The Weak Link Theory of Economic Development

Charles I. Jones*

Department of Economics, U.C. Berkeley and NBER
E-mail: chad@econ.berkeley.edu
http://www.econ.berkeley.edu/~chad

October 17, 2006– Version 0.45

Per capita income in the richest countries of the world exceeds that in the poorest countries by more than a factor of 50. What explains these enormous differences? This paper returns to two old ideas in development economics and proposes that complementarity and linkages are at the heart of the explanation. First, just as a chain is only as strong as its weakest link, problems at any point in a production chain can reduce output substantially if inputs enter production in a complementary fashion. Second, linkages between firms through intermediate goods deliver a multiplier similar to the one associated with capital accumulation in a neoclassical growth model. Because the intermediate goods' share of revenue is about 1/2, this multiplier is substantial. The paper builds a model with complementary inputs and links across sectors and shows that it can easily generate 50-fold aggregate income differences.

*I would like to thank Daron Acemoglu, Pol Antras, Bill Easterly, Luis Garicano, Chang Hsieh, Pete Klenow, Alwyn Young and seminar participants at Berkeley, the Chicago GSB, Toulouse, USC, and the World Bank for helpful comments. I am grateful to the Hong Kong Institute for Monetary Research for hosting me during the early stages of this research and to the Toulouse Network for Information Technology for financial support.

1. INTRODUCTION

By the end of the 20th century, per capita income in the United States was more than 50 times higher than per capita income in Ethiopia and Tanzania. Dispersion across the 95th-5th percentiles of countries was more than a factor of 32. What explains these profound differences in incomes across countries?[1]

This paper returns to two old ideas in the development economics literature and proposes that complementarity and linkages are at the heart of the explanation. Because of complementarity, high productivity in a firm requires a high level of performance along a large number of dimensions. Textile producers require raw materials, knitting machines, a healthy and trained labor force, knowledge of how to produce, security, business licenses, transportation networks, electricity, etc. These inputs enter in a complementary fashion, in the sense that problems with any input can substantially reduce overall output. Without electricity or production knowledge or raw materials or security or business licenses, production is likely to be severely hindered.

Intermediate goods provide links between sectors that create a productivity multiplier. Low productivity in electric power generation reduces productivity in banking and construction. But this reduces the ease with which the electricity industry can build new dams and therefore further reduces output in electric power generation. This multiplier effect is similar to the multiplier associated with capital accumulation in a neoclassical growth model. However, because the intermediate goods' share of revenue is approximately 1/2, the intermediate goods multiplier is large.

The metaphor that works best to describe this paper is the old adage, "A chain is only as strong as its weakest link." Complementarity and linkages in the

[1]Recent work on this topic includes Romer (1994), Klenow and Rodriguez-Clare (1997), Prescott (1998), Hall and Jones (1999), Parente and Prescott (1999), Acemoglu, Johnson and Robinson (2001), Klenow and Rodriguez-Clare (2005), Manuelli and Seshadri (2005), Caselli and Coleman (2006), Armenter and Lahiri (2006), Erosa, Koreshkova and Restuccia (2006), and Marimon and Quadrini (2006).

economy mean that problems at any point in the production chain can sharply reduce overall output. The strength of a typical link need not differ by a large amount between rich and poor countries. Instead, what differs is the strength of the weakest links.

The contribution of this paper is to build a model in which these ideas can be made precise. We show that complementarity and linkages amplify small differences across economies. With plausible average differences in productivity across countries, we are able to explain 50-fold differences in per capita income.

2. LINKAGES AND COMPLEMENTARITY

We begin by discussing briefly the two key mechanisms at work in this paper, linkages and complementarity. These mechanisms are conceptually distinct — one can have linkages without complementarity. The linkage mechanism turns out to be quite simple and powerful, so very little time (perhaps too little) will be needed to convey its role. The complementarity mechanism is more difficult to model and hence consumes more than its share of words and effort in this paper.

2.1. Linkages through Intermediate Goods

The notion that linkages across sectors can be central to economic performance dates back at least to Leontief (1936), which launched the field of input-output economics. Hirschman (1958) emphasized the importance of complementarity and linkages to economic development. A large subsequent empirical literature constructed input-output tables for many different countries and computed sectoral multipliers.

In what may prove to be an ill-advised omission, these insights have not generally be incorporated into modern growth theory. Linkages between sectors through intermediate goods deliver a multiplier very much like the multiplier associated with capital in the neoclassical growth model. More capital leads to more output, which in turn leads to more capital. This geometric series sums to a multiplier equal to $\frac{1}{1-\alpha}$, if α is capital's share of overall revenue. Because the capital share is only about 1/3, this multiplier is relatively small: differences in investment rates are too small to explain large income differences, and large total factor productivity residuals are required. This led a number of authors to broaden the definition of capital, say to include human capital or organizational capital. It is generally recognized that if one can get the capital share up to something like 2/3 — so the multiplier is 3 — large income differences are much easier to explain without appealing to a large residual.[2]

Intermediate goods generate this same kind of multiplier. An increase in productivity in the transportation sector raises output in the capital equipment sector which may in turn raise output in the transportation sector. In simple models, this multiplier depends on $\frac{1}{1-\sigma}$, where σ is the share of intermediate goods in total revenues. In the United States, this share is approximately 1/2, delivering a multiplier of 2. In the model, the overall multiplier on productivity is the product of the intermediate goods and capital multipliers: $\frac{1}{1-\sigma} \times \frac{1}{1-\alpha} = 2 \times 3/2 = 3$. Combining a neoclassical story of capital accumulation with a standard treatment of intermediate goods therefore delivers a very powerful engine for explaining income differences across countries. Related insights pervade the older development literature but have not had a large influence on modern growth theory. The main exception is Ciccone (2002), which appears to be underappreciated.[3]

[2] Mankiw, Romer and Weil (1992) is an early example of this approach to human capital. Chari, Kehoe and McGrattan (1997) introduced "organizational capital" for the same reason. More recently, Manuelli and Seshadri (2005) and Erosa et al. (2006) have resurrected the human capital story in a more sophisticated fashion.

[3] Ciccone develops the multiplier formula for intermediate goods and provides some quantitative examples illustrating that the multiplier can be large. The point may be overlooked by readers of his paper because the model also features increasing returns, externalities, and multiple equilibria. Interestingly, the intermediate goods multiplier shows up most clearly in the economic fluctuations literature; see Long and Plosser (1983), Basu (1995), Horvath (1998), Dupor (1999), Conley and Dupor (2003), and Gabaix (2005). See also Hulten (1978).

2.2. The Role of Complementarity

A large multiplier in growth models is a two-edged sword. On the one hand, it is extremely useful in getting realistic differences in investment rates, productivity, and distortions to explain large income differences. However, the large multiplier often has a cost. In particular, theories of economic development often suffer from a "magic bullet" critique. If the multiplier is so large, then solving the development problem is often quite easy. For example, this is a potential problem in the Manuelli and Seshadri (2005) paper: small subsidies to the production of output or small improvements in a single (exogenous) productivity level have enormous long-run effects on per capita income in their model. If there were a single magic bullet for solving the world's development problems, one would expect that policy experimentation across countries would hit on it, at least eventually. The magic bullet would become well-known and the world's development problems would be solved.

This is where the second insight of this paper plays it role. Because of complementarity, the development problem may be hard to solve. In any production process, there are ten things that can go wrong that will sharply reduce the value of production. In rich countries, there are enough substitution possibilities that these things do not often go wrong. In poor countries, on the other hand, any one of several problems can doom a project. Obtaining the instruction manual for how to produce socks is not especially useful if the import of knitting equipment is restricted, if cotton and polyester threads are not available, if property rights are not secure, and if the market to which these socks will be sold is unknown. Complementarity is at the heart of the O-ring theory put forward by Kremer (1993). The idea in this paper is similar, but the papers differ substantially in crucial ways. These differences will be discussed in detail below.

Linkages through intermediate goods provide a large multiplier, while complementarity means that there is typically not a single magic bullet that can exploit this multiplier. Occasionally, of course, there is. Fixing the last bottleneck to development can have large effects on incomes, which may help us to understand growth miracles.

2.3. An Example of Complementarity

Standard models of production emphasize the substitutability of different inputs. While substitution will play an important role in the model that follows, so will complementarity. Since this is less familiar, we begin by focusing our attention on complementary inputs.

For this purpose, it is helpful to begin with a simple example. Suppose you'd like to set up a factory in China to make socks. The overall success of this project requires success along a surprisingly large number of different dimensions. These different activities are complementary, so that inefficiencies on any one dimension can sharply reduce overall output.

As one example, the managers of the firm require knowledge of exactly how to manufacture socks. This kind of knowledge plays a central role in the endogenous growth literature following Romer (1990).

Second, the firm needs the basic inputs of production. These include cotton, silk, and polyester; the sock-knitting machines that spin these threads into socks; a competent, healthy, and motivated workforce; a factory building; electricity and other utilities; a means of transporting raw materials and finished goods throughout the factory, etc.

Apart from the physical production of socks, other activities are required to turn raw materials into revenue. The entire production process must be kept secure from theft or expropriation. The sock manufacturer must match with buyers, perhaps in foreign markets, and must find a way to deliver the socks to these buyers. Legal requirements must also be met, both domestically and in foreign markets. Firms must acquire the necessary licenses and regulatory approval for production and trade.

The point of this somewhat tedious enumeration is that production — even of something as simple as a pair of socks — involves a large number of necessary activities. If any of these activities are performed inefficiently, overall output can be reduced considerably. Without a reliable supply of electricity, the sock-making machines cannot be utilized efficiently. If workers are not adequately trained or are unhealthy because of contaminated water supplies, productivity will suffer. If export licenses are not in order, the socks may sit in a warehouse rather than being sold. If property is not secure, the socks may be stolen before they can reach the market.

2.4. Modeling Complementary Inputs

A natural way to model the complementarity of these activities is with a CES production function:

$$Y = \left(\int_0^1 z_i^\rho di \right)^{1/\rho}. \tag{1}$$

We use z_i to denote a firm's performance along the i^{th} dimension, and we assume there are a continuum of activities indexed on the unit interval that are necessary for production. In terms of our sock example, z_a could be the quality of the instructions the firm has for making socks. z_b could be number of sock-making machines, z_c might represent the extent to which the relevant licenses have been obtained, etc.

The elasticity of substitution among these activities is $1/(1-\rho)$, so the degree of complementarity is a parameter. With $\rho = 0$, the elasticity of substitution is one and the production function is Cobb-Douglas. But if $\rho < 0$, inputs are even more complementary and the elasticity of substitution is less than one.

Complementarity puts extra "weight" on the activities in which the firm is least successful. This is easy to see in the limiting case where $\rho \to -\infty$; in this case, the CES function converges to the minimum function, so output is equal to the smallest of the z_i.

This intuition can be pushed further by noting that the CES combination in equation (1) is called the *power mean* of the underlying z_i in statistics. The power mean is just a generalized mean. For example, if $\rho = 1$, Y is the arithmetic mean of the z_i. If $\rho = 0$, output is the geometric mean (Cobb-Douglas). If $\rho = -1$, output is the harmonic mean, and if $\rho \to -\infty$, output is the minimum of the z_i. From a standard result in statistics, these means decline as ρ becomes more negative. Economically, a stronger degree of complementarity puts more weight on the weakest links and reduces output.

The essence of the story pursued here is this. On average, rich countries like the United States are only a little bit better — maybe by a factor of two — than the poorest countries in their underlying productivity at performing the key activities of production. Because of complementarity, however, it is not the average that matters. Instead, a chain is only as strong as its weakest link. Poor countries are poor because very low productivity at one or more essential activities reduces overall output. In a sense that will be made more precise below, poor countries have a thicker lower tail in the distribution of productivities, and complementarity among activities inflates these differences in the lower tail.

2.5. Comparing to Kremer's O-Ring Approach

It is useful to compare the way we model complementarity to the O-ring theory of income differences put forward by Kremer (1993). Superficially, the theories are similar, and the general story Kremer tells is helpful in understanding the current paper: the space shuttle Challenger and its seven-member crew are destroyed because of the failure of a single, inexpensive rubber seal.

This paper differs crucially, however, in terms of how the general idea gets implemented. In particular, Kremer's modeling approach assumes a large degree of increasing returns, which is difficult to justify.

To see this, recall that Kremer assumes there are N different tasks that must be completed for production to succeed. Suppose workers have a probability of success q at any task, and assume these probabilities are independent. Expected output is then given by $Q = q^N$. Suppose the richest countries are flawless in production, so $q^{rich} = 1$, while the poorest countries are successful in each task 50 percent of the time, so $q^{poor} = 1/2$. The ratio of incomes between rich and poor countries is therefore on the order of 2^N. If there are five different tasks in production, it is quite easy to explain a 32-fold difference in incomes across countries.

A problem with this approach is that the O-ring logic implies complementarity, but it does not imply the huge degree of increasing returns assumed in Kremer's $Q = q^N$ formulation. For example, an alternative production function that is also perfectly consistent with the O-ring story is $Q = q_1^{1/N} q_2^{1/N} \cdot ... \cdot q_N^{1/N}$ — that is, a Cobb-Douglas combination of tasks with constant returns. Notice that the O-ring complementarity applies here as well: if any q_i is zero, then $Q = 0$ and the entire project fails. With symmetry so that $q_i = q$, this approach leads to $Q = q$, so that a 2-fold difference in success on each task only translates into a 2-fold difference in incomes across countries.

While the O-ring story is quite appealing, Kremer's formulation relies on an arbitrary and exceedingly strong degree of increasing returns — which is not part of the O-ring logic — to get big income differences. The approach taken here is to drop the large increasing returns inherent in Kremer's formulation and to emphasize complementarity instead.

3. SETTING UP THE MODEL

We now apply this basic discussion of complementarity and linkages to construct a theory of economic development.

3.1. The Economic Environment

A single final good in this economy is produced using a continuum of activities that enter in a complementary fashion, as discussed above:[4]

$$Y = \zeta \cdot \left(\int_0^1 Y_i^\rho di \right)^{1/\rho}, \quad \rho < 0. \tag{2}$$

In this expression, Y_i denotes the activity inputs, and ζ is a constant that we will use to simplify some expressions later.[5]

Activities are themselves produced using a relatively standard Cobb-Douglas production function:

$$Y_i = A_i \left(K_i^\alpha H_i^{1-\alpha} \right)^{1-\sigma} X_i^\sigma, \tag{3}$$

where α and σ are both between zero and one. K_i and H_i are the amounts of physical capital and human capital used to produce activity i, and A_i is an exogenously-given productivity level. The novel term in this production specification is X_i, which denotes the quantity of intermediate goods used to produce activity i.

Before discussing the role of X_i, it is convenient to specify the three resource constraints that face this economy:

$$\int_0^1 K_i di \leq K, \tag{4}$$

$$\int_0^1 H_i di \leq H, \tag{5}$$

and

$$C + \int_0^1 X_i di \leq Y. \tag{6}$$

[4]Becker and Murphy (1992) consider a production function that combines a continuum of tasks in a Leontief way to produce output. They use this setup to study the division of labor and argue that it is limited by problems in coordinating the efforts of specialized workers.

[5]In particular, we assume $\zeta = \sigma^{-\sigma}$, where σ will be defined below.

The first two constraints are straightforward. We assume the economy is endowed with an exogenous amount of physical capital, K, and human capital, H, that can be used in production. Later on, we will endogenize K and H in standard ways, but it is convenient to take them as exogenous for now.

The last resource constraint says that final output can be used for consumption, C, or for the X_i intermediate goods. One unit of the final good can be used as one unit of the intermediate input in any activity.[6]

One can think of this as follows. Consider the production of the i^{th} activity Y_i, which we might take to be transportation services. Transportation is produced using physical capital, human capital, and some intermediate goods from other sectors (such as fuel). The share of intermediate goods in the production of the i^{th} activity is σ. To keep the model simple and tractable, we assume that the same bundle of intermediate goods are used in each activity, and that these intermediate goods are just units of final output.

The parameter σ measures the importance of *linkages* in our economy. If $\sigma = 0$, the productivity of physical and human capital in each activity depends only on A_i and is independent of the rest of the economy. To the extent that $\sigma > 0$, low productivity in one activity feeds back into the others. Transportation services may be unproductive in a poor country because of inadequate fuel supplies or repair services. Low productivity in the telecommunications sector reduces output throughout the economy.

3.2. Substitution and Complementarity

This basic setup is not necessarily the most natural way to formulate the model. In particular, one could imagine directly replacing X_i in equation (3) with a

[6] An issue of timing arises here. To keep the model simple and because we are concerned with the long run, we make the seemingly strange assumption that intermediate goods are produced and used simultaneously. A better justification goes as follows. Imagine incorporating a lag so that today's final good is used as tomorrow's intermediate input. The steady state of that setup would then deliver the result we have here.

CES combination of the different activities. Separately, the final good could be produced as a Cobb-Douglas function of the activities, as opposed to (2). Intermediate goods would then involve substantial complementarity (think of materials and energy), but when activities combine to produce the consumption good, there would be more substitutability. For example, computer services are today nearly an essential input into semiconductor design, banking, and health care, but there may be substantial substitution between computer games and other sources of entertainment in consumption. In order to produce within a firm, there are a number of complementary steps that must be taken. At the final consumption stage, however, there appears to be a reasonably high degree of substitution across goods.

Unfortunately, this more natural formulation does not lead to closed-form solutions. The simplification here replaces these two conceptually distinct production functions with the single CES combination. This makes sense at the level of the activity production function in (3), but it is a stretch when applied to the final good in (2). Nevertheless, this is the trick needed to make progress analytically. While it generally works well, we will see that this formulation does have some minor drawbacks.

4. ALLOCATING RESOURCES AND SOLVING

Taking the aggregate quantities of physical and human capital as given, we consider two alternative ways of allocating resources. The first is a symmetric allocation of resources across the activities. This allocation is not optimal, but it is quite easy to solve for and allows us to get quickly to some of the important results in this paper. Second, we consider the optimal allocation of resources, an obvious allocation of interest. These two allocations are defined in turn.

DEFINITION 4.1. The *symmetric misallocation of resources* in this economy has $K_i = K$, $H_i = H$, $X_i = X$, and $X = \bar{s}Y$, where $0 < \bar{s} < 1$. Moreover, we assume $\bar{s} = \sigma$, which turns out to be the optimal share of output to use as intermediate goods. Y and Y_i are then determined from the production functions in (2) and (3).

The optimal allocation of resources is the choice of K_i, H_i, and X_i that maximizes consumption:

DEFINITION 4.2. The *optimal allocation of resources* in this economy consists of values for the six endogenous variables $Y, C, \{Y_i, K_i, H_i, X_i\}$ that solve

$$\max_{\{X_i, K_i, H_i\}} C \equiv Y - \int_0^1 X_i di$$

subject to

$$Y = \zeta \cdot \left(\int_0^1 Y_i^\rho di \right)^{1/\rho}$$

$$Y_i = A_i \left(K_i^\alpha H_i^{1-\alpha} \right)^{1-\sigma} X_i^\sigma$$

$$\int_0^1 K_i di = K$$

$$\int_0^1 H_i di = H$$

where the productivity levels A_i are given exogenously.[7]

We report the solution of the model under these two allocations in a series of propositions, not because the results are especially deep, but because this helps organize the algebra in a useful way, both for presentation and for readers who wish to solve the model themselves. (Outlines of the proofs are in the Appendix.)

4.1. Solving With Misallocation

For expositional reasons and because it is easy to solve for, we begin with the symmetric misallocation. In the symmetric misallocation, $Y_i = A_i m$, where

[7] In terms of counting equations and unknowns, notice that we get three sets of first order conditions from the maximization, and then we have three main equations determining consumption, output, and the activities. The last two resource constraint equations do not really count as they give a single restriction but there are a continuum of capital allocations to be chosen.

$m \equiv (K^\alpha H^{1-\alpha})^{1-\sigma}(\bar{s}Y)^\sigma$ is constant across activities. Therefore final output just depends on the CES combination of the A_i with curvature parameter ρ, as stated in the following proposition:

PROPOSITION 4.1. *(The Symmetric Misallocation.) Under the symmetric misallocation of resources, total production of the final good is given by*

$$Y = Q_m^{\frac{1}{1-\sigma}} K^\alpha H^{1-\alpha}, \tag{7}$$

where

$$Q_m \equiv \left(\int_0^1 A_i^\rho di \right)^{\frac{1}{\rho}}. \tag{8}$$

The model delivers a simple expression for final output. Y is the familiar Cobb-Douglas combination of aggregate physical and human capital with constant returns to scale.

Two novel results also emerge, and both are related to total factor productivity. The first illustrates the role of complementarity, while the second reveals the multiplier associated with linkages through intermediate goods.

Total factor productivity is a CES combination of the productivities of the individual activities. Activities enter production in a complementary fashion, and this complementarity shows up in the aggregate production function in TFP.

To interpret this result, it is helpful to consider the special case where $\rho \to -\infty$. In this case, the CES function becomes the minimum function, so that $Q_m = \min\{A_i\}$. Aggregate TFP then depends on the smallest level of TFP across the activities of the economy. That is, aggregate TFP is determined by the weakest link. Firms in the United States and Kenya may not differ that much in average efficiency, but if the distribution of Kenyan firms has a substantially worse lower tail, overall economic performance will suffer because of complementarity.

The second property of this solution worth noting is the multiplier associated with intermediate goods. Total factor productivity is equal to the CES combination of underlying productivities raised to the power $\frac{1}{1-\sigma} > 1$. A simple example should make the reason for this transparent. Suppose $Y_t = aX_t^\sigma$ and $X_t = sY_{t-1}$; output depends in part on intermediate goods, and the intermediate goods are themselves produced using output from the previous period. Solving these two equations in steady state gives $Y = a^{\frac{1}{1-\sigma}} s^{\sigma/1-\sigma}$, which is a simplified version of what is going on in our model. Obviously, this is very similar to the multiplier that emerges because of capital accumulation in a standard neoclassical growth model, where the term $\frac{1}{1-\alpha}$ appears frequently.

The economic intuition for this multiplier is also straightforward. Low productivity in electric power generation reduces output in the banking and construction industries. But problems in these industries hinder the financing and construction of new dams and electric power plants, further reducing output in electric power generation. Linkages between sectors within the economy generate an additional multiplier through which productivity problems get amplified.

At some level, the paper could end here. The main points of the model both appear in the symmetric allocation: the role of complementarity and the multiplier associated with intermediate goods. The remainder of the paper develops these points further, adds a few insights, and considers some numerical examples.

4.2. Solving for the Optimal Allocation

The optimal allocation is more tedious to solve for, but it is a natural one to focus on in this environment. Resources can be misallocated in many ways, and there is nothing to recommend our symmetric misallocation other than its simplicity. The optimal allocation shows the best that a country can do given its endowments of inputs and exogenous productivities. The solution of the model in this case is next.

PROPOSITION 4.2. *(The Optimal Allocation.) When physical capital, human capital, and intermediate goods are allocated optimally across activities, total production of the final good is given by*

$$Y = Q_o^{\frac{1}{1-\sigma}} K^\alpha H^{1-\alpha}, \tag{9}$$

where

$$Q_o \equiv \left(\int_0^1 A_i^{\frac{\rho}{1-\rho}} di \right)^{\frac{1-\rho}{\rho}}. \tag{10}$$

The optimal allocations for K_i, H_i, and X_i are [zzz to be filled in].

It is useful to compare this result with the previous proposition. The aggregate production function takes the same form, and the multiplier associated with intermediate goods appears once again.

The essential difference relative to the previous result is that the curvature parameter determining the productivity aggregate is now $\frac{\rho}{1-\rho}$ rather than the original ρ. Notice that if the domain of ρ is $[0, -\infty)$, the domain of $\frac{\rho}{1-\rho}$ is $[0, -1)$, which means there is less complementarity in determining Q_o than there was in the original CES combination of activities. The reason is that the optimal allocation strengthens weak links by allocating more resources to activities with low productivity. If the transportation sector has especially low productivity, the optimal allocation will put extra physical and human capital in that sector to help offset its low productivity and prevent this sector from becoming a bottleneck. Interestingly, this shows up in the math by raising the effective elasticity of substitution used to aggregate the underlying productivities.

This result can be illustrated with an example. Suppose $\rho \to -\infty$. In this case, the symmetric misallocation depends on the smallest of the A_i, the pure weak link story. In contrast, the optimal allocation depends on the harmonic mean of the productivities, since $\frac{\rho}{1-\rho} \to -1$. Disasterously low productivity in a single activity is fatal in the symmetric allocation. In the optimal allocation, resources can substitute for low productivity, and weak links get strengthened.

5. EVALUATING TFP

The expressions for Q_m and Q_o above are nice, but it is not immediately obvious how to use them to quantify TFP differences across countries. At the moment, we have a continuum of exogenous productivity levels, A_i. In this section, we parameterize this continuum parsimoniously for the purpose of quantifying the predictions of the model. This should be viewed as a convenient simplifying device rather than as something fundamental in the model.

In this spirit, we now assume the A_i are distributed independently according to a Weibull distribution. That is,

$$\text{Pr}\left[A_i \leq a\right] \equiv F(a) = 1 - e^{-(a/\beta)^{\theta}}. \tag{11}$$

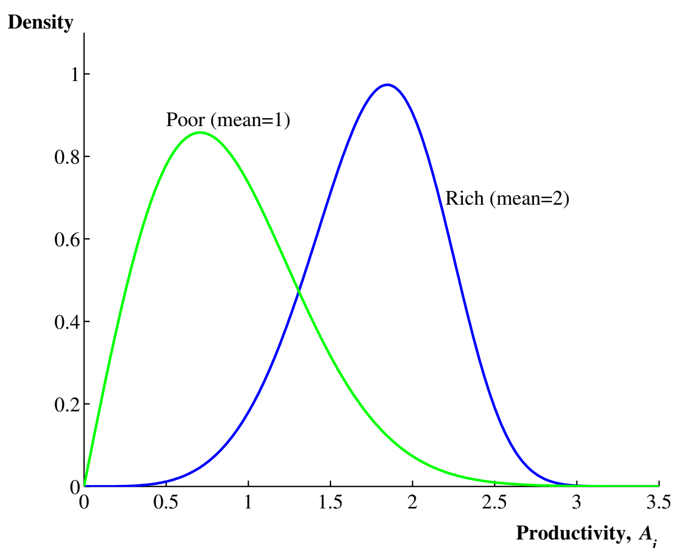
The mean of this distribution is $\beta\Gamma(1 + 1/\theta)$, where $\Gamma(\cdot)$ is Euler's factorial function (which will be discussed in more detail below). The Weibull distribution is chosen because it is very flexible and yet can be transformed and integrated up in nice ways.

To capture differences in the A_i, we assume the parameters of this distribution — β and θ — differ across countries. Figure 1 shows an example.

As a rough rule of thumb, one can think of β as determining the mean and θ as determining the thickness of the lower tail. For example, if $\theta = 1$, the Weibull distribution is an exponential distribution, and therefore has lots of mass in the lower tail. For $\theta > 1$, the Weibull looks something like a log-normal distribution.

In Figure 1, the "rich" country has $\beta = 1.93$ and $\theta = 5$, while the "poor" country has $\beta = 1$ and $\theta = 2$. The average value of productivity in the rich country works out to be twice that in the poor country, showing the role of β. The poor country has a thicker lower tail, as reflected in the θ parameters. Our two countries have different underlying productivities, but on average they

FIGURE 1. The Weibull Distribution of A_i



Note: The Weibull density for the "rich" country has $\beta = 1.93$ and $\theta = 5$, while the density for the poor country has $\beta = 1$ and $\theta = 2$.

are not that different. However, it is not the average that matters. Because of complementarity in production, bad draws from the distribution get magnified.

5.1. Derivation

Our assumption that the A_i productivities are drawn from a Weibull distribution allows us to solve for Q_o or Q_m as functions of the parameters of the distribution, leading to a more parsimonious expression. We do this now. Since this argument is less familiar than the algebra needed to understand the previous propositions, we go through the reasoning in more detail.

Let η represent the absolute value of the curvature parameter in determining the productivity aggregate Q_o or Q_m. For the optimal allocation of resources, $\eta = -\frac{\rho}{1-\rho}$, so that $\eta \in [0, 1)$ is a positive curvature parameter. For the symmetric misallocation, $\eta = -\rho \in [0, \infty)$. Also, define $z_i \equiv A_i^{-\eta}$. Finally, let Q denote the productivity aggregate, either Q_o or Q_m. With this notation, we have

$$Q = \left(\int_0^1 A_i^{-\eta} di \right)^{-\frac{1}{\eta}} = \left(\int_0^1 z_i di \right)^{-\frac{1}{\eta}}. \tag{12}$$

Applying the law of large numbers to our model, Q can be viewed as the mean of the z_i across our continuum of sectors, raised to the power $-1/\eta$. To compute this mean, notice that

$$\begin{aligned} \Pr[z_i \le z] &= \Pr[A_i^{-\eta} \le z] \\ &= \Pr[A_i \ge z^{-1/\eta}] \\ &= 1 - F(z^{-1/\eta}) \\ &= e^{-\left(\frac{1}{\beta} \cdot z^{-1/\eta}\right)^{\theta}} \\ &= e^{-(\beta^{\eta} z)^{-\theta/\eta}}. \end{aligned}$$

This last expression is the cumulative distribution function for a Frechet random variable, which has a mean given by $\beta^{-\eta}\Gamma(1-\eta/\theta)$. This leads to the following proposition:

PROPOSITION 5.1. *(The Solution for Q) If the underlying productivities A_i are distributed according to a Weibull distribution, as in equation (11), then the aggregate productivity term Q is given by*

$$Q^* = \beta \left(\Gamma(1 - \frac{\eta}{\theta}) \right)^{-1/\eta} \tag{13}$$

where $\Gamma(\cdot)$ is Euler's factorial function.

5.2. The Gamma Function

All of the ingredients we need to understand large differences in incomes across countries are now in place. We will conduct a full quantitative analysis of the model in a later section after we have endogenized physical and human capital. However, we pause now to show the complementarity and linkage mechanisms at work.

To begin, it is helpful to get more familiar with the $\Gamma(\cdot)$ function.[8] Figure 2 shows the gamma function. In particular, notice that $\Gamma(n)$ diverges to infinity as n falls to zero.

Now recall the solution for Q in equation (13): $Q = \beta \left(\Gamma(1 - \frac{\eta}{\theta}) \right)^{-1/\eta}$. For our problem to yield an interior solution, we require the term inside the gamma function to be positive, which is equivalent to $\eta < \theta$; we will see below what

[8] The gamma function is defined as

$$\Gamma(n) \equiv \int_0^\infty x^{n-1} e^{-x} dx$$

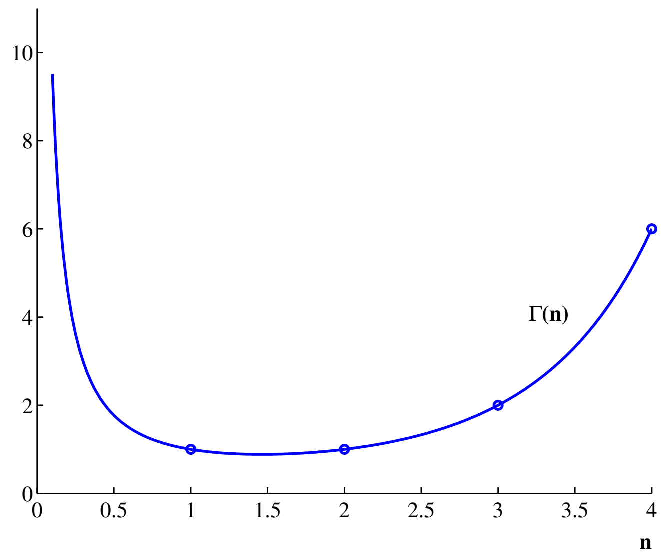
for $n > 0$. Some useful properties of this function are $\Gamma(1) = 1$, and $\Gamma(n+1) = n\Gamma(n)$, so that $\Gamma(n+1) = n!$ if n is a positive integer. This is why the gamma function is sometimes referred to as Euler's extension of the factorial.

For our purposes, we are more concerned with the behavior of the factorial function for n between zero and one. To see what happens here, it is helpful to rewrite the equation above as

$$\Gamma(n) = \frac{\Gamma(n+1)}{n}.$$

Because $\Gamma(1) = 1$, this expression tells us that $\Gamma(n)$ diverges to $+\infty$ as n falls to zero.

FIGURE 2. The Gamma Function



Note: This figure plots Euler's factorial function $\Gamma(n)$. If n is a positive integer, $\Gamma(n+1) = n!$. The lowest value of n shown in the plot is 0.1, and $\Gamma(0.1) \approx 9.5$.

happens if this condition is not met. Recall that $0 \leq \eta < 1$ for the optimal allocation of resources and $\theta > 0$, so there is plenty of room in the parameter space for this to occur. For the symmetric misallocation, we have only that $\eta > 0$, so there is more room for a corner solution.

We think of rich and poor countries as having different distributions of underlying productivity. In particular, a rich country may be expected to have a higher value of θ than a poor country, let's suppose, corresponding to a thinner lower tail.

If the poor country has a sufficiently low value of θ — that is, lots of mass at low productivity levels — then η/θ gets close to one, and $\Gamma(x) \rightarrow \Gamma(0) = +\infty$. Because Q depends on the inverse of the gamma function, this means that Q

gets arbitrarily close to zero. This is the mathematics that allows us to explain large income differences.

What is the economics? A lower value of θ corresponds to a thicker lower tail, and a higher value of η corresponds to more complementarity in production. In this framework, the poorest countries of the world are poor because they have a number of weak links that play an important role because of complementarity. With the misallocation of resources, complementarity is even stronger because weak links are not reinforced. The parameter η is larger and this makes it more likely that the gamma function blows up.

5.3. A Numerical Example

A simple numerical example shows how this works. Suppose $\rho = -1$ so that the elasticity of substitution between activities is $\frac{1}{1-\rho} = 1/2$, midway between Cobb-Douglas and Leontief. With the optimal allocation of resources, $\eta \equiv -\frac{\rho}{1-\rho} = 1/2$. Allocating capital, labor, and intermediate inputs efficiently therefore raises the relevant elasticity of substitution to $\frac{1}{1+\eta} = 2/3$. Assume a rich country has $Q_{rich} = 1$, and a poor country has $\beta = 1$, so differences are driven solely by θ. Finally, consider a share of intermediate goods in production of $\sigma = 1/2$. Table 1 shows the implied TFP differences in this example.

Before looking closely at the table, let us stipulate that to explain income differences of a factor of 32, one needs TFP differences of about 4 in a framework like this. This is the kind of number one gets from Klenow and Rodriguez-Clare (1997) or Hall and Jones (1999), and more details will be provided later.

Table 1 shows that enormous TFP differences can be obtained if the lower tail of the underlying productivity distribution is sufficiently thick, if activities are sufficiently complementary, and if linkages between sectors are sufficiently strong. This conclusion is especially true if resources are misallocated so that weak links are not strengthened. Notice that for the first two rows — corresponding to $\theta = 0.75$ and $\theta = 1$, the condition $\eta < \theta$ is violated. In this case,

TABLE 1.
TFP Differences: A Numerical Example

	— Optimal Allocation —		— Symmetric Misallocation —	
θ	$\frac{Q_o^{rich}}{Q_o^{poor}}$	$\left(\frac{Q_o^{rich}}{Q_o^{poor}}\right)^{\frac{1}{1-\sigma}}$	$\frac{Q_m^{rich}}{Q_m^{poor}}$	$\left(\frac{Q_m^{rich}}{Q_m^{poor}}\right)^{\frac{1}{1-\sigma}}$
0.75	7.2	51.5	∞	∞
1.0	3.1	9.9	∞	∞
1.5	1.8	3.4	2.7	7.2
2.0	1.5	2.3	1.8	3.1
4.0	1.2	1.4	1.2	1.5

Note: This example assumes $\rho = -1$ so that $\eta = 1/2$ for the optimal allocation and $\eta = 1$ for the symmetric misallocation. Q for the rich country is taken to be 1.0, and Q for the poor country is calculated according to equation (13), with $\beta = 1$.

the lower tail of the distribution is so thick that the mean of z_i (approximately the inverse of A_i) does not exist, driving output in the poor country to zero.

6. ENDOGENIZING K AND H

The remainder of the paper proceeds in two steps. In this section, we enrich the model slightly by endogenizing a country's stocks of physical and human capital. The former gives us another multiplier in a familiar fashion, while the latter gives us another factor of 2. Both of these are useful in explaining large income differences across countries. The last main section of the paper will then turn to a full calibration exercise.

6.1. Endogenizing Physical Capital

We endogenize physical capital in a standard fashion. In particular, we assume that capital can be rented from the rest of the world at a constant and exogenous real rate of return, $\bar{r}$. This rate of return includes both the real interest rate and whatever country-specific distortions there are in the capital market. This parameter will therefore vary across countries.

The optimal allocation then hires capital until the marginal product of capital falls to equal this real rate of return (which includes depreciation). Given our Cobb-Douglas expression for output in equation (9), this condition is

$$\alpha \frac{Y^*}{K^*} = \bar{r}. \tag{14}$$

This equation implicitly determines the capital stock in a country.

6.2. Endogenizing Human Capital (Schooling)

We turn now to the human capital of the labor force, modeled as schooling. This is useful for two reasons. First, it allows us to present a very simple, tractable model of human capital that can be embedded in any theory of development. Second, it allows us to make additional quantitative predictions about the role of human capital in development. The specification below is closest to that in Mincer (1958). Richer models of human capital include Ben-Porath (1967), Bils and Klenow (2000), and Manuelli and Seshadri (2005). The approach here is purposefully stripped-down, trading generality and realism for simplicity and tractability.

Aggregate human capital H is labor in efficiency units: $H = hL$, where h is human capital per worker and L is the number of workers. Assume the (constant) population in a country is distributed exponentially by age and faces a constant death rate $\delta > 0$: the density is $f(a) = \delta e^{-\delta a}$. A person attending school for S years obtains human capital $h(S)$, a smooth increasing function. The representative individual's problem is to choose S to maximize the expected present discounted value of income:

$$\max_{S} \int_S^\infty w_t h(S) e^{-(\bar{r}+\delta)t} dt, \tag{15}$$

where the base wage w_t is assumed to grow exponentially at rate $\bar{g}$.

Solving this maximization problem leads to the Mincerian return equation:

$$\frac{h'(S^*)}{h(S^*)} = \tilde{r} \equiv \bar{r} - \bar{g} + \delta. \tag{16}$$

The left side of this equation is the standard Mincerian return: the percentage increase in the wage if schooling increases by a year. The first order condition says that the optimal choice of schooling equates the Mincerian return to the effective discount rate. In this case, the effective discount rate is the interest rate, adjusted for wage growth and the probability of death. The original Mincer (1958) specification pinned down the Mincerian return by the interest rate. The generalization here shows the additional role played by economic growth and limited horizons. Rather than being an exogenous parameter, as in the simple version of Bils and Klenow (2000) used by Hall and Jones (1999) and others, the Mincerian return in this specification is related to fundamental economic variables.

More progress can be made by assuming a functional form for $h(S)$. Consider the constant elasticity form $h(S) = S^{\phi}$. In this case, the Mincerian return is $h'(S)/h(S) = \phi/S$, so the Mincerian return falls as schooling rises. The first-order condition in equation (16) then implies the optimal choice for schooling is

$$S^* = \frac{\phi}{\bar{r} - \bar{g} + \delta}, \tag{17}$$

and the human capital of the labor force in efficiency units is

$$h^* = \left(\frac{\phi}{\bar{r} - \bar{g} + \delta}\right)^{\phi}. \tag{18}$$

We assume ϕ is the same across countries, so differences in schooling can be explained in this simple framework by differences in the effective discount rate. A higher interest rate, slower growth, and a higher death rate all translate into lower educational attainment.

People in this world go to school for the first S^* years of their lives and then work for the remainder of their lives. Anyone working has S^* years of schooling and therefore supplies h^* efficiency units of labor for production.

6.3. Solving the Extended Model

The Cobb-Douglas expression for output in equation (9) can be combined with the solutions for K^* and h^* in equations (14) and (18) to yield the following solution of the model:

PROPOSITION 6.1. *(The Solution for Y/L) In this weak link theory of economic development, output per worker is given by*

$$
\begin{aligned}
y^* \equiv \frac{Y^*}{L^*} &= Q^{*\frac{1}{1-\sigma}\frac{1}{1-\alpha}} \left(\frac{K^*}{Y^*}\right)^{\frac{\alpha}{1-\alpha}} h^* \\
&= Q^{*\frac{1}{1-\sigma}\frac{1}{1-\alpha}} \left(\frac{\alpha}{\bar{r}}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\phi}{\bar{r}-\bar{g}+\delta}\right)^{\phi}, \qquad (19)
\end{aligned}
$$

where Q is either Q_o or Q_m, depending on whether resources are allocated optimally or symmetrically.

The wealth of nations is explained by two sets of parameters. Differences in $\bar{r}$ and the human capital parameters reflect the standard neoclassical forces. Now however, we also have differences in TFP arising from complementary activities. The parameters β and θ reflect the differences in underlying productivities across countries.

The form of this solution should be familiar. Output per worker is determined by productivity, the cost of physical capital, and the factors that influence the accumulation of human capital. For the usual reasons, there is a $\frac{1}{1-\alpha}$ multiplier (exponent) associated with capital accumulation: anything that increases output leads to additional capital accumulation, which further increases output, etc. The sum of this geometric series is $\frac{1}{1-\alpha}$.

7. QUANTITATIVE ANALYSIS

We now explore the model's quantitative predictions: can it help us to understand 50-fold differences in incomes across countries?

TABLE 2.
Baseline Parameter Values

Parameter	Rich Country	Poor Country	Comments
β	1.93	1	Weibull location parameter
θ	5	2	Weibull curvature parameter
$\bar{r}$	.06	.12	Interest rate
$\bar{g}$	.02	0	Growth rate
δ	.01	.02	Death rate
α	1/3		Capital share
σ	1/2		Share of intermediate goods
ρ	-1		EofS=1/2 (midway)
ϕ	0.6		To match Mincerian returns

7.1. Calibration

There are five country-specific parameters in equation (19) that need to be calibrated: the Weibull distribution parameters β and θ, the interest rate $\bar{r}$, the growth rate $\bar{g}$, and the death rate δ. There are also four parameters that are assumed to be common across countries: the capital exponent α, the share of intermediate goods in production σ, the complementarity parameter ρ, and the schooling elasticity ϕ. Our benchmark values for all of these parameters are reported in Table 2.

Values for the parameters of the Weibull distribution are by far the most difficult to obtain. Fortunately, the main results of the paper will not turn out to depend crucially on specific values.

In principle, these parameters might be estimated by looking at plant- or industry-level data on total factor productivity in different countries. In practice, however, the necessary research has not yet been done. In the absence of compelling evidence, we proceed in the following way. We pick what appear to be relatively small and plausible differences in the distribution of the A_i across countries. We then see how the model amplifies these small differences to deliver large income differences.[9]

The specific parameter values we use have already been discussed briefly; they are from the example shown back in Figure 1. The parameter choices imply that the mean of the distribution of total factor productivity in the rich country is only twice that in the poor country, so in some average sense the countries do not look so different.

This factor of 2 difference in means largely pins down the β parameters. Pinning down the θ parameters that govern the thickness of the lower tail is harder. After some experimentation, we've chosen $\theta^r = 5$ and $\theta^p = 2$.[10]

The most direct way to interpret these parameter choices is to see what they imply about the distributions of A_i in the rich and poor country. As discussed above, the means of these distributions differ by a factor of 2. Differences in θ then drive differences in the thickness of the tails. In particular, the 10th percentile in the rich country turns out to be 3.8 times higher than the 10th percentile in the poor country in this calibration. If we are thinking about the United States versus Kenya or Ethiopia, these differences appear to be reasonable and, if anything, might underestimate differences in TFP across activities.

The remaining parameters are much easier to calibrate. First, there is a set of parameters related to schooling. We assume the interest rate for discounting

[9]Recent work by Hsieh and Klenow (2006) comes the closest to giving us these parameter values. That paper looks at plant level TFP in China and India, but focuses on analyzing TFP *within* narrowly-defined industries. For our purposes, it is necessary to look *across* industries as well, taking into account the input-output structure of the economy. Another difficulty raised by Foster, Haltiwanger and Syverson (2005) and Hsieh and Klenow (2006) is the critical importance of firm-specific price measures. The most common way of computing total factor productivity uses industry-level price deflators. But if the producer of a specific variety of capital equipment is extremely productive, this will show up as a low firm-specific price. With industry-level deflators, the true variation in total factor productivity can be completely missed.

[10]These values of $\theta^r = 5$ and $\theta^p = 2$ are consistent with Hsieh and Klenow's evidence on the distribution of TFPR *within* 4-digit sectors in China and India, and with the Syverson (2004) evidence for the United States. Of course, for the present purposes, it would be better to have data on the distribution of TFPQ *across* sectors, and the lack of firm-specific prices in these distribtuion numbers is problematic.

future wages is 6% in the rich country and 12% in the poor country. Such values are well within the range of plausibility; see, for example, Caselli and Feyrer (2005). The parameter $\bar{r}$ plays two roles in the model, as the domestic cost of capital and the interest rate for discounting future wages. In theory, these interest rates could be determined by different forces. For example, the cost of capital could be higher because of capital taxation, while the (after tax) interest rate for discounting wages could be higher because of borrowing constraints. The two-fold difference assumed here seems perfectly reasonable given the distortions to capital markets in Kenya or Ethiopia versus the United States.

We take a growth rate of 2% per year for the rich country and a growth rate of zero for the poor country. Many of the poorest countries of the world have exhibited essentially zero growth for the last forty years.

For the death rate, we assume $\delta = 1\%$ per year in the rich country and 2% per year in the poor country. With this constant probability of death, life expectancy is 50 years in the poor country and 100 years in the rich country.

These parameter values imply a Mincerian return to schooling of 5% in the rich country and 14% in the poor country. We also take $\phi = 0.6$. Together with the other parameter values, this implies people in the rich country get 12 years of schooling, while people in the poor country get 4.3 years of schooling. These numbers are not a perfect match of the data (one might want a slighly smaller gap in the Mincerian returns and a slightly larger gap in the years of schooling, as documented by Bils and Klenow 2000), but they are certainly in the right ballpark, which is a nice accomplishment for the simple schooling framework used here.

The remaining parameters are common across countries. The most important of these is σ, which equals the share of intermediate goods in total output. Basu (1995) recommends a value of 0.5 based on the numbers from Jorgenson, Gollop and Fraumeni (1987) for the U.S. economy since between 1947 and 1979. Ciccone (2002), citing the extensive analysis in Chenery, Robinson and

Syrquin (1986), argues that the intermediate goods share rises with the level of development. However, the numbers cited for South Korea, Taiwan, and Japan in the early 1970s are all substantially higher than the U.S. number, ranging from 61% to 80%. A larger intermediate goods share makes the results in this paper even stronger, so the choice of $\sigma = 1/2$ appears conservative. Notice that this choice implies a substantial multiplier that works through intermediate goods: $\frac{1}{1-\sigma} = 2$.

The complementarity parameter is another parameter that is quite important but difficult to calibrate. Recall that we want ρ to be negative in the complementarity story. We take $\rho = -1$, which corresponds to an elasticity of substitution of 1/2, midway between Cobb-Douglas and Leontief. Once inputs are allocated optimally across sectors to reinforce weak links, this delivers a value for $\eta \equiv -\frac{\rho}{1-\rho} = 1/2$ and therefore an effective elasticity of substitution of $\frac{1}{1+\eta} = 2/3$. Obviously it would be desirable to obtain better evidence on the extent of complementarity of activities in production. But given the stories we told to motivate this paper, this value of ρ does not seem extreme.

Finally, we pick $\alpha = 1/3$ to match the empirical evidence on capital shares; see Gollin (2002).

7.2. Results

To emphasize how this model explains differences in incomes between rich and poor countries, we evaluate the solution for output per worker in equation (19) for two countries and compute the ratio. Let the superscript r denote a rich country and the superscript p denote a poor country. Then income ratios are given by

$$\frac{y^{r*}}{y^{p*}} = \left[\underbrace{\left(\frac{\beta^r}{\beta^p} \cdot \left(\frac{\Gamma(1-\eta/\theta^p)}{\Gamma(1-\eta/\theta^r)} \right)^{1/\eta} \right)^{\frac{1}{1-\sigma}}}_{\text{TFP}} \underbrace{\left(\frac{\bar{r}^p}{\bar{r}^r} \right)^{\alpha}}_{\text{K/Y}} \underbrace{\left(\frac{\bar{r}^p - \bar{g}^p + \delta^p}{\bar{r}^r - \bar{g}^r + \delta^r} \right)^{\phi(1-\alpha)}}_{\text{h}} \right]^{\frac{1}{1-\alpha}}$$

Using the baseline parameters from Table 2, the terms in this equation can be quantified as follows. First, for the optimal allocation of resources:

$$\begin{aligned}\frac{y^{r*}}{y^{p*}} &\approx (6.44 \times 1.26 \times 1.51)^{1.5} \approx (12.3)^{1.5}\\ &\approx 16.4 \times 1.41 \times 1.85\\ &\approx 42.9\end{aligned}$$

And next for the symmetric misallocation:

$$\begin{aligned}\frac{y^{r*}}{y^{p*}} &\approx (8.64 \times 1.26 \times 1.51)^{1.5} \approx (16.4)^{1.5}\\ &\approx 25.4 \times 1.41 \times 1.85\\ &\approx 66.6\end{aligned}$$

The standard neoclassical terms for physical capital and schooling imply a difference in incomes of a factor of $1.41 \times 1.85 = 2.6$. This is smaller than the 4-fold difference between the 5 richest and 5 poorest countries documented by Hall and Jones (1999). With larger differences in $\bar{r}$, we could increase the difference in the model, but to be conservative, we keep these values.

Here, of course, we also have a theory of TFP differences, and the story goes as follows. Rich and poor countries are not that different on average in the efficiency with which they produce various activities (a factor of two, recall). However, these small differences get amplified in two distinct ways. First, activities enter production in a complementary fashion, so that problems in one area reduce the value of overall output. Second, intermediate goods provide linkages between activities. Low productivity in one activity leads to low productivity in the others.

The TFP differences in our calibration can be decomposed as follows. The basic factor of two is reflected in $\beta^r/\beta^p = 1.93$. This term is multiplied by the ratio of the gamma functions, reflecting complementarity. This ratio is (only) 1.3 for the optimal allocation and 1.5 for the symmetric misallocation. The ratio of the Q productivity aggregates is then $1.93 \times 1.3 = 2.5$ for the optimal allocation and $1.93 \times 1.5 = 2.9$ for the symmetric misallocation. Because the intermediate goods share in production is 1/2, these numbers get squared in order to yield the basic TFP differences: 6.44 and 8.64. Capital accumulation provides further amplification, raising each of these numbers to the 3/2 power to yield 16.4 and 25.4.

The overall income difference predicted by this simple calibration is then the product of this TFP factor with the roughly 3-fold neoclassical effect. The model predicts differences between rich and poor countries of about 43 times for the optimal allocation and 67 times for the symmetric misallocation. These numbers can be compared to a 95th/5th percentile ratio for GDP per capita of 32.1 for the year 1999. The mechanisms at work in this paper, then, seems to be perfectly capable of explaining the large income differences observed in the data.

7.3. Robustness

There are a number of parameter values in this quantitative exercise whose values we do not know especially well. This section shows the robustness of the results to changes in some of these parameter values. In particular, we consider changing the complementarity parameter ρ, the Weibull distribution parameters θ_p and θ_r, and the share of intermediate goods in the economy, σ.

The results of these robustness checks are shown in Table 3. The first scenario simply repeats the baseline results, for comparison. The last column of the table shows the results when resources are allocated optimally in the rich country but symmetrically in the poor country.

The second and third scenarios explore changes in the degree of complementarity in the economy. The baseline value for ρ is -1; we consider -1/2 and -2 as

TABLE 3.
Output per Worker Ratios: Robustness Results

Scenario	Optimal Allocation	Symmetric Misallocation	Rich=optimal Poor=symm.
1. Baseline simulation	42.9	66.6	70.6
2. Less complementarity: $\rho = -1/2$	38.4	42.9	43.7
3. More complementarity: $\rho = -2$	48.7	∞	∞
4. Thicker tail in poor country (5.5): $\theta^p = 1.5$	82.6	243.0	257.5
5. Thinner tail in poor country (2.6): $\theta^p = 3$	26.8	30.4	32.2
6. Thicker tail in rich country (3.4): $\theta^r = 4$	39.7	59.4	65.2
7. Thinner tail in rich country (4.5): $\theta^r = 8$	46.8	75.3	76.9
8. Lower intermediate share: $\sigma = 1/4$	16.9	22.7	23.5
9. Zero intermediate share: $\sigma = 0$	10.6	13.2	13.6

Note: The table reports income ratios between rich and poor countries. The baseline case uses the parameter values from Table 2: $\rho = -1, \theta^p = 2, \theta^r = 5$, and $\sigma = 1/2$. Other scenarios change one parameter at a time. The numbers in parentheses in the descriptive column in Scenarios 4 through 7 represent the ratio of A_i at the 10th percentile between the rich and poor countries, which equals 3.8 in the baseline case. The parameter β^r is changed when necessary to keep average underlying productivity twice as high in the rich country. The last column shows the results when resources are allocated optimally in the rich country but symmetrically in the poor country.

alternatives. Large income differences are clearly preserved by this change, and the differences explode to infinity at $\rho = -2$ for the case of misallocation.

The next four scenarios consider variations in the θ parameters, which govern the thickness of the lower tail of the Weibull distribution. Once again, large income differences are easily preserved for the range of values considered.

Finally, the last two rows show what happens when the share of intermediate goods in production is reduced. The case of $\sigma = 0$ illustrates the first-order impact of the multiplier associated with intermediate goods. Reasonable people might argue about the importance of complementarity, particularly since the parameter values we have chosen to illustrate this mechanism are not currently well-established empirically. The role played by intermediate goods is on a much firmer foundation. Intermediate good shares in modern economies are high, and this mechanically delivers a substantial amplification of productivity differences.

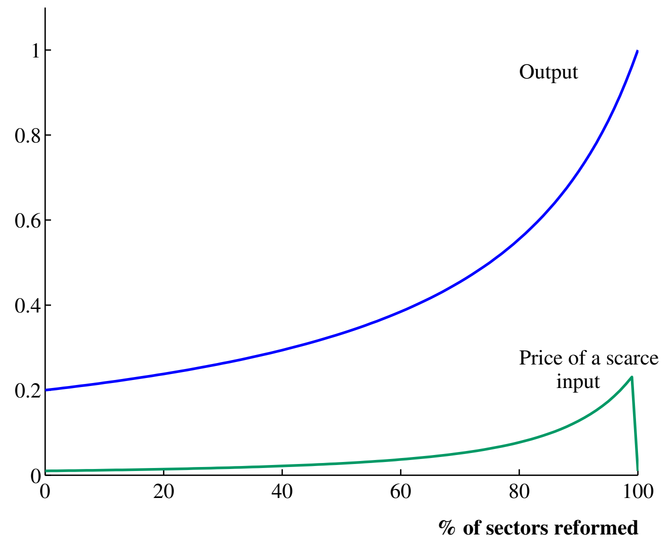
7.4. Discussion

The model possesses two key features that seem desirable in any theory designed to explain the large differences in incomes across countries. First, relatively small and plausible differences in underlying parameters can yield large differences in incomes. That is, the model generates a large multiplier.

Second, improvements in underlying productivity along any single dimension have relatively small effects on output. If a chain has a number of weak links, fixing one or two of them will not change the overall strength of the chain.

This principle is clearly true in the extreme Leontief case, but it holds more generally as well. To see this, consider a simple exercise. Suppose output is given by a symmetric CES combination of 100 inputs, for example as in equation (1). Initially, all inputs take the value 0.2 and therefore output is equal to 0.2 as well. A sequence of "reforms" then leads the inputs to increase one at a time to their rich-country values of 1.0. After 100 reforms, all inputs take the value of 1.0

FIGURE 3. The Cumulative Effect of Reforms



Note: Output is given by a symmetric CES combination of 100 inputs, with an elasticity of substitution equal to 1/2. Initially, all inputs take the value 0.2. A sequence of reforms lead the inputs to increase to the value 1.0, one at a time.

and output is equal to 1.0. Figure 3 shows the sequence of output levels that result from the reforms for the case $\rho = -1$, as well as the marginal product of an unreformed input.

Notice that output is relatively flat for much of the graph. The first doubling of output does not occur until more than 60% of the sectors are reformed. In addition, the marginal product of the input in an unreformed sector remains low for a long time. When the economy suffers from many problems, reforms that address only a few may have small effects.

Interestingly, the sharp curvature of these paths suggests that the pressure for reform can accelerate. This general setup may then help us to understand why some countries remain unreformed and poor for long periods while others — those that are close to the cusp — experience growth miracles.

Multinational firms can help to solve these problems. For example, they may bring with them knowledge of how to produce, access to transportation and foreign markets, and the appropriate capital equipment. And yet domestic weak links may still be a problem. A lack of contract enforcement may make intermediate inputs and other activities hard to obtain. Weak property rights may lead to expropriation. Inadequate energy supplies may reduce productivity. Indeed many of the examples we know of where multinationals produce successfully in poor countries effectively give the multinational control on as many dimensions as possible: consider the maquiladoras of Mexico and the special economic zones in China and India.

The development problem is hard because there are ten things that can go wrong in any production process. In the poorest countries of the world, productivity is low at many different stages, and complementarity means that reforms targetted at one or two problems have only modest effects.

8. CONCLUSION

In the weak link theory of economic development, relatively small differences in total factor productivity at the firm or activity level translate into large differences in aggregate output per worker. There are two reasons for this. First, production at the firm level involves complementarity. For virtually any good, there is a list of intermediate inputs — transportation, electricity, raw materials — that are essential for production. Replacement parts are an absolute requirement when machines break down. Business licenses and security are necessary for success. Because of complementarity, output does not depend on average productivity but rather hinges on the strength of the weakest links.

Formalizing the consequences of complementarity occupied a majority of the space in this paper, but the second amplification force is both simpler and potentially more important. The presence of intermediate goods leads to a multiplier that depends on the share of intermediate goods in firm revenue. Low productivity in transportation reduces the output of many other sectors, including the truck manufacturing sector and the fuel sector. This in turn will reduce output in the transportation sector. This vicious cycle is the source of the multiplier associated with intermediate goods.

In the neoclassical growth model, the multiplier associated with capital accumulation depends on $\frac{1}{1-\alpha}$, where α is the capital share. Similarly, the multiplier here depends on $\frac{1}{1-\sigma}$, where σ is the intermediate goods share. The overall multiplier in this model is the product of these two terms. With a capital share of 1/3 and an intermediate goods share of 1/2, the multiplier is $\frac{1}{1-\sigma} \times \frac{1}{1-\alpha} = 2 \times 3/2 = 3$. If TFP at the firm level in a rich country is twice that in a poor country — say because of distortions — the aggregate long-run income difference will be a factor of $2^3 = 8$ because of the multiplier associated with intermediate goods and capital accumulation. Combined with a factor of 4 from differences in investment rates and human capital, a simple model along these lines can easily generate 32-fold differences in output per worker.

These amplification channels imply that the model makes a simple, testable prediction. In particular, if we look at total factor productivity at the micro level — say at the gross output production function for a plant or firm — we should see something that at first appears puzzling: total factor productivity for firms in China or India, for example, should not be that different on average from total factor productivity in the richest countries in the world. Given the large income differences and large aggregate TFP differences, one might have expected to see large TFP differences at the firm level. To the extent that complementarity and the multiplier associated with intermediate goods are important, we should find that firm-level TFP in the poorest countries of the world is typically only 1/2 as low as in the United States. Firms in poor countries should look surprisingly efficient.

A casual reading of McKinsey studies of productivity suggests that this may be true. More directly, the analysis of large firm-level data sets in China and India by Hsieh and Klenow (2006) suggests that this prediction can be tested in the near future.

Another important channel for future research concerns the role of intermediate goods. The present model simplifies considerably by taking the intermediate input to be units of the final output good. The input-output matrix in this model is very special. This is a good place to start. However, it is possible that the rich input-output structure in modern economies delivers a multiplier smaller than $\frac{1}{1-\sigma}$ because of "zeros" in the matrix. In work in progress, Jones (2006) explores this issue. The preliminary results are encouraging. For example, if the share of intermediate goods in each sector is σ but the composition of this share varies arbitrarily, the aggregate multiplier is still $\frac{1}{1-\sigma}$. More generally, I plan to use actual input-output tables for the United States and some developing countries to compute the associated multipliers. I believe this will confirm the central role played by intermediate goods in amplifying distortions.

Finally, the approach taken in this paper can also be compared with the recent literature on political economy and institutions; for example, see Acemoglu and Johnson (2005) and Acemoglu and Robinson (2005). This paper is more about mechanics: can we develop a plausible mechanism for getting a big multiplier, so that relatively modest distortions lead to large income differences? The modern institutions approach builds up from political economy. This is useful in explaining why the allocations in poor countries are inferior — for example, why investment rates in physical and human capital are so low — but the institutions approach ultimately still requires a large multiplier to explain income differences. As just one example, even if a political economy model explains observed differences in investment rates across countries, the model cannot explain 50-fold income differences if it is embedded in a neoclassical framework. The political economy approach explains why resources are misallocated; the approach here explains why misallocations lead to large income differences. Clearly, both steps are needed to understand development.

APPENDIX: PROOFS OF THE PROPOSITIONS

Proposition 4.1: The Symmetric Misallocation

Proof. Follows directly from the fact that $Y_i = A_i m$, where $m = (K^\alpha H^{1-\alpha})^{1-\sigma} X^\sigma$ is constant across activities. ▮

Proposition 4.2: The Optimal Allocation

Proof. In deriving the aggregate production function, it is helpful to proceed in two steps. First, consider the optimal allocation of the intermediate goods, and then consider the optimal allocation of physical and human capital.

Define $a_i \equiv A_i (K_i^\alpha H_i^{1-\alpha})^{1-\sigma}$, so that $Y_i = a_i X_i^\sigma$. Then, the optimal allocation of X_i solves

$$\max_{\{X_i\}} C = \zeta \left(\int_0^1 a_i^\rho X_i^{\rho\sigma} di \right)^{1/\rho} - \int_0^1 X_i di.$$

Solving this problem and substituting the solution back into the production function in equation (2) gives

$$Y = \left(\int_0^1 a_i^\lambda di \right)^{\frac{1}{\lambda} \cdot \frac{1}{1-\sigma}}, \tag{A.1}$$

where $\lambda \equiv \frac{\rho}{1-\rho\sigma}$.(This is where the judicious definition of ζ comes in handy.)

Using this expression, the optimal allocations of K_i and H_i solve

$$\max_{\{K_i, H_i\}} \int_0^1 a_i^\lambda di$$

subject to the resource constraints in equations (4) and (5), where $a_i \equiv A_i (K_i^\alpha H_i^{1-\alpha})^{1-\sigma}$. Solving this problem and substituting the result back into equation (A.1) gives the aggregate production function. ▮

Proposition 5.1: The Solution for Q

Proof. Given in the text. ▮

Proposition 6.1: The Solution for Y/L

Proof. Given in the text. ▮

REFERENCES

Acemoglu, Daron and James Robinson, *Economic Origins of Dictatorship and Democracy*, Cambridge University Press, 2005.

—— and Simon Johnson, "Unbundling Institutions," *Journal of Political Economy*, 2005, *forthcoming.*

——, ——, and James A. Robinson, "The Colonial Origins of Camparative Development: An Empirical Investigation," *American Economic Review*, December 2001, *91* (5), 1369–1401.

Armenter, Roc and Amartya Lahiri, "Endogenous Productivity and Development Accounting," 2006. University of British Columbia working paper.

Basu, Susanto, "Intermediate Goods and Business Cycles: Implications for Productivity and Welfare," *American Economic Review*, June 1995, *85* (3), 512–531.

Becker, Gary S. and Kevin M. Murphy, "The Divsion of Labor, Coordination Costs, and Knowledge," *Quarterly Journal of Economics*, November 1992, *107* (4), 1137–1160.

Ben-Porath, Yoram, "The Production of Human Capital and the Life Cycle of Earnings," *Journal of Political Economy*, August 1967, *75* (4), 352–365.

Bils, Mark and Peter Klenow, "Does Schooling Cause Growth?," *American Economic Review*, December 2000, *90*, 1160–1183.

Caselli, Francesco and James Feyrer, "The Marginal Product of Capital," August 2005. NBER Working Paper 11551.

—— and Wilbur John Coleman, "The World Technology Frontier," *American Economic Review*, June 2006, *96* (3), 499–522.

Chari, V.V., Pat Kehoe, and Ellen McGrattan, "The Poverty of Nations: A Quantitative Investigation," 1997. Working Paper, Federal Reserve Bank of Minneapolis.

Chenery, Hollis B., Sherman Robinson, and Moshe Syrquin, *Industrialization and Growth: A Comparative Study*, New York: Oxford University Pres, 1986.

Ciccone, Antonio, "Input Chains and Industrialization," *Review of Economic Studies*, July 2002, *69* (3), 565–587.

Conley, Timothy G. and Bill Dupor, "A Spatial Analysis of Sectoral Complementarity," *Journal of Political Economy*, April 2003, *111* (2), 311–352.

Dupor, Bill, "Aggregation and irrelevance in multi-sector models," *Journal of Monetary Economics*, April 1999, *43* (2), 391–409.

Erosa, Andres, Tatyana Koreshkova, and Diego Restuccia, "On the Aggregate and Distributional Implications of Productivity Differences Across Countries," 2006. University of Toronto working paper.

Foster, Lucia, John Haltiwanger, and Chad Syverson, "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?," August 2005. NBER Working Paper 11555.

Gabaix, Xavier, "The Granular Origins of Aggregate Fluctuations," 2005. MIT working paper.

Gollin, Douglas, "Getting Income Shares Right," *Journal of Political Economy*, April 2002, *110* (2), 458–474.

Hall, Robert E. and Charles I. Jones, "Why Do Some Countries Produce So Much More Output per Worker than Others?," *Quarterly Journal of Economics*, February 1999, *114* (1), 83–116.

Hirschman, Albert O., *The Strategy of Economic Development*, New Haven, CT: Yale University Press, 1958.

Horvath, Michael T.K., "Cyclicality and Sectoral Linkages: Aggregate Fluctuations from Independent Sectoral Shocks," *Review of Economic Dynamics*, October 1998, *1* (4), 781–808.

Hsieh, Chang-Tai and Peter J. Klenow, "Misallocation and Manufacturing TFP in China and India," May 2006. University of California at Berkeley working paper.

Hulten, Charles R., "Growth Accounting with Intermediate Inputs," *Review of Economic Studies*, 1978, *45* (3), 511–518.

Jones, Charles I., "Intermediate Goods, Multipliers, and Income Differences," 2006. U.C. Berkeley, work in progress.

Jorgenson, Dale W., Frank M. Gollop, and Barbara M. Fraumeni, *Productivity and U.S. Economic Growth*, Cambridge, MA: Harvard University Press, 1987.

Klenow, Peter J. and Andres Rodriguez-Clare, "The Neoclassical Revival in Growth Economics: Has It Gone Too Far?," in Ben S. Bernanke and Julio J. Rotemberg, eds., *NBER Macroeconomics Annual 1997*, Cambridge, MA: MIT Press, 1997.

—— and ——, "Extenalities and Growth," in Philippe Aghion and Steven Durlauf, eds., *Handbook of Economic Growth*, Amsterdam: Elsevier, 2005.

Kremer, Michael, "The O-Ring Theory of Economic Development," *Quarterly Journal of Economics*, August 1993, *108* (4), 551–576.

Leontief, Wassily, "Quantitative Input and Output Relations in the Economic System of the United States," *Review of Economics and Statistics*, 1936, *18* (3), 105–125.

Long, John B. and Charles I. Plosser, "Real Business Cycles," *Journal of Political Economy*, February 1983, *91* (1), 39–69.

Mankiw, N. Gregory, David Romer, and David Weil, "A Contribution to the Empirics of Economic Growth," *Quarterly Journal of Economics*, May 1992, *107* (2), 407–438.

Manuelli, Rodolfo and Ananth Seshadri, "Human Capital and the Wealth of Nations," March 2005. University of Wisconsin working paper.

Marimon, Ramon and Vincenzo Quadrini, "Competition, Innovation and Growth with Limited Commitment," 2006. U.S.C. working paper.

Mincer, Jacob, "Investment in Human Capital and Personal Income Distribution," *Journal of Political Economy*, August 1958, *66* (4), 281–302.

Parente, Stephen L. and Edward C. Prescott, "Monopoly Rights: A Barrier to Riches," *American Economic Review*, December 1999, *89* (5), 1216–1233.

Prescott, Edward C., "Needed: A Theory of Total Factor Productivity," *International Economic Review*, August 1998, *39* (3), 525–51.

Romer, Paul M., "Endogenous Technological Change," *Journal of Political Economy*, October 1990, *98* (5), S71–S102.

___ , "New Goods, Old Theory, and the Welfare Costs of Trade Restrictions," *Journal of Development Economics*, 1994, *43*, 5–38.

Syverson, Chad, "Market Structure and Productivity: A Concrete Example," *Journal of Political Economy*, December 2004, *112* (6), 1181–1222.