UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202B
Spring 2008
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Problem Set 6
Due in lecture Thursday, March 6

1. Consider the version of the Romer model presented in lecture. Suppose, however, that households have constant relative risk aversion utility with a coefficient of relative risk aversion of θ. Find the equilibrium level of labor in the R&D sector, L_A.

2. (This is based on Jones, AER, 2002.) Consider the model presented in equations (3.45)-(3.49) of Romer, with the assumption that $G(E) = e^{\phi E}$. Suppose, however, that E, rather than being constant, is increasing steadily: $\dot{E}(t) = m$, where $m > 0$. (Assume that, despite the steady increase in the amount of education people are getting, the growth rate of the number of workers is constant and equal to n, as in the basic model.)

a. With this change in the model, what is the long-run growth rate of output per worker?

b. In the United States over the past century, if we measure E as years of schooling, $\phi \approx 0.1$ and $m \approx 1/15$. Overall growth of output per worker has been about 2 percent per year. In light of your answer to (a), approximately what fraction of this overall growth has been due to increasing education?

c. Can $\dot{E}(t)$ continue to equal $m > 0$ forever? Explain.

3. Hall and Jones measure the contribution of differences in education to cross-country income differences by:

A. Running a cross-country regression (that is, a regression where the unit of observation is a country) of log output per worker on average years of education and other variables, using OLS.

B. Running a cross-country regression of log output per worker on average years of education and other variables, using IV, where the instruments are distance from the equator, a measure of trade openness based on geographic characteristics, and measures of native speakers of major European languages.

C. Combining micro evidence about how individuals' earnings depend on their years of education with data on the average years of education of workers in different countries.

D. Using micro evidence about the earnings of immigrants from different countries in the United States.

4. (A different form of measurement error.) Suppose the true relationship between social infrastructure (SI) and log income per person (y) is $y_i = \alpha + \beta SI_i + \varepsilon_i$. SI and ε are uncorrelated. Unfortunately, there are two components of social infrastructure, SI^A and SI^B (with $SI_i = SI_i^A + SI_i^B$), and we only have data on one of the components, SI^A. Both SI^A and SI^B are uncorrelated with ε. We are considering running an OLS regression of y on a constant and SI^A.

(OVER)

a. Derive an expression of the form, $y_i = \alpha + \beta SI_i^A +$ other stuff.

b. Use your answer to part (a) to determine whether an OLS regression of y on a constant and SI^A will produce an unbiased estimate of the impact of social infrastructure on income if:

i. SI^A and SI^B are uncorrelated.

ii. SI^A and SI^B are positively correlated.

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

5. Consider a cross-country regression of log output per worker on a measure of social infrastructure,

$$\ln(Y_i/L_i) = a + bSI_i + e_i. \qquad (*)$$

A variable Z is a good instrument for SI if it is correlated with SI and if:

A. It is uncorrelated with the fitted residuals when we estimate (*) by OLS.

B. We know from auxiliary evidence that Z is not affected by social infrastructure,

C. If we regress Z on a constant and SI we obtain a coefficient that is not significantly different from zero, and so we cannot reject the null hypothesis that Z is not affected by SI.

D. (A) or (B).

E. (A) or (C).

F. (B) or (C).

G. None of the above.

6. Consider a monopolist who can produce at a constant marginal cost of c and who faces the demand curve $Q = BP^{-\eta}$, $\eta > 1$. Show that the profit-maximizing price is $[\eta/(\eta - 1)]c$.

EXTRA EXTRA PROBLEMS (NOT TO BE HANDED IN/NO ANSWERS WILL BE PROVIDED)

7. Consider the version of the Paul Romer model presented in lecture. Consider allocations where $L_A(t)$ is constant over time. Find the lifetime utility of the representative household as a function of L_A (and of anything else that is relevant). Find the level of L_A that maximizes the utility of the representative household.

8. Consider the version of the Paul Romer model presented in lecture. Set up the problem of choosing the path of $L_A(t)$ to maximize the lifetime utility of the representative household. What is the control variable? What is the state variable? What is the current value Hamiltonian? Find the conditions that characterize the optimum, and show that they imply that the optimal allocation has L_A constant over time.