price distribution, and derive $P(c)$. Since in a symmetric equilibrium firms set identical prices in all states of the world, consumers always choose the product closest to their taste. Hence, as in Section 4, $G(\cdot)$ is the uniform distribution on the interval $[0, 1/(2n)]$. Furthermore, Equation (5) implies that the derivative of firm 1's demand exists at $P(c)$ and is equal to

$$D_1(P(c), P(c)) = -\frac{1}{t} \cdot \frac{2 + (\lambda - 1)F(P(c))}{1 + \lambda} = -\frac{1}{t} \cdot \frac{2 + (\lambda - 1)\Theta(c)}{1 + \lambda}, \tag{7}$$

[17] If the price distribution has atoms at p or p', so that residual demand is not differentiable, the argument still works by considering—instead of first-order conditions—incentives to lower the higher price as compared to incentives to raise the lower price.

where $F(P(c)) = \Theta(c)$ because $P(\cdot)$ is non-decreasing and $P(c)$ is not a pricing atom. For $P(c)$ to be a profit-maximizing choice, we must have

$$D(P(c), P(c)) + (P(c) - c)D_1(P(c), P(c)) = 0.$$

Substituting Equation (7), using that $D(P(c), P(c)) = 1/n$, and rearranging yields

$$P(c) = c + \frac{t}{n} \cdot \frac{2 + (\lambda - 1)}{2 + (\lambda - 1)\Theta(c)} \equiv \Phi(c). \tag{8}$$

Based on this formula, Proposition 3 characterizes all symmetric market equilibria. We will develop the implications of and intuition for the proposition in a number of steps below. Because the statement is somewhat cumbersome, the reader may want to delay looking at the proposition carefully until after considering some of the specific cases below.

Proposition 3. *Suppose firms have identical marginal costs distributed according to Θ on $[\underline{c}, \overline{c}]$. A pricing function $P : [\underline{c}, \overline{c}] \to \mathbb{R}$ is a symmetric-market-equilibrium pricing function if and only if all of the following are satisfied:*

1. *$P(\cdot)$ is continous and non-decreasing.*
2. *There are disjoint intervals $[f_1, f_1'], [f_2, f_2'], \dots \subset [\underline{c}, \overline{c}]$ such that $P(\cdot)$ is constant on all $[f_i, f_i']$ and not constant on any interval not contained in any $[f_i, f_i']$.*
3. *$P(c) = \Phi(c)$ for any $c \notin \cup_i [f_i, f_i']$.*
4. *$P(\underline{c}) \leq \Phi(\underline{c})$ and $P(\overline{c}) \geq \Phi(\overline{c})$.*

To start identifying the implications of Proposition 3 in specific cases, suppose that $\Phi(\cdot)$ is strictly increasing. A pricing function $P(c) = \Phi(c)$ is then consistent with Lemma 1, and because it generates no pricing atoms, our analysis above implies that no firm would have an incentive to deviate locally from it. In fact, in this case Proposition 3 implies:

Corollary 1. *Under the conditions of Proposition 3, if $\Phi(c)$ is strictly increasing, the unique symmetric market equilibrium has pricing strategies $P(c) = \Phi(c)$. Otherwise, a symmetric equilibrium with strictly increasing pricing strategies does not exist.*

But $\Phi(\cdot)$ is not necessarily strictly increasing. Differentiating Equation (8) with respect to c,

$$\Phi'(c) = 1 - \frac{t}{n} \cdot \frac{(1+\lambda)(\lambda-1)\theta(c)}{[2+(\lambda-1)\Theta(c)]^2}, \tag{9}$$

which is negative if $\theta(c)$ is very high. Since by Lemma 1 the pricing strategy is non-decreasing, this is an indication that it must be constant:

Corollary 2. *Under the conditions of Proposition 3, if $\Phi(c)$ is non-increasing, any symmetric market equilibrium is a focal-price equilibrium. Otherwise, symmetric equilibria other than focal-price equilibria exist.*

The intuition for this result is easiest to see by first assuming that consumers expect the firm's prices to be strictly increasing in cost. If the density of the cost distribution is high, a small increase in c implies a large increase in $F(P(c))$ and hence a large increase in the comparison effect and the corresponding responsiveness of demand. Such a large increase in marginal revenue in response to a small increase in marginal cost is inconsistent with equilibrium: a firm can increase profits either by decreasing its higher prices and attracting substantial extra demand, or by increasing its lower prices without losing many consumers. Since this is true for any strictly increasing pricing strategy, the equilibrium price must be constant.

When $\Phi(\cdot)$ is neither strictly increasing nor non-increasing, Proposition 3 implies that market equilibria will generally be "hybrids" in the following sense. The pricing function starts weakly below $\Phi(\underline{c})$ and ends up weakly above $\Phi(\overline{c})$ (Property 4), and is composed of flat parts (Property 2) pasted together continuously (Property 1) with strictly increasing parts over which prices are given by $\Phi(\cdot)$ (Property 3). Figure 3 illustrates a non-monotonic $\Phi(\cdot)$ along with two market-equilibrium pricing functions, $P^1(\cdot)$ and $P^2(\cdot)$. First, we argue that no firm has an incentive to slightly raise or lower its price from $P^1(c)$; the appendix establishes that there also no profitable non-local deviations. In the two strictly increasing regions, $[\underline{c}, f_1)$ and $(f_1', \overline{c}]$, where $P^1(\cdot)$ coincides with $\Phi(\cdot)$, we have shown above that there is no profitable local deviation. In addition, the same analysis implies that for $c = f_1$, there is no profitable local price decrease. This implies that there is also no profitable local price decrease for any higher cost $c \in [f_1, f_1']$. Similarly, since it is unprofitable to locally increase the price for $c = f_1'$, it is also unprofitable for $c \in [f_1, f_1']$. Using the same argument, $P^2(\cdot)$ is also a market-equilibrium pricing function.

Now we argue that any market-equilibrium pricing function looks like $P^1(\cdot)$ and $P^2(\cdot)$. For price decreases from $P(\underline{c})$ to be unprofitable, we must have $P(\underline{c}) \leq \Phi(\underline{c})$; and for price increases from $P(\overline{c})$ to be unprofitable, we must have $P(\overline{c}) \geq \Phi(\overline{c})$. This implies that for $c \in [\underline{c}, c']$ and $c \in [c'', \overline{c}]$ the function cannot have a flat part, because that could not be pasted continuously with a strictly increasing part (where $P(\cdot)$ must coincide with $\Phi(\cdot)$). Hence, in these regions $P(\cdot)$ is strictly increasing and therefore equal to $\Phi(\cdot)$. Since there is a region where $\Phi(\cdot)$ is decreasing, however, the monotonicity requirement on $P(\cdot)$ implies that $P(\cdot)$ and $\Phi(\cdot)$ cannot coincide globally. Instead, the pricing function must be "ironed out" over the range where $\Phi(\cdot)$ is decreasing. Furthermore, because at the ends of a flat interval, $P(\cdot)$ connects continuously to increasing parts of $\Phi(\cdot)$, there is exactly one flat part.

In combination with Equation (8), Proposition 3 has a number of important implications for symmetric equilibria. First, the firms' markup is greater than t/n—the markup in the standard Salop model—for $c < \overline{c}$, and greater than or equal to t/n for $c = \overline{c}$. Hence, as in focal-price equilibria (Proposition 2), loss aversion increases the price level. The consumers a firm attracts by lowering its price experience a pure loss in product satisfaction (from choosing a product unexpectedly far from ideal), and unless $c = \overline{c}$, only some combination of gain and avoided loss in money. Hence, they are more difficult to attract than in the standard setting, decreasing competition and increasing prices.

Second, while in the standard Salop model the markup is constant in c, with loss aversion it is strictly decreasing in c:

Corollary 3. *Under the conditions of Proposition 3, in any symmetric market equilibrium, $P(c) - c$ is strictly decreasing in c.*

This prediction of our theory is potentially relevant for understanding macroeconomic fluctuations. Extensive evidence reviewed by Rotemberg and Woodford (1999) indicates that costs are strongly procyclical. Hence, our model implies markups are countercyclical.[18] Intuitively, recall

[18] Of course, if one measures countercyclicality using the Lerner index $\frac{p-c}{p}$, the Salop model without loss aversion also features countercyclical markups. But that model does not feature countercyclical markups if markups are defined as $p - c$.

that due to the comparison effect, consumers are more responsive to price changes at higher than at lower prices within the price distribution. Since inframarginal demand is constant across the price distribution, this means that firms compete more fiercely at higher prices, reducing their markup.

Third, a decrease in the competitiveness of the market has a systematic effect on price variation. The more concentrated is the industry or the greater is product differentiation (the lower is n or the greater is t), the lower is the derivative of $\Phi(c)$ at any $c \in [\underline{c}, \overline{c}]$. As a result, the more countercyclical are markups—the faster $P(c) - c$ decreases with c—when price is strictly increasing in cost, and the more likely it is that any symmetric equilibrium is a focal-price one. When average markups are higher—as would be the case in a less competitive industry—the increased ability to attract consumers at higher prices has a greater impact on firms' incentive to cut prices, generating markups that decrease faster in cost. As t/n gets sufficiently large, the impact of an increase in demand responsiveness on firms' incentive to cut prices is so great that firms are unwilling to raise their price at all—they charge a focal price.

In fact, Proposition 3 allows us to more fully describe how pricing patterns change as the industry moves from very competitive ($t/n \approx 0$) to very uncompetitive (t/n very large). If competition is sufficiently strong, the unique symmetric market equilibrium features a strictly increasing pricing function, which is close to marginal-cost pricing if competition is very strong. As competition softens, markups increase and become more countercyclical. As competition softens further, the price becomes sticky near regions where the cost distribution is relatively dense, but may remain strictly increasing in cost in other regions. At even lower levels of competition, the price becomes focal.

6 Idiosyncratic Cost Shocks

In this section, we identify conditions under which firms suppress *idiosyncratic* costs shocks and adhere to focal pricing in any market equilibrium even though they might be ex ante asymmetric and asymmetric strategies are allowed. To our knowledge, no price-setting model predicts focal prices so robustly.

We establish our main result in two parts. First, we show that if the supports of firms' cost distributions overlap and they all set deterministic prices, they set a focal price. That is, there cannot be an equilibrium with stable but different prices. Then, we identify conditions under which all firms set deterministic prices.

Proposition 4. *Suppose* $\cap_{i \in N}[\underline{c}_i, \overline{c}_i] \neq \varnothing$. *If all firms set a deterministic price and either*

$$\lambda \leq 1 + \frac{2}{n-1}\left(1 + \sqrt{1 + 2n(n-1)}\right) \tag{10}$$

or $n = 2$, *the market equilibrium is a focal-price equilibrium.*

For an intuition, suppose a highest-price firm H and a lowest-price firm L charge prices p_H and $p_L < p_H$, and consider cost realizations c_H and c_L with $c_H \leq c_L$ (which do not have to occur in the same state of the world). With these cost realizations, firm H has lower demand and a higher markup than firm L. Furthermore, the comparison effect implies that there is a tendency for the increase in demand in response to a marginal price decrease from p_H to be at least as great as the decrease in demand in response to a marginal price increase from p_L. Therefore, in contradiction to equilibrium, either firm H wants to deviate by lowering its price, or firm L wants to deviate by raising its price.

This intuition, however, ignores an effect that makes it necessary to impose Condition (10) in the proposition. As a firm changes its price, the distribution of marginal consumers in its two markets changes. As explained in Section 3, this typically changes the price responsiveness of its residual demand. If demand responsiveness could change very fast relative to the change in inframarginal demand, the firm's problem would be badly behaved in a number of ways. To rule out such possibilities, Proposition 4 above and Proposition 5 below impose restrictions on λ.

But Condition (10) is relatively weak. It only applies when $n > 2$, and it is satisfied for any number of firms whenever $\lambda \leq 1 + 2\sqrt{2} \approx 3.8$. Since the conventional assumption of two-to-one loss aversion is equivalent to $\lambda = 3$, the condition seems unproblematic.

As a second ingredient for our main result, we give conditions such that all firms charge a deterministic price. To model idiosyncratic cost shocks, we suppose that marginal costs are independently distributed, with c_i's distribution on $[\underline{c}_i, \overline{c}_i]$ being given by Θ_i.[19] Define the parameter $\gamma = (\overline{c} - \underline{c})/(t/n)$.

[19] Assuming independence greatly simplifies some technical issues that arise when firms do not always set the same

Proposition 5. *Suppose costs are independently distributed with $c_i \sim \Theta_i[\underline{c}_i, \overline{c}_i]$. If $38 > \lambda > 1$ and $\gamma < (3+\lambda)/(1+\lambda)$, there is a real number $\underline{\psi}(\lambda, \gamma)$ such that if*

$$\frac{t}{n^2} \cdot \theta_i(c) \geq \underline{\psi}(\lambda, \gamma)$$

for all $c \in [\underline{c}_i, \overline{c}_i]$, then firm i sets a deterministic price in any interior equilibrium.

The intuition for Proposition 5 is closely related to that for Corollary 2, and is easiest to see by first assuming that the consumer expects firm i's prices to be continuously distributed. Recall that due to the comparison effect, the consumer's price sensitivity at a price p is increasing in the probability with which she expected lower prices than p. Hence, at least to the extent that she expected to buy from firm i, her demand becomes more price responsive as the price moves through firm i's price distribution. Because the optimal markup depends on demand responsiveness, a firm with a sufficiently dense cost distribution wants to deviate from the consumer's price expectations either by decreasing its higher prices or by increasing its lower prices. Since this is true for any pricing strategy that varies with cost, the price in fact cannot vary with cost.

Combining the two propositions above, we get the most important result in this section:

Corollary 4. *If the conditions of Propositions 4 and 5 hold, any interior market equilibrium is a focal-price equilibrium.*

7 Robustness to Alternative Specifications

Because in many situations consumers are unsure either about what they want or about what is available, we have assumed a dispersed prior on χ. But most of our results do not depend on this assumption. Even if χ is known perfectly, our results in Sections 4 and 5 remain unchanged.[20] Consider when there is a focal-price equilibrium. If a firm raises its price, its consumers (who expected to buy the product closest to χ) must choose between two losses, a surprisingly high

price. If costs are not independent, a change in c_i changes the distribution of competitors' prices conditional on c_i and hence also the distribution of residual demand for a given p_i. We do not know how to deal with this complication.

[20] Technically, if different consumers have different information about their preferred location, there cannot be a representative consumer. The definition of market equilibrium has to be modified accordingly to account for such heterogeneity.

price and a surprisingly bad product match. If a firm lowers its price, the consumers it attracts experience a gain in money and a loss in satisfaction. Locally, both of these effects are the same as in our analysis above. And because the tradeoff is the same for non-marginal consumers, the residual demand curve is two-piece linear (see Figure 2), making a non-local deviation unprofitable whenever a local one is. Hence, a focal-price equilibrium with price p^* exists when consumers know χ if and only if it exists in our model above. By a similar reasoning, the two models are also locally identical in the case of symmetric equilibria with industry-wide cost shocks. Furthermore, because the sensitivity to price changes increases in price, when consumers know χ demand is concave. Hence, again a global deviation is unattractive whenever a local one is, and our results survive unchanged.

When firms might charge different prices, the possible multiplicity of personal equilibria when χ is known significantly changes the model. Nevertheless, for a completely different reason, Proposition 4 still holds. Suppose firms 1 and 2 charge prices p_1 and $p_2 < p_1$. If a consumer who prefers product 1 expects to take product 2 at the cheaper price p_2, to avoid a loss in money she typically strictly prefers not to take product 1. If she expects to take the better product, to avoid a loss in satisfaction she typically strictly prefers not to take the other one. With consumers "locked in," both firms want to raise their price, contradicting equilibrium.

Our methods in Proposition 5 do not extend to the case when χ is known with certainty, and we conjecture that the result is not true in that case. The logic of our proof, however, only seems to use that sufficiently many *marginal* consumers are sufficiently uncertain about their taste that they are unsure as to which product they will buy. More precisely, for consumers who have some uncertainty in x and who might be marginal with all relevant price realizations, the variational methods we have developed should still work.

Our results on focal pricing and reduced price variation more generally (but not our result that loss aversion increases prices) hold in a model in which consumers are loss averse only in money.[21] This assumption would, in fact, substantially simplify some of our proofs. Methodologically, how-

[21] In such a model, however, the money dimension is relatively more important than the good dimension: even if consumers were not loss averse ($\lambda = 1$), their demand responsiveness would be twice as high as in the standard Salop (1979) model. This artificial increase in price sensitivity would lead to fiercer competition and hence reduced prices in comparison to our model and the standard model.

ever, we believe that the spirit of our research—deriving insights by enriching the consumer side of models traditionally focused on the supply side—requires us to use a psychologically, experimentally, and theoretically well-motivated consumer model rather than constructing a different ad-hoc model for our application. Hence, we make the far more plausible and experimentally well-motivated assumption that consumers are loss averse also in the product dimension.

Our definition of market equilibrium assumes that all consumers play the same personal equilibrium. Relaxing this assumption does not affect our results. In all situations in Sections 4 and 5, selection is a non-issue simply because the personal equilibrium is unique. Our proofs in Section 6 work by estimating how a firm's demand responsiveness changes across the price distribution. Since our bounds hold for any personal equilibrium a person might be playing, they also hold if consumers play a variety of equilibria.

The results in this paper are also robust to heterogeneity in loss aversion among consumers. Our estimation methods would have to account for such heterogeneity, but as long as there is some loss aversion in the population, the results would survive in some form.

We assume above that each firm sells exactly one product. As long as no firm owns neighboring products, our results carry over unchanged to multiproduct firms.[22] In interior equilibria, the incentives for locally changing one product's price are unaffected by how many non-neighboring products a firm owns. But global deviations are weakly less profitable for a multiproduct firm because such a firm might be cannibalizing its own market. Even if firms can own two neighboring products, all the forces behind our results are still present, so that focal pricing will often be an equilibrium, and often the only type of equilibrium. Our conditions and proofs, however, would have to account for the decreased competition between firms and for the fact that products may differ in how many neighboring products they compete with.[23]

The analogue of our focal-price equilibrium above for multiproduct firms is a market equilibrium in which all products have the same price. In such an equilibrium, each multiproduct firm charges the same price for its different products, so that our model predicts uniform pricing.

[22] In this case, however, intensity of competition, which drives many of our results, is determined by the number of products rather than the number of firms.

[23] When a firm can own three neighboring products, the middle one faces no immediate external competition, so the firm always wants to set a higher price for it.

In our model, we assume that firms' situations are symmetric and stable with respect to demand, while costs are possibly different and uncertain. In most industries, firms also differ in the elasticity of residual demand they face (even if all firms set the same price). One way to formalize such differences is to assume that marginal costs are zero, but there is a possibly different and randomly determined mass of "loyal" consumers at each of the locations $0, 1/n, \ldots, (n-1)/n$. Loyal consumers either buy their firm's product, or do not consume, with their maximum willingness to pay chosen so that it is unprofitable to sell only to them. Because a change in the number of loyal consumers has similar implications for the firm's behavior as a change in marginal cost, we confidently conjecture that all of our results and methods of proof carry over to this alternative model.

While we have assumed that industry structure is exogenous, our model can be extended to allow for endogenous entry. Suppose industry concentration is determined by a fixed cost firms must pay to enter the market. Since the fixed cost determines the number of firms but has no impact on market equilibria given the number of firms, our qualitative results on the effect of industry concentration on market equilibria survive.

8 Conclusion

A primitive of our model is the time at which the expectations determining the reference point are formed. For example, in the case of industry-wide cost shocks we assume that the reference point does not adjust to such shocks at all. In some cases—such as regulatory intervention widely publicized in advance—consumers' expectations regarding market prices may adjust early enough to change their reference point. In this case, it is appropriate to model the intervention as a change in the entire cost distribution rather than as a shift within the same cost distribution. While Kőszegi and Rabin (2005) discuss some psychological principles related to such timing issues, it is ultimately not something we can formally elucidate in this paper.

In contrast to our results, which predict reduced price variation in a number of senses, in industries where firms can obscure price and product differences there often seems to be excess price variation between even identical products.[24] Ellison (2005) and Gabaix and Laibson (2004)

[24] Baye, Morgan, and Scholten (2004), for example, document that for many products, different internet-based

identify competitive advantages of making price comparisons difficult, and the model of Spiegler (2004) suggests that this could lead to price variation.[25] While we have no general theory of when prices would be obscure and when they would not, we believe that our theory applies better when prices are more transparent.

retailers charge very different prices.

[25] Gabaix and Laibson (2004) show that in the presence of some consumers who ignore add-on costs, firms have no incentive to make add-on prices transparent, even when it is very cheap to do so. Ellison (2005) gives natural conditions under which rational consumers who are unresponsive to an ex-ante hidden add-on price also decrease competition in transparent aspects of the product.

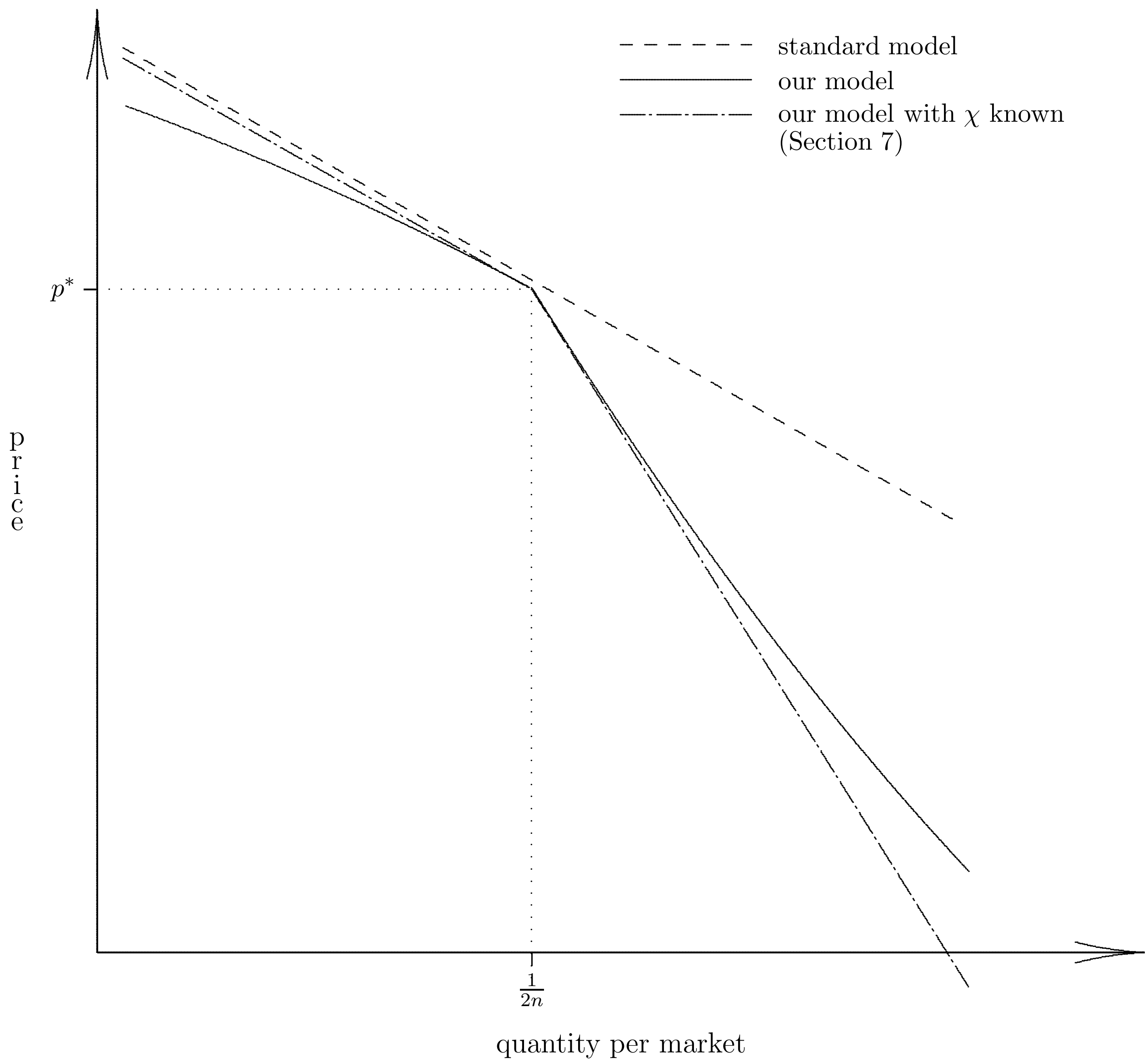

Figure 2: A Firm's Residual Demand When Neighbors Charge p^*

For better visibility, overlapping curves are drawn as close parallel curves. Note that the standard residual demand curve is linear. The demand curve in our model is kinked at p^*, and is tangent to the standard demand curve at p^* for price increases. In addition, it is concave for price increases from p^*, and convex for price decreases from p^*. The demand curve in our model with χ known is two-piece linear with a kink at p^*, and is tangent to the demand curve with χ unknown in both directions from p^*.

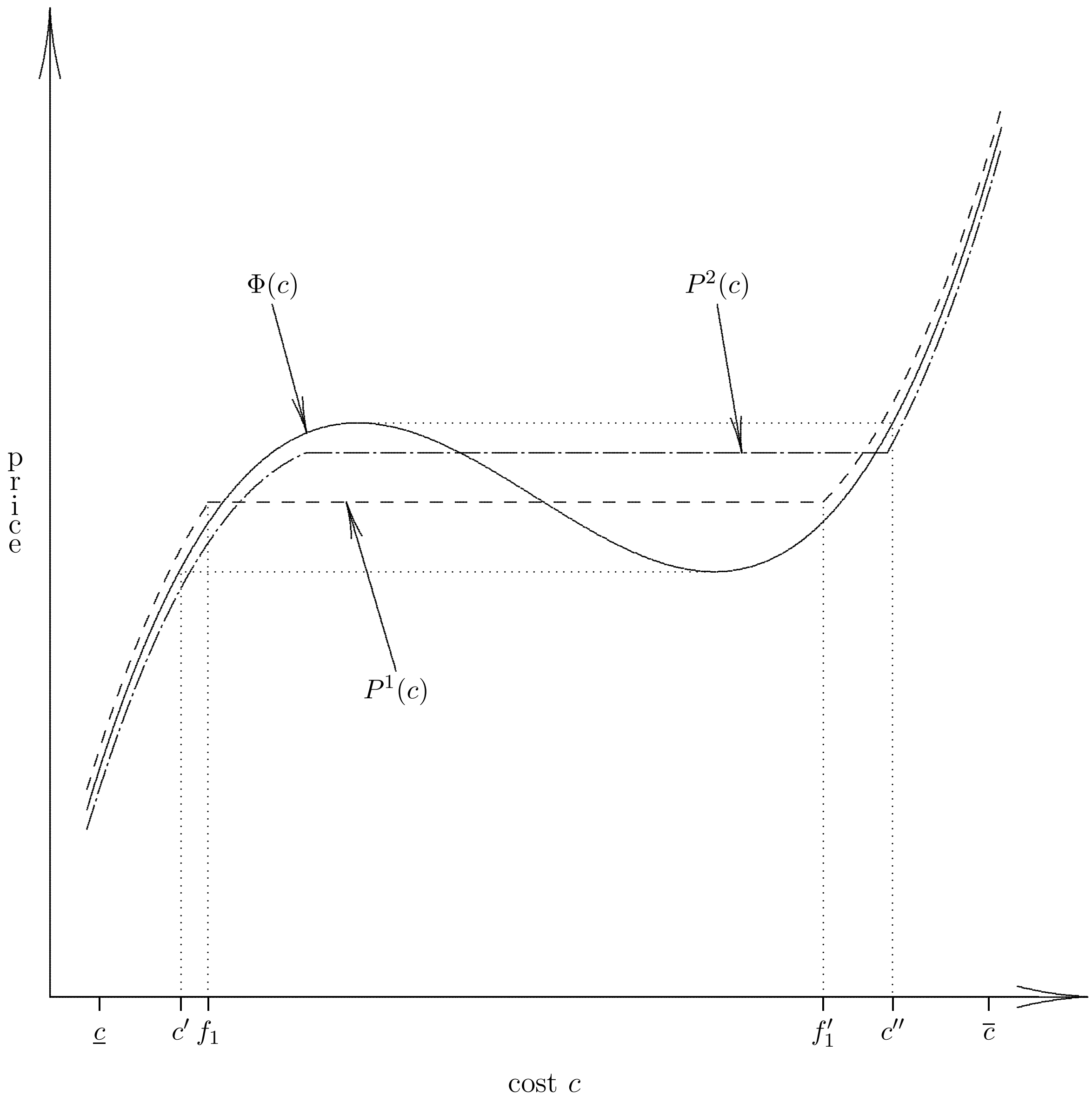

Figure 3: Determination of Equilibrium Pricing Functions with Industry-Wide Shocks

For better visibility, overlapping curves are drawn as close parallel curves. The function $\Phi(\cdot)$ is the solution to the first-order condition for optimal pricing at any price that is not an atom of the price distribution. Since a market-equilibrium pricing function is non-decreasing and continuous, it consists of constant parts pasted together with strictly increasing parts that coincide with Φ. Two market-equilibrium pricing functions are $P^1(c)$ and $P^2(c)$.

Appendix A: Labor-Market Reinterpretation

Suppose employees' "ideal jobs" are located on the unit circle, where firms' jobs are also located. If an employee with job-taste x takes job y, her intrinsic utility is $v + w - t \cdot d(x, y)$, where v is her intrinsic (dis)utility from working, w is her wage, and $t \cdot d(x, y)$ is her disutility from taking a job different from her ideal. Company i's profit from attracting an employee is $\psi_i - w_i$, where ψ_i is the company's labor productivity and w_i is its wage. For any distribution of costs Θ in the price-setting game and any constant Ψ, we can define a corresponding wage-setting game by letting $\psi_i \equiv \Psi - c_i$ for any realization c_i (and by requiring that $w_i \leq \Psi \ \forall i$). In this game, employees first form expectations regarding the type of job they will get and the wage. Next, each firm's productivity is realized and firms simultaneously set wages. Finally, consumers decide which job to take (we assume employees find jobs sufficiently valuable that they always work in equilibrium). A wage-setting equilibrium is defined as a situation where employees play a personal equilibrium correctly anticipating the distribution of wages in the market, and firms maximize profits, correctly anticipating employee behavior. We have the following:

Proposition 6. *The pricing functions $\{P_1, P_2, \ldots, P_n\}$ are part of a pricing equilibrium if and only the wage-setting functions $W_i : \psi_i \mapsto w_i$ defined by $W_i(\psi_i) = \Psi - P_i(\Psi - \psi_i)$ are part of a wage-setting equilibrium in the wage-setting game.*

Proof. Obvious. □

Appendix B: Proofs

Proof of Proposition 1. If the condition in the proposition is satisfied, then there is a p^* satisfying $p^* - \frac{t}{n}\frac{1+\lambda}{2} \leq c \leq p^* - \frac{t}{n}$ for all $c \in [\underline{c}, \overline{c}]$. That this is a necessary and sufficient condition for local deviations to be unprofitable has been established in the text.

We now show that under the above condition, non-local deviations are also unprofitable. We start with increases in the price. First, note that the firm will never charge a price so high that it would be charging itself out of one market: if a deviating firm is charging itself out of one market, it is charging itself out of both, earning zero profits. Therefore, we only need to consider deviations for which $x \in \left(0, \frac{1}{2n}\right)$. Recall Equation 4:

$$\frac{dx}{dp_1} = -\frac{1}{2t}\left[\frac{2 + (\lambda - 1)F(p_1)}{\left\{2 + \frac{\lambda-1}{2}[G(x) + G((1/n) - x)]\right\}}\right]$$

Since $F(p_1) = G\left(\frac{1}{n} - x\right) = 1$ and $G(x)$ is increasing in x, in the range $x \in \left(0, \frac{1}{2n}\right)$, firm 1's demand (as a function of p_1) is convex. This implies that if local deviations are unprofitable, non-local increases in the price are also unprofitable.

Next, we rule out the possibility that firm 1 might like to charge a price so that $x \in \left[\frac{1}{2n}, \frac{1}{n}\right]$. In that case, Equations (3) and (4) imply that

$$-xt - p_1 + (p^* - p_1) - \lambda t\left(x - \frac{1}{4n}\right) = -\left(\frac{1}{n} - x\right)t - p^* - \lambda t \cdot 2n \cdot \left(\frac{1}{n} - x\right)\frac{\frac{1}{n} - x}{2} + t \cdot 2n \cdot \left(x - \frac{1}{2n}\right)\frac{x - \frac{1}{2n}}{2}.$$

Solving for p_1 gives

$$p_1 = p^* - \frac{1}{2}t\underbrace{\left[(\lambda + 1)\left(2x - \frac{1}{n}\right) + (\lambda - 1)\left(x - \frac{1}{4n} - nx^2\right)\right]}_{\equiv \kappa}.$$

To show that lowering the price to p_1 is not a profitable deviation, it is equivalent to show that

$$\frac{1}{n}(p^* - c) \geq 2x(p_1 - c) = 2x\left(p^* - c - \frac{1}{2}t\kappa\right).$$

Rearranging and using that $p^* - c \leq \frac{t(1+\lambda)}{2n}$ gives that it is sufficient to show that

$$\left(2x - \frac{1}{n}\right)\frac{1 + \lambda}{n} \leq 2x\kappa, \tag{11}$$

or equivalently

$$(\lambda + 1)\left(2x - \frac{1}{n}\right)^2 \geq (\lambda - 1)2x\left(nx^2 + \frac{1}{4n} - x\right) = (\lambda - 1)2x\left(2x - \frac{1}{n}\right)\left(\frac{nx}{2} - \frac{1}{4}\right).$$

This simplifies to

$$(\lambda + 1)\left(2x - \frac{1}{n}\right) \geq (\lambda - 1)2x\left(\frac{nx}{2} - \frac{1}{4}\right).$$

Notice that in the above inequality, the left-hand side is equal to the right-hand side for $x = \frac{1}{2n}$ and greater for $x = \frac{1}{n}$. Furthermore, the left-hand side is linear, while the right-hand side is quadratic and convex. This implies that the left-hand side is no less for all $\frac{1}{2n} \leq x \leq \frac{1}{n}$.

For $n > 2$, we are left to rule out that firm 1 undercuts its rival and steals more than the entire adjacent market. We begin by ruling out deviations in which the firm captures less than two adjacent markets on each side. Let p_1' be the price at which the consumer located at $\frac{1}{n}$ is indifferent between buying from firm 1 and buying from firm 2. This consumers utility of buying from firm 1 is

$$v - \frac{1}{n}t - p_1' + (p^* - p_1') - \lambda t \left[\frac{1}{n} - \frac{1}{4n}\right].$$

In case he buys from firm 2, her utility is

$$v - p^* + t\frac{1}{4n}.$$

Thus, if the consumer is indifferent

$$p^* - p_1' = \frac{t}{2n}\left[2 + \frac{3}{4}(\lambda - 1)\right].$$

Consider the maximum price at which a local deviation is unprofitable; for this price $p^* - \underline{c} = \frac{t}{2n}[2 + \lambda - 1]$ and in this case $p_1' - \underline{c} = \frac{t}{2n}\left[\frac{1}{4}(\lambda - 1)\right]$. Thus even if firm 1 would get the entire two adjacent markets when setting p_1', this is unprofitable as $\frac{1}{n}(p^* - \underline{c}) > \frac{4}{n}(p_1 - \underline{c})$.[26] Obviously undercutting is (weakly) less profitable for any lower focal price.

We are left to consider the case in which $n > 4$, and firm 1 steals more than two adjacent markets on each side. We show that this is unprofitable because it requires firm 1 to price below marginal cost. For the consumer located at $\frac{2}{n}$ to weakly prefer buying from firm 1 rather than firm 3, it must be that

$$v - \frac{2}{n}t - p_1 + (p^* - p_1) - \lambda t \left[\frac{2}{n} - \frac{1}{4n}\right] \geq v - p^* + t\frac{1}{4n}.$$

Hence, in this case $p^* - p_1 \geq \frac{t}{2n}[4 + (\lambda - 1)\frac{7}{4}] > \frac{t}{2n}[2 + \lambda - 1] \geq p^* - c$, which completes the proof. $\square$

Proof of Proposition 2. We have shown in the text that local deviations are unprofitable if and only if

$$p^* - \frac{t}{n} \cdot \frac{1 + \lambda}{2} \leq c_i \leq p^* - \frac{t}{n}$$

[26] Clearly if $n = 3$, the firm cannot attract two adjacent markets on each side, as there are only three local markets. Nevertheless, the upper bound on profitability we use is still valid.

for all i and $c_i \in [\underline{c}_i, \overline{c}_i]$. It follows from the proof of Proposition 1 that if local deviations are unprofitable, so are global ones.

It remains to show the second part of the proposition. In the standard Salop model, for the consumer x between firms 1 and 2 who is indifferent between the two products,

$$x = \frac{\frac{t}{n} + p_2 - p_1}{2t}.$$

Hence, for realized cost c, firm 1's problem is

$$\max_{p_1} \ \frac{p_1 - c}{2t} \cdot \left(\frac{2t}{n} - 2p_1 + E[p_2 + p_n | c] \right).$$

This implies that

$$P_1(c) = \frac{t}{2n} + \frac{E[p_2 + p_n | c]}{4} + \frac{c}{2}.$$

Suppose that the supremum of prices charged by firms 1, 2, and n are $\overline{p}_1$, $\overline{p}_2$, and $\overline{p}_n$, respectively. Suppose without loss of generality that $\overline{p}_1$ is the supremum of market-equilibrium prices of all firms. Then for any $c \in [\underline{c}_1, \overline{c}_1]$,

$$P_1(c) \leq \frac{t}{2n} + \frac{\overline{p}_2 + \overline{p}_n}{4} + \frac{c}{2}. \tag{12}$$

Taking the supremum of both sides implies

$$\overline{p}_1 \leq \frac{t}{2n} + \frac{\overline{p}_1 + \overline{p}_1}{4} + \frac{\overline{c}}{2}.$$

Rearranging gives the upper bound in the proposition.

Finally, we show that this upper bound can only be attained at $\overline{c}$. If no firm's price attains $\overline{p}_1$, we are done. Next, suppose that for a price $c < \overline{c}$, $P_1(c) = \overline{p}_1$. By Inequality (12), again we are done. □

Proof of Lemma 1. We begin with proving continuity. Suppose by contradiction that $c^i \to c$ but $P(c^i) \nrightarrow P(c)$. Then, since the pricing function is obviously bounded, we can choose the sequence so that $P(c^i)$ converges; let $P(c^i) \to P' \neq P(c)$. Furthermore, suppose that $P' > P(c)$; the proof for the other case is analogous.

Since $P(c^i)$ is optimal when the marginal cost is c^i, a firm cannot benefit from marginally lowering its price. Using Equation 4 to express the firm's marginal profit from lowering its price, this implies that

$$\frac{1}{2n} - \left(P(c^i) - c^i\right) \cdot \frac{2 + (\lambda - 1)F_{\uparrow}\left(P(c^i)\right)}{1 + \lambda} \geq 0. \tag{13}$$

Similarly, since $P(c)$ is optimal when the marginal cost is c, a firm cannot benefit from marginally raising its price. Using Equation 4, this implies that

$$\frac{1}{2n} - (P(c) - c) \cdot \frac{2 + (\lambda - 1)F\left(P(c)\right)}{1 + \lambda} \leq 0. \tag{14}$$

Subtracting Inequality 13 from Inequality 14 gives

$$\left(P(c^i) - c^i\right) \cdot \frac{2 + (\lambda - 1)F_{\uparrow}\left(P(c^i)\right)}{1 + \lambda} - (P(c) - c) \cdot \frac{2 + (\lambda - 1)F\left(P(c)\right)}{1 + \lambda} \leq 0.$$

The limit of the left-hand side of this inequality as $i \to \infty$ is positive, a contradiction.

Next, we prove by contradiction that $P(c)$ is non-decreasing. Suppose that $c' > c$ and $P(c') < P(c)$. Since $P(c)$ is optimal when the marginal cost is c, a firm cannot benefit from marginally lowering its price. As above, this implies that

$$\frac{1}{2n} - (P(c) - c) \cdot \frac{2 + (\lambda - 1)F_{\uparrow}\left(P(c)\right)}{1 + \lambda} \geq 0. \tag{15}$$

Similarly, since $P(c)$ is optimal when the marginal cost is c, a firm cannot benefit from marginally raising its price. Therefore,

$$\frac{1}{2n} - \left(P(c') - c'\right) \cdot \frac{2 + (\lambda - 1)F\left(P(c')\right)}{1 + \lambda} \leq 0. \tag{16}$$

Subtracting Inequality 15 from Inequality 16 gives

$$(P(c) - c) \cdot \frac{2 + (\lambda - 1)F_{\uparrow}\left(P(c)\right)}{1 + \lambda} - \left(P(c') - c'\right) \cdot \frac{2 + (\lambda - 1)F\left(P(c')\right)}{1 + \lambda} \leq 0,$$

a contradiction. □

Proof of Proposition 3. We first show that any symmetric equilibrium pricing function satisfies the above properties. Property 1 follows from Lemma 1. Since $P(\cdot)$ is increasing and continuous, $P^{-1}(p)$ is a closed interval for any p on the range of $P(\cdot)$. Let $p_1, p_2, \ldots$ be the (at most countable) set of prices p_i such that $P^{-1}(p_i)$ is a non-trivial interval, and let $[f_i, f_i'] = P^{-1}(p_i)$. These $[f_i, f_i']$ satisfy Property 2 by construction. Also, for any $c \notin [f_i, f_i']$, $P(c)$ is not an atom of the pricing distribution, so a firm's demand is differentiable, and hence $P(c)$ must satisfy Equation 8. This implies that Property 3 holds. Notice that $D_{1\uparrow}(P(c), P_{-1}(c)) = -\frac{1}{t}\frac{2}{1+\lambda}$, so firm 1 does not want to decrease its price at $\underline{c}$ only if $(P(\underline{c}) - \underline{c})\frac{1}{t}\frac{2}{1+\lambda} \leq \frac{1}{n}$, which implies the first part of Property 4. Also, $D_{1\downarrow}(P(c), P_{-1}(c)) = -\frac{1}{t}$. So for raising the price marginally to be unprofitable, we must have $(P(\overline{c}) - \overline{c})\frac{1}{t} \geq \frac{1}{n}$, which implies the second part of Property 4.

We now argue that if $P(\cdot)$ satisfies the properties in the Proposition, it is an equilibrium pricing strategy. Notice that for any $c \in (\underline{c}, \overline{c}), c \notin [f_i, f_i')$, demand is differentiable from the right. Since $P(c) = \Phi(c)$ for all such c, our analysis in the text implies that there is no profitable local price increase. We are left to consider non-local price increases. Analogously to Proposition 1, since the demand curve is concave for price increases, the result is immediate.

Now for any $c \in (\underline{c}, \overline{c}), c \notin (f_i, f_i']$, demand is differentiable from the left. Furthermore, since $P(c) = \Phi(c)$ for all such c, our analysis in the text implies that local price decreases are unprofitable. We now consider non-local price decreases.

The proof mirrors the proof of Proposition 1. Suppose the realized cost is c, so that the firm's price in the posited equilibrium is $P(c)$. At this price, consumers' marginal utility in money from a price decrease is $2 + (\lambda - 1)F_{\uparrow}(P(c))$. We will use that as the price decreases, this marginal utility in money also decreases.

We first rule out the possibility that firm 1 might like to charge a price p_1 so that the indifferent consumer is $x \in \left[\frac{1}{2n}, \frac{1}{n}\right]$. Equating Expressions (3) and (4), setting $p_2 = P(c)$, and replacing the difference in money utilities,

$$P(c) - p_1 + \left[-\lambda \int_0^{p_1} (p_1 - p)\, dF(p) + \int_{p_1}^{\infty} (p - p_1)\, dF(p)\right] - \left[-\lambda \int_0^{P(c)} (P(c) - p)\, dF(p) + \int_{P(c)}^{\infty} (p - P(c))\, dF(p)\right],$$

with its upper bound $(2 + (\lambda - 1)F_{\uparrow}(P(c)))(P(c) - p_1)$, gives that for the indifferent consumer x

$$\begin{aligned} & -xt + (P(c) - p_1)(2 + (\lambda - 1)F_{\uparrow}(P(c))) - \lambda t \left(x - \frac{1}{4n}\right) \\ \geq \quad & -\left(\frac{1}{n} - x\right) t - \lambda t \cdot 2n \cdot \left(\frac{1}{n} - x\right) \frac{\frac{1}{n} - x}{2} + t \cdot 2n \cdot \left(x - \frac{1}{2n}\right) \frac{x - \frac{1}{2n}}{2}, \end{aligned}$$

so that

$$P(c) - p_1 \geq \frac{t}{2 + (\lambda - 1)F_{\uparrow}(P(c))} \underbrace{\left[(\lambda + 1) \left(2x - \frac{1}{n} \right) + (\lambda - 1) \left(x - \frac{1}{4n} - nx^2 \right) \right]}_{\equiv \kappa}. \tag{17}$$

To show that lowering the price to p_1 is not a profitable deviation, it is sufficient to show that

$$\frac{1}{n}(P(c) - c) \geq 2x(p_1 - c).$$

Using Inequality (17), it is sufficient to show that

$$\frac{1}{n}(P(c) - c) \geq 2x \left(P(c) - c - \frac{t}{2 + (\lambda - 1)F_{\uparrow}(P(c))} \kappa \right).$$

Rearranging and using that $P(c) - c = \frac{t(1+\lambda)}{n(2+(\lambda-1)F_{\uparrow}(P(c)))}$ gives

$$\left(2x - \frac{1}{n} \right) \frac{1 + \lambda}{n} \leq 2x\kappa,$$

which is equivalent to Inequality (11), which we verified in the proof of Proposition 1.

For $n > 2$, we are left to rule out that firm 1 undercuts its rival and steals more than the entire adjacent market. We begin by ruling out deviations in which the firm captures less than two adjacent markets. Let p_1 be the price at which the consumer located at $\frac{1}{n}$ is indifferent between buying from firm 1 and buying from firm 2. Substituting $x = 1/n$ into Equation 17 gives

$$P(c) - p_1 \geq \frac{t}{(2 + (\lambda - 1)F_{\uparrow}(P(c)))n} \left[2 + \frac{3}{4}(\lambda - 1) \right].$$

Using the expression for $P(c) - c$ we get $p_1 - c \leq \frac{t}{(2+(\lambda-1)F_{\uparrow}(P(c)))n} \left[\frac{1}{4}(\lambda - 1) \right]$. Thus, even if firm 1 would get the entire two adjacent markets when setting p_1', this is unprofitable as $\frac{1}{n}(P(c) - c) > \frac{4}{n}(p_1 - c)$.

We are left to consider the case when $n > 4$ and firm 1 steals at least two adjacent markets on each side. We show that this is unprofitable because it requires firm 1 to price below marginal cost. For the consumer located at $\frac{2}{n}$ to weakly prefer buying from firm 1 rather than firm 3, it must be that

$$P(c) - p_1 \geq \frac{t}{(2 + (\lambda - 1)F_{\uparrow}(P(c)))n} [4 + (\lambda - 1)\frac{7}{4}] > \frac{t}{(2 + (\lambda - 1)F_{\uparrow}(P(c)))n} [2 + \lambda - 1] = P(c) - c.$$

This completes the proof that non-local price decreases are unprofitable.

We have established that there is no profitable deviation for $c \in (\underline{c}, \overline{c}), c \notin [f_i, f_i']$. For any $c \in (\underline{c}, \overline{c}), c \in [f_i, f_i']$, we have $P(f_i) = P(c) = P(f_i')$. Since it is not profitable to lower the price at f_i, it is also not profitable to lower it for c, and since it is not profitable to raise the price for f_i', it is also not profitable to raise it for c.

We are left to prove that there are no profitable deviations for $\underline{c}$ and $\overline{c}$. Our analysis of non-local deviations above (which only used that $P(c) = \Phi(c)$) implies that for $P(\underline{c}) = \Phi(\underline{c})$, there is no profitable deviation. Now suppose that $P(\underline{c}) < \Phi(\underline{c})$. Demand responsiveness to price decreases from $P(\underline{c})$ is then the same as when $P(\underline{c}) = \Phi(\underline{c})$. Hence, with the markup being lower, the incentive to lower the price is smaller than for $P(\underline{c}) = \Phi(\underline{c})$, so there is no profitable price decrease. Next, we deal with price increases from $\underline{c}$. Since $P(\underline{c}) < \Phi(\underline{c})$, we consider two cases. First, suppose that $P(c)$ is a constant p^*. Then, using that by Property 4 in the proposition $\Phi(\underline{c}) \geq p^* \geq \Phi(\overline{c})$, and Equation 8, the condition in Proposition 2 is satisfied. Hence, p^* is a market-equilibrium focal price. If $P(c)$ is not constant, there is a largest interval $[\underline{c}, f_1']$ for which it is constant, and where $f_1' < \overline{c}$. In this case, our argument in the previous paragraph applies. Finally, a similar argument works for price deviations from $\overline{c}$. □

Proof of Corollary 1. Suppose by contradiction that there is a constant interval $[f_1, f_1']$. By Conditions 3 and 4 of Proposition 3, we must have $P(f_1) \leq \Phi(f_1)$. But by the same two conditions, we must also have $P(f_1') \geq \Phi(f_1')$, which is impossible since $\Phi(\cdot)$ is strictly increasing on the interval while $P(\cdot)$ is constant. □

Proof of Corollary 2. We first prove by contradiction that if $\Phi(c)$ is weakly decreasing, then any symmetric equilibrium is a focal-price one. Suppose the price is not deterministic. Then, by the continuity of the pricing function, there are cost levels $c, c' > c$ such that $P(c)$ and $P(c')$ are not atoms of the price distribution. Thus, for these cost levels, the chosen price must satisfy Equation

8. Using that $\Phi(c)$ is strictly decreasing, this means that $P(c') < P(c)$, contradicting that the pricing function is non-decreasing.

If $\Phi(c)$ is not weakly decreasing, then there are obviously non-constant $P(\cdot)$ satisfying Proposition 3. $\square$

Proof of Corollary 3. The statement is true on both the constant and strictly increasing parts of the pricing function. $\square$

Proof of Proposition 4. Posit a candidate market equilibrium in which all firms set a deterministic price and in which the highest price p_H is strictly greater than the lowest price p_L. We prove that if the condition in the Proposition is satisfied, either (one of) the highest price firm(s) has a strict incentive to lower its price or (one of) the lowest price firm(s) has a strict incentive to raise its price, contradicting equilibrium.

We establish that the marginal profit of lowering the highest price is weakly greater than the marginal profit of raising the lowest price for all given cost realizations c. This is sufficient because it implies that the high-price firm has a strict incentive to lower its price when it has its lowest cost realization, or the low-price firm has a strict incentive to raise its price when it has its highest cost realization (which is higher than the high-price firm's lowest cost realization because the supports of the cost distributions overlap), contradicting equilibrium. Let x_H^+ and x_H^- be one of the highest cost firm's demands on its right and left, respectively. Define x_L^+ and x_L^- similarly. We want to establish that

$$(p_H - c)\left[\underbrace{\frac{1}{2 + \frac{\lambda-1}{2}[G(x_H^+) + G(\frac{1}{n} - x_H^+)]}}_{\equiv\frac{1}{z_H^+}} + \underbrace{\frac{1}{2 + \frac{\lambda-1}{2}[G(x_H^-) + G(\frac{1}{n} - x_H^-)]}}_{\equiv\frac{1}{z_H^-}}\right] \times [2 + F_{\uparrow}(p_H)(\lambda - 1)]$$

$$\geq (p_L - c)\left[\frac{1}{z_L^+} + \frac{1}{z_L^-}\right] \times [2 + F(p_L)(\lambda - 1)], \tag{18}$$

where z_L^+ and z_L^- are defined analogously to z_H^+ and z_H^-. For brevity, let $\eta_H \equiv [2 + F_\uparrow(p_H)(\lambda - 1)]$ and let $\eta_L \equiv [2 + F(p_L)(\lambda - 1)]$.

Notice that either $\left(z_L^+ \frac{1}{z_H^+} + z_L^- \frac{1}{z_H^-}\right) \leq \frac{1}{2}(z_L^+ + z_L^-)\left(\frac{1}{z_H^+} + \frac{1}{z_L^-}\right)$ or $\left(z_L^- \frac{1}{z_H^+} + z_L^+ \frac{1}{z_H^-}\right) \leq \frac{1}{2}(z_L^+ + z_L^-)\left(\frac{1}{z_H^+} + \frac{1}{z_L^-}\right)$. We distinguish two cases depending on whether the former (Case I) or the latter (Case II) holds.

Case I. We rewrite Equation 18 as

$$\eta_H \left(z_L^+ \frac{z_L^-}{z_H^+} + z_L^- \frac{z_L^+}{z_H^-}\right) \geq \left(1 - \frac{p_H - p_L}{p_H - c}\right) \eta_L \; (z_L^+ + z_L^-). \tag{19}$$

Equation 19 is equivalent to

$$\eta_H \left(z_L^+ \left(1 - \frac{z_H^+ - z_L^-}{z_H^+}\right) + z_L^- \left(1 - \frac{z_H^- - z_L^+}{z_H^-}\right)\right) \geq \left(1 - \frac{p_H - p_L}{p_H - c}\right) \eta_L \; (z_L^+ + z_L^-).$$

As $\eta_H > \eta_L$ a sufficient condition for Equation (18) to hold is that

$$\eta_H \left(z_L^+ \frac{z_H^+ - z_L^-}{z_H^+} + z_L^- \frac{z_H^- - z_L^+}{z_H^-}\right) \leq \frac{p_H - p_L}{p_H - c} \; \eta_L \; (z_L^+ + z_L^-). \tag{20}$$

Using that

$$|z_H^+ - z_L^-| = \frac{\lambda - 1}{2} \left| \left[G(x_H^+) - G(x_L^-)\right] - \left[G\left(\frac{1}{n} - x_L^-\right) - G\left(\frac{1}{n} - x_H^+\right)\right] \right|,$$

that $g(\cdot)$ is bounded by $2n$, and that for all $p < p_H$

$$\left|\frac{dx_\downarrow}{dp}\right|, \left|\frac{dx_\uparrow}{dp}\right| \leq \frac{1}{2t} \cdot \frac{2 + (\lambda - 1)F_\uparrow(p_H)}{2 + \frac{\lambda - 1}{2}},$$

we get that

$$|z_H^+ - z_L^-| \leq \frac{\lambda - 1}{2} 2n \; |x_H^+ - x_L^-| \leq \frac{\lambda - 1}{2} 2n \; (p_H - p_L) \left(\frac{2 + (\lambda - 1)F_\uparrow(p_H)}{2 + \frac{\lambda - 1}{2}} \; \frac{1}{2t}\right),$$

and by a similar logic $|z_H^- - z_L^+|$ has the same upper bound. Combining these with Equation 20 implies that it is sufficient to prove

$$\frac{1}{p_H - c} \eta_L (z_L^+ + z_L^-) \geq (\eta_H)^2 \frac{\lambda - 1}{2} 2n \; \frac{1}{2 + \frac{\lambda - 1}{2}} \frac{1}{2t} \left(z_L^+ \frac{1}{z_H^+} + z_L^- \frac{1}{z_H^-}\right).$$

Using that $\left(z_L^+ \frac{1}{z_H^+} + z_L^- \frac{1}{z_H^-}\right) \leq \frac{1}{2}(z_L^+ + z_L^-)\ (\frac{1}{z_H^+} + \frac{1}{z_H^-})$ it is sufficient to prove

$$\frac{1}{p_H - c}\eta_L \geq (\eta_H)^2 \frac{\frac{\lambda-1}{2}}{2 + \frac{\lambda-1}{2}} \frac{n}{2t} \left(\frac{1}{z_H^+} + \frac{1}{z_H^-}\right). \tag{21}$$

Since the high-price firm's demand is always less than or equal $\frac{1}{n}$, the fact that it does not want to lower its price implies

$$1 \geq \frac{n}{2t}(p_H - c)\eta_H \left(\frac{1}{z_H^+} + \frac{1}{z_H^-}\right).$$

Hence, a sufficient condition Equation 21 to hold is that

$$\eta_L \geq \eta_H \ \frac{\frac{\lambda-1}{2}}{2 + \frac{\lambda-1}{2}}.$$

For $n = 2$, $F(p_L) = F_\uparrow(p_H)$, so the above is satisfied for any $\lambda > 1$. For $n > 2$, using that $F(p_L) \geq 1/n$ and $F_\uparrow(p_H) \leq 1 + \lambda$, a sufficient condition for the above inequality to hold is that

$$(4 + \lambda - 1)(2n + \lambda - 1) \geq n(2 + \lambda - 1)(\lambda - 1).$$

Setting $a = \lambda - 1$, this can be rewritten as

$$0 \geq (n-1)a^2 - 4a - 8n.$$

Since this quadratic has one positive and one negative root, if a is positive and

$$a \leq \frac{2}{n-1}\left(1 + \sqrt{1 + 2n(n-1)}\right),$$

the inequality is satisfied. This gives the bound in the proposition.

Case II. In this case, we rewrite Equation 18 as

$$\eta_H \left(z_L^- \frac{z_L^+}{z_H^+} + z_L^+ \frac{z_L^-}{z_H^-}\right) \geq \left(1 - \frac{p_H - p_L}{p_H - c}\right) \eta_L \ (z_L^+ + z_L^-).$$

The remaining steps are analogous to Case I and thus omitted. $\square$

Proof of Proposition 5. We begin by proving that each firm's pricing function is continuous in cost. This fact follows from the following lemma.

Lemma 2. *In any interior equilibrium with* $\lambda < 38$*, firm* i*'s expected profit is single-peaked in its price for any cost realization* c_i*.*

Proof. Notice that (using Equation 4) profits are differentiable whereever the market price distribution does not have an atom—which is almost everywhere—and continuous. Furthermore, at prices where the profit function is not differentiable, it has a concave kink.

Suppose by contradiction that the profit function is not single-peaked. This implies that the profit function must have a trough. At this trough, it obviously cannot have a concave kink, so it is differentiable. To arrive at a contradiction, we prove that if firm 1's first-order condition is satisfied at some price p_1, profits are lower slightly to the right of p_1.

Let the subscript 1 x^+ and x^- denote partial derivative with respect to firm 1's price. Note that for each $x(p_1, p_{-1}) \in \{x^-(p_1, p_{-1}), x^+(p_1, p_{-1})\}$ one has

$$\limsup_{p_1' \searrow p_1} \frac{1}{p_1' - p_1} \cdot \left(x_1(p_1', p_{-1}) - x_1(p_1, p_{-1})\right) = -\frac{1}{2t} \frac{(\lambda - 1)F'(p_1)}{2 + \frac{\lambda-1}{2}(G(x(p_1, p_{-1})) + G(\frac{1}{n} - x(p_1, p_{-1})))}$$

$$+\frac{1}{2t} \frac{\frac{\lambda-1}{2}(2 + (\lambda-1)F(p_1)) \cdot \limsup_{p_1' \searrow p_1} \frac{G(x(p_1,p_{-1})) - G(x(p_1', p_{-1})) + G(1/n - x(p_1, p_{-1})) - G(1/n - x(p_1', p_{-1}))}{p_1' - p_1}}{[2 + \frac{\lambda-1}{2}(G(x(p_1, p_{-1})) + G(\frac{1}{n} - x(p_1, p_{-1})))]^2}$$

$$\leq (x_1(p_1, p_{-1}))^2 \; \frac{\frac{\lambda-1}{2} 2n}{2 + \frac{\lambda-1}{2}(G(x(p_1, p_{-1})) + G(\frac{1}{n} - x(p_1, p_{-1})))} \leq (x_1(p_1, p_{-1}))^2 \; \frac{\frac{\lambda-1}{2}2n}{2 + \frac{\lambda-1}{2}}. \tag{22}$$

Let $\pi(p) = (p - c)E[x^+(p, p_{-1}) + x^-(p, p_{-1})]$. We will prove that

$$\limsup_{p_1' \searrow p_1} \frac{\pi'(p_1') - \pi'(p_1)}{p_1' - p_1} < 0.$$

This is sufficient because it shows that the derivative of the profit function is negative to the right of and sufficiently close to p_1, so that profits are smaller there.

By Equation 22, it is sufficient to prove

$$(p_1 - c) \; \underbrace{E\left[\left(x_1^-(p_1, p_{-1})\right)^2 + \left(x_1^+(p_1, p_{-1})\right)^2\right]}_{I} \; \frac{\frac{\lambda-1}{2}2n}{2 + \frac{\lambda-1}{2}} + 2\,\underbrace{E\left[x_1^-(p_1, p_{-1}) + x_1^+(p_1, p_{-1})\right]}_{II} < 0. \tag{23}$$

To bound the above, we begin showing that I divided by II is less than or equal to $\frac{1}{2}\frac{(k+1)^2}{4k}$, where

$$k \equiv \frac{2 + \lambda - 1}{2 + \frac{\lambda-1}{2}}.$$

Observe that

$$\frac{\max\{x_1^-(p_1,p_{-1}),x_1^+(p_1,p_{-1})\}}{\min\{x_1^-(p_1,p_{-1}),x_1^+(p_1,p_{-1})\}} \leq \frac{2+\lambda-1}{2+\frac{\lambda-1}{2}} = k.$$

Now we use the following fact.

Fact 1. *Suppose $\tilde{a}_+$ and $\tilde{a}_-$ are positive random variables such that*

$$\frac{\sup\{\tilde{a}_+,\tilde{a}_-\}}{\inf\{\tilde{a}_+,\tilde{a}_-\}} \leq k$$

Then

$$\frac{E[\tilde{a}_+^2+\tilde{a}_-^2]}{E[\tilde{a}_++\tilde{a}_-]^2} \leq \frac{1}{2}\frac{(k+1)^2}{4k}. \tag{24}$$

Proof. Suppose without loss of generality that $1 \leq \tilde{a}_+,\tilde{a}_- \leq k$. Since the quadratic function is convex, the ratio on the left-hand side of Inequality 24 is maximized if the support if $\tilde{a}_+,\tilde{a}_-$ consists of the extremal values $1,k$. Thus, the left-hand side of the inequality is less than or equal to the maximum of

$$\max_{b^+,b^-\in[0,1]} \frac{b^+\left(k^2-1\right)+1+b^-\left(k^2-1\right)+1}{\left(b^+(k-1)+1+b^-(k-1)+1\right)^2},$$

which is equivalent to maximizing

$$\max_{b^+,b^-} \frac{1}{2}\left[\frac{\frac{b^++b^-}{2}\left(k^2-1\right)+1}{\left(\frac{b^++b^-}{2}(k-1)+1\right)^2}\right].$$

For brevity, let $b=\frac{b^++b^-}{2}$. Then the first order condition is satisfied if and only if

$$(b(k-1)+1)^2\left(k^2-1\right)-(b\left(k^2-1\right)+1)2(k-1)(b(k-1)+1)=0,$$

which yields $b=\frac{1}{k+1}$. Substituting this into the maximant and rewriting gives the desired inequality.

□

Hence, Inequality 23 gives

$$(p-c)\left|E\left[x_1^-(p_1,p_{-1})+x_1^+(p_1,p_{-1})\right]\right|\frac{1}{2}\frac{(k+1)^2}{4k}\frac{\frac{\lambda-1}{2}2n}{2+\frac{\lambda-1}{2}} < 2.$$

If the firm only prices its neighbors out of the market with probability zero one has

$$(p-c)\left|E\left[x_1^-(p_1,p_{-1})+x_1^+(p_1,p_{-1})\right]\right| \le \frac{2}{n}.$$

In this case, the above condition simplifies to

$$\frac{(k+1)^2}{4k}\frac{\frac{\lambda-1}{2}}{2+\frac{\lambda-1}{2}} < 1.$$

This condition holds for any $\lambda < 38$. □

From the above lemma, the following corollary is obvious.

Corollary 5. *In an interior equilibrium with $\lambda < 38$, the pricing function is continuous in cost for each firm.*

We are now ready to prove the statement of the proposition. We prove by contradiction; suppose that there exists (at least one) firm that does not charge a deterministic price. Corollary 5 implies that there must exist a nontrivial interval of prices, each of which the firm charges for some cost. On this interval, consider a price p^0 and a sequence of prices $p^i \searrow p^0$ such that i.) none of these prices are atoms of F; ii.) the pricing function $p(\cdot)$ is differentiable at p^0 with a strictly positive derivative.[27] Let the corresponding costs be c^0 and $c^i \searrow c^0$.

Using the same calculation as in the proof Lemma 2, we have

$$0 < p'(c^0) < \frac{1}{2-\frac{(k+1)^2}{2k}\frac{\frac{\lambda-1}{2}}{2+\frac{\lambda-1}{2}}}.$$

Let $x^+(p_1,p_{-1})$ and $x^-(p_1,p_{-1})$ be the demand of firm 1 in its two markets, as a function of its price and others' prices. By the firm's maximization problem,

$$(p^i-c^i)E\left[x_1^-(p^i,p_{-1})+x_1^+(p^i,p_{-1})\right]+E[x^+(p^i,p_{-1})+x^-(p^i,p_{-1})]=0 \tag{25}$$

for each i, and a similar condition holds at p^0.

[27] Given our estimation in Lemma 2 (which we also use again below to bound the derivative of $p(\cdot)$), we can show that $p(\cdot)$ is Lipschitz continuous. Hence, we can apply the Fundamental Theorem of Calculus to conclude that its derivative must be strictly positive on a set of positive measure.

Fix any p_{-1}. We find a condition under which for $x(\cdot,\cdot) \in \{x^{+}(\cdot,\cdot)x^{-}(\cdot,\cdot)\}$,

$$\limsup_{c^i \to c^0} \frac{\big[(p^i - c^i)x_1(p^i, p_{-1}) + x(p^i, p_{-1})\big] - \big[(p^0 - c^0)x_1(p^0, p_{-1}) + x(p^0, p_{-1})\big]}{c^i - c^0} < 0.$$

This is sufficient for a contradiction because it implies that the first-order condition 25 cannot hold for all p^i, p^0 (since the difference between the integrands is negative at all points, it is negative in expectation).

Notice that the above limsup is equal to

$$\begin{aligned}
& \lim_{c^i \to c^0} \frac{x(p^i, p_{-1}) - x(p^0, p_{-1})}{c^i - c^0} + \lim_{c^i \to c^0} \frac{[(p^i - p^0) - (c^i - c^0)]}{c^i - c^0} \cdot x_1(p^0, p_{-1}) \\
+ \;& \limsup_{c^i \to c^0} \;(p^i - c^i) \cdot \frac{x_1(p^i, p_{-1}) - x_1(p^0, p_{-1})}{c^i - c^0} \\
= \;& x_1(p^0, p_{-1}) \cdot (2p'(c_0) - 1) + (p^0 - c^0) \cdot \limsup_{c^i \to c^0} \;\frac{x_1(p^i, p_{-1}) - x_1(p^0, p_{-1})}{c^i - c^0}. \qquad (26)
\end{aligned}$$

Now we work on the last term above, which is equal to

$$\begin{aligned}
- \;& \frac{p^0 - c^0}{2t} \cdot \limsup_{c^i \to c^0} \frac{1}{c^i - c^0} \left\{ \left[\frac{(\lambda - 1)(F(p^i) - F(p^0))}{\{2 + \frac{\lambda-1}{2}[G(x(p^0, p_{-1})) + G((1/n) - x(p^0, p_{-1}))]\}} \right] \right. \\
+ \;& \left. \left[\frac{(2 + (\lambda - 1)F(p^i))}{\{2 + \frac{\lambda-1}{2}[G(x(p^i, p_{-1})) + G((1/n) - x(p^i, p_{-1}))]\}} - \frac{(2 + (\lambda - 1)F(p^i))}{\{2 + \frac{\lambda-1}{2}[G(x(p^0, p_{-1})) + G((1/n) - x(p^0, p_{-1}))]\}} \right] \right\} \\
= \;& -\frac{p^0 - c^0}{2t} \cdot \frac{(\lambda - 1)F'(p^0)}{\{2 + \frac{\lambda-1}{2}[G(x(p^0, p_{-1})) + G((1/n) - x(p^0, p_{-1}))]\}} - \frac{p^0 - c^0}{2t} \cdot \limsup_{c^i \to c^0} \frac{2 + (\lambda - 1)F(p^i)}{c^i - c^0} \\
\times \;& \frac{\frac{\lambda-1}{2} \cdot \big(G(x(p^0, p_{-1})) - G(x(p^i, p_{-1})) + G(x(1/n - p^0, p_{-1})) - G(1/n - x(p^i, p_{-1})) \big)}{\{2 + \frac{\lambda-1}{2}[G(x(p^0, p_{-1})) + G((1/n) - x(p^0, p_{-1}))]\} \cdot \{2 + \frac{\lambda-1}{2}[G(x(p^i, p_{-1})) + G((1/n) - x(p^i, p_{-1}))]\}} \\
\leq \;& (p^0 - c^0) \left. \frac{\partial x(p_1, p_{-1})}{\partial p_1} \right|_{p_1 = p^0} \left[\frac{\lambda - 1}{\lambda + 1} p'(c^0) F'(p^0) \right. \\
+ \;& \left. \limsup_{c^i \to c^0} \frac{1}{c^i - c^0} \frac{\frac{\lambda-1}{2} \cdot \big(G(x(p^0, p_{-1})) - G(x(p^i, p_{-1})) + G(x(1/n - p^0, p_{-1})) - G(1/n - x(p^i, p_{-1})) \big)}{\{2 + \frac{\lambda-1}{2}[G(x(p^0, p_{-1})) + G((1/n) - x(p^0, p_{-1}))]\}} \right].
\end{aligned}$$

Now, notice that one of $G(x(p^0, p_{-1})) - G(x(p^i, p_{-1}))$ and $G(x(1/n - p^0, p_{-1})) - G(1/n - x(p^i, p_{-1}))$ is greater than or equal to zero, and since $G(s) - G(s') \leq 2n(s - s')$ for any $s > s'$, the other one is greater than or equal to $-2n\left|x(p^i, p_{-1}) - x(p^0, p_{-1})\right|$. Using also $G(x(p^0, p_{-1})) + G(1/n - x(p^0, p_{-1})) \geq 1$, this implies that the above is less than or equal to

$$(p^0 - c^0)p'(c^0) \left(x_1(p^0, p_{-1}) \frac{\lambda - 1}{\lambda + 1} F'(p^0) + \big(x_1(p^0, p_{-1})\big)^2 \frac{(\lambda - 1)n}{2 + \frac{\lambda-1}{2}} \right).$$

Substituting into Expression 26 and using that $\left|x_1(p^0, p_{-1})\right| \leq \frac{1+\lambda}{2+\frac{\lambda-1}{2}} = \frac{k}{2t}$ implies that it is sufficient to prove

$$1 + k^2 \frac{\lambda - 1}{\lambda + 1} \frac{n}{2t}(p^0 - c^0)p'(c^0) \leq 2p'(c^0) + (p^0 - c^0)p'(c^0)\frac{\lambda - 1}{\lambda + 1}F'(p^0).$$

Using that $F'(p^0) \geq D(p^0)\frac{\theta(c^0)}{p'(c^0)}$ and that

$$p^0 - c^0 = \frac{D(p^0)}{-D'(p^0)} \leq \frac{\frac{2}{n}}{\frac{1}{2t}\frac{2}{1+\lambda}} = \frac{2t}{n}(1 + \lambda),$$

the above becomes

$$1 + k^2(\lambda - 1)p'(c^0) \leq 2p'(c^0) + (p^0 - c^0)D(p^0)\frac{\lambda - 1}{\lambda + 1}\theta(c^0). \tag{27}$$

To finish our proof, we put a bound on the firm's profits $(p^0 - c^0)D(p^0)$. In a market equilibrium, no firm charges a price less than $\underline{c}$, so firm 1's profits are at least as much as it would make if both of its neighbors charge $\underline{c}$ with probability one. If firm 1 also charges $\underline{c}$, its demand in each of its two markets is $\frac{1}{2n}$. Now

$$|x_1(p_1, p_{-1})| \leq \frac{1}{2t}\frac{1 + \lambda}{2 + \frac{\lambda-1}{2}} = \frac{k}{2t}.$$

This implies that a sufficient condition for the firm to be able to sell profitably is

$$\overline{c} - \underline{c} < \frac{\frac{1}{2n}}{\frac{k}{2t}} = \frac{t}{n}\frac{1}{k}.$$

Furthermore, if this is the case, its profits are at least

$$2(p - c)\left(\frac{1}{2n} - \frac{k}{2t}(p - \underline{c})\right) = (p - c)\left(\frac{1}{n} - \frac{k}{t}(p - \underline{c})\right).$$

Maximizing this expression with respect to p and setting $c = \overline{c}$ gives

$$(p^0 - c^0)D(p^0) \geq \frac{k}{4t}\left(\frac{t}{n}\frac{1}{k} - (\overline{c} - \underline{c})\right)^2 = \frac{k}{4}\frac{t}{n^2}\left(\frac{1}{k} - \gamma\right)^2.$$

Now we have two cases.

Case I: $k^2(\lambda - 1) \leq 2$. In this case, a sufficient condition for Inequality 27 to hold is

$$\frac{t}{n^2}\theta(c^0) \geq \frac{\lambda + 1}{\lambda - 1}\frac{4k}{1 - k\gamma}.$$

Case II: $k^2(\lambda - 1) > 2$. Then, substituting our bound for $p'(c^0)$ into Inequality 27 and rearranging gives that a sufficient condition is

$$\frac{t}{n^2}\theta(c^0) \geq \frac{4k}{1-k\gamma} \cdot \frac{(1+\lambda)k^2 - \frac{(k+1)^2}{4}}{2 - \frac{\lambda-1}{\lambda+1}\frac{(k+1)^2}{4}}.$$

This completes our proof. □

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