UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202B
Spring 2008
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Problem Set 1
Due in lecture Thursday, January 31

1. In an economy described by the Solow model where initially capital per unit of effective labor is below its balanced-growth-path value, as the economy converges to its balanced growth path:

A. The marginal product of capital is falling over time.
B. Consumption per unit of effective labor is rising over time.
C. Investment per unit of effective labor is rising over time.
D. (A) and (B).
E. (A) and (C).
F. (B) and (C).
G. All of the above.

2. In the Solow model, the long-run effect of an increase in the rate of population growth (relative to what would have happened without the change in population growth) is to:

A. Lower income per person.
B. Have no impact on the growth rate of income per person.
C. Increase the growth rate of total income.
D. (A) and (B).
E. (A) and (C).
F. (B) and (C).
G. (A), (B), and (C).

3. (The Solow model with Leontief production.) Consider an economy that is described by the Solow model, except that: (1) The production function is $Y(t) = \min[K(t), A(t)L(t)]$; (2) $n + g + \delta$ is assumed to satisfy $0 < n + g + \delta < s$.

a. As in the usual Solow model, define $k \equiv K/(AL)$, $y \equiv Y/(AL)$. Find the function $y = f(k)$ that gives y as a function of k. Does $f(\bullet)$ satisfy the Inada conditions? Why or why not?

b. Draw the Solow diagram (that is, a diagram showing $sf(k)$ and $(n + g + \delta)k$ as functions of k) for this model.

c. In the standard Solow model, there is a k^* such that for any $k(0) > 0$, k converges to k^* and then remains there. Is this true in this model? Explain your answer.

d. Assume the economy starts on a balanced growth path with $k > 0$, and that there is a permanent increase in s. Is the long-run impact of this change on k positive, negative, or zero (or impossible to tell)? Is the long-run impact on y positive, negative, or zero (or impossible to tell)? Explain your answers.

(OVER)

e. Let us change the model to make technological change capital-augmenting rather than labor-augmenting. Specifically, assume $Y(t) = \min[B(t)K(t), L(t)]$. Assume the dynamics of B are given by $\dot{B}(t) = gB(t)$, and assume that $sB(0) > n + \delta$. The other assumptions are the same as in the standard Solow model.

i. Show that, if we define $y \equiv Y/L$ and $k \equiv K/L$, the dynamics of k are given by $\dot{k}(t) = sy(t) - (n + \delta)k(t)$.

ii. True or false: For any $k(0) > 0$, the economy converges to a situation where Y/L is growing at a constant rate. Explain your answer.

4. Consider the Solow model. Let z(t) denote the marginal product of capital, $\partial F(K(t), A(t)L(t))/\partial K(t)$.

a. Show that $z(t) = f'(k(t))$.

b. Let y* denote the balanced-growth-path value of y. If y(t) is greater than y*, is $\dot{z}(t)$ positive, negative, zero, or of ambiguous sign? Explain your answer.

c. Let z* denote the balanced-growth-path value of z. Find an expression of the form $\dot{z}(t) \simeq a + b[z(t) - z^*]$ in the vicinity of $z = z^*$.

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

5. Describe how, if at all, each of the following developments affects the break-even and actual investment lines in our basic diagram for the Solow model:

a. The rate of population growth falls.

b. The rate of technological progress rises.

c. The production function is Cobb-Douglas, $F(K, AL) = K^{\alpha}(AL)^{1-\alpha}$, and capital's share, α, rises.

d. Workers exert more effort, so that output per unit of effective labor for a given value of capital per unit of effective labor is higher than before.

6. Consider an economy described by the Solow model that is on its balanced growth path. Now suppose that, because of acid rain, the depreciation rate rises permanently.

Sketch the resulting path of log output per worker and what that path would have been if the depreciation rate had not changed. Explain your answer.

7. Consider an economy described by the Solow model. Assume that initially capital and output per unit of effective labor are less than their balanced-growth-path values. Now suppose that in this situation, the saving rate rises permanently.

Sketch the resulting path of the log of output per worker and what that path would have been if the saving rate had not changed. Explain your answer.

8. Consider the Solow model. Find an expression for the elasticity of the balanced-growth-path level of output per unit of effective labor with respect to $n + g + \delta$. Simplify your expression as much as possible.