Econ 196
Topic: Behavioral Economics

Quasi-Endowment in Private-Value Auctions

Introduction

The analysis of symmetric, private-value auctions pioneered by Vickrey (1961) and Myerson (1981) assumes bidders are risk neutral, a crucial assumption upon which the famous revenue-equivalence principle relies. However, despite their elegance, these models have seen little empirical validation. Neither these models nor subsequent extensions to the case of risk-averse bidders have been able to account for overbidding in auctions, a phenomenon widely documented in empirical research.

Overbidding in auctions has long been an object of interest, even to non-economists. For instance, most are familiar with the colloquial explanation of overbidding known as "auction fever," which holds that bidders become emotionally caught up in the bidding process. One of the more novel explanations for overbidding was recently proposed by Ariely et al (2004). They provide experimental evidence for a "quasi-endowment" effect, whereby bidders develop a sense of attachment to the object during the bidding process and come to see the object as an item that already belongs to them. This suggests that bidders influenced by this effect would bid higher than they otherwise would have, in order to retain what they perceive as an endowment.

This result confounds most of the theoretical literature on auctions, under which no such behavior should occur. In this literature, auctions are modeled as games of imperfect information, and bidders are assumed to be rational, their strategies dictated by the game's Bayesian-Nash equilibrium. In particular, the payoff for a particular bidder is assumed to be a function of only the value of the object and the expected payment, the function linear for the case of risk neutrality and concave for risk aversion.

This paper departs from the literature on auctions first by incorporating loss aversion into specifications of bidder utility. In doing so, we provide some theoretical basis for the quasi-endowment effect, which can be framed as loss aversion. For first- and second-price auctions,[1]

[1] In a first-price auction, the winner pays the highest bid (her bid). In a second-price auction, the winner pays the second-highest bid. English auctions are the familiar oral auctions in which an auctioneer calls out ascending prices, and bidders signal their willingness to pay.

we present equilibrium bidding strategies for a fixed, exogenous reference point. We also extend the revenue-equivalence principle to bidders with loss aversion and thus determine the expected payment of such bidders for a wide class of auction formats.

We next attempt to incorporate some of the insights from the empirical research on quasi-endowment in dynamic auctions into two models of English auctions. One model follows the standard model of Milgrom and Weber (MW) (1982) for interdependent-value auctions, and the second employs an "alternating recognition" (AR) model developed by Harstad and Rothkopf (2000). Unlike the MW model, the AR model gives theoretical meaning to the idea of being a leading bidder. In both models, rather than assuming loss aversion, we let each bidder's valuation increase as other bidders exit the auction, but in the AR model, we allow only the "leading" bidder's valuation to increase the "longer" she is in the lead. This is an attempt to incorporate the intuition behind the result found in Ariely et al (2004) that bidders submit higher rebids the longer they are in the lead. That is, the quasi-endowment effect is stronger for bidders that remain in the lead for a greater amount of time. We also discuss the technical difficulties of solving for equilibrium in these auctions.

The outline of the paper is as follows. Section one presents results from the rational model of bidders in symmetric, private-value auctions. Section two surveys relevant empirical results on overbidding. Section three presents the equilibrium solutions for first- and second-price auctions. Section four discusses models of quasi-endowment for English auctions.

1. Rational Bidders

Following Vickrey (1961), private-value auctions are modeled as games of imperfect information. There are N risk-neutral bidders, each bidder denoted by i, each with type $X_i \subseteq \Re_+$ which represents the bidder's valuation of the object. The X_i's are independently and identically distributed on $[0, \omega]$ with density f. Every bidder knows her realized value x_i and has no information about the values of the other bidders, except their densities f. The payoff functions are contingent on the particular auction format. In a second-price, sealed-bid auction[2],

[2] In a sealed-bid auction, all bidders submit a single bid, and the winner is determined from these bids. In this paper, we only consider first-price and second-price auctions that are sealed-bid. In a dynamic auction, there are multiple rounds of bidding, the most well-known example being the English or oral ascending auction.

the player who submits the highest bid pays the second highest bid, so ignoring zero-probability ties, the payoff function is

$$\Pi^{II}(x_i, x_{-i}) = \begin{cases} x_i - \max_{j \neq i} b_j & \text{if} \quad b_i > \max_{j \neq i} b_j \\ 0 & \phantom{\text{if}} \quad b_i < \max_{j \neq i} b_j \end{cases} \quad (1)$$

where b_i is player i's bid and $x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_N)$. Players bid according to a strategy β_i: $[0, \omega] \rightarrow \Re_+$ which is a function that assigns a bid b_i to every private value x_i. These auctions typically have a multitude of equilibria, but papers often restrict attention to symmetric equilibria, since bidders are symmetric. In this paper, we will follow this convention.

One can prove that in a second-price, sealed-bid auction, truth-telling, or bidding one's private value, is a weakly dominant strategy. Additionally, it turns out that English auctions are equivalent to second-price auctions in a weak sense. To see this, notice that it cannot be profitable for a bidder to drop out before the auctioneer reaches her value or to stay in after the posted price exceeds her value. Thus, as soon as the bidder with the second-highest value drops out, which is precisely when the posted price meets her value, the bidder with the highest value wins the object at that price, which is exactly the second-highest bid in a second-price auction. Hence, bidding one's value is also a weakly dominant strategy for English auctions.

The bidding strategies in first-price auctions are slightly more involved and are not weakly dominant. Let Y_1 denote the first-order statistic of the values for all bidders besides bidder *i.* Then one can show that in a first-price auction, $\beta_i(x_i) = E(Y_1 \mid Y_1 < x_i)$. In other words, bidders bid the expected value of the highest competing bid, conditional on being the highest bidder. One can rewrite this formula to obtain $\beta_i(x_i) = x_i - \int_0^{x_i} \left[\frac{F(y)}{F(x_i)} \right]^{N-1} dy$ where F is the CDF of f. Thus, relative to the second-price auction, bidders shade their bids below their true valuation by a value equal to the integral in the expression.

Given this difference in bidding strategies between the first- and second-price auctions, it is perhaps remarkable that these auctions – indeed, a wide class of auctions – yield the same amount of revenue to the seller in expectation and that bidders have the same expected payment in each auction. This is the substance of the Revenue Equivalence Principle, due to Myerson (1981).

Theorem (Revenue Equivalence): Suppose bidders are risk neutral, bidder values are i.i.d., and the expected payment of a bidder with value zero is zero. Then for any symmetric equilibrium of a standard auction (in which highest bidder wins the object), the expected payment for any bidder *i* is given by $m_i(x_i) = G(x_i) * E(Y_1 \mid Y_1 < x_i)$, where G is the CDF of Y_1 and thus the expected revenue to the seller is the same.

This surprising result shows that, loosely speaking, from the seller's perspective, all the "usual" auction types (i.e., auctions satisfying the theorem's assumptions) are the same. Whether the seller chooses to use the first price, second price, English ascending, Dutch ascending, third price, or even the all pay auction[3] is essentially immaterial; they all yield the same revenue in expectation. Indeed, even buyers are indifferent between the formats, since they pay the same amount in expectation in any of these auctions. Crucial, of course, is the assumption of risk neutrality.

This theory of bidding in private-value auctions, while certainly elegant, has not seen much empirical validation, which is perhaps unsurprising, given that game theory in general tends to have a rather lackluster empirical track record. Indeed, a wealth of empirical evidence documents systematic deviations from the rational model in the form of overbidding.

2. The Quasi-Endowment Effect in Auctions

This paper is concerned with a particular kind of overbidding—that induced by the "quasi-endowment" effect, sometimes also called the "pseudo-endowment" effect. The name of the effect refers to a closely related phenomenon, the endowment effect, or the finding that a person's valuation for an item is different depending on whether or not she owns the item. Contrary to economic theory, ownership itself is said to influence an individual's subjective valuation of an object, perhaps due to a sense of attachment.

The endowment effect is considered an instance of a more general psychological phenomenon known as loss aversion. The idea is that losses loom larger than same-sized gains;

[3] In all-pay auctions, every bidder pays the amount she bids, regardless of whether or not she wins. In the Dutch auction, the auctioneer gradually lowers the price until a bidder signals her willingness to purchase the object at the price, at which point the object ends. It is simple to see that this auction is strategically equivalent to the first-price auction.

the magnitude of a utility gain from obtaining an item is smaller than the magnitude of the utility loss from losing that item. In terms of the endowment effect, the minimum amount that people are willing to accept in exchange for an object they own is less than the amount they are willing to pay to obtain the same object. This is because not purchasing the object is viewed as a forgone gain, while selling the object is viewed as a loss of endowment, a framing effect that makes no material difference under rational economic theory but that has important consequences under the theory of loss aversion.

Under a simplified version of this theory, utility functions take the following form:

$$U(c) = \begin{cases} v(c) + \varepsilon(v(c) - v(p) & \text{if} \quad v(c) \geq v(p) \\ v(c) + \lambda\varepsilon(v(c) - v(p)) & \quad\quad v(c) < v(p) \end{cases}$$

where v is a standard rational utility function, c is some amount of consumption, p is the "reference point," $\lambda > 1$ is the loss aversion coefficient, and $\varepsilon \in \Re_+$ is the degree of reference-dependence. In the case where $\varepsilon = 0$, this reduces to the rational model. The notion of relative gains/losses is captured by the difference $v(c) - v(p)$, $v(p)$ being a reference amount of utility. When $v(c)$ exceeds $v(p)$, this is a relative gain. Notice however that when $v(c)$ is below $v(p)$, this relative loss is penalized by the factor λ, which captures loss aversion. Thus, when $v(c) = 5$ and $v(p) = 4$, the agent gains ε. But when $v(c) = 3$ and $v(p) = 4$, the agent loses $\lambda\varepsilon > \varepsilon$. Losses hurt more than same-size gains.

A crucial issue is the determination of *p.* The problem of reference-point specification is a common one, and the psychological research in this area has not provided much in the way of answers. The reference point is frequently taken to be the status quo in experimental literature, a reference point which makes no sense for a theory of quasi-endowment, as we will discuss below. Recent theoretical research has pointed out the limitations of a status-quo reference point. Koszegi and Rabin (2006) provide the example that a person expecting to undergo an uncomfortable dental procedure may feel a sense of gain if she learns it is no longer necessary. Yet, this only makes sense if the reference point is related to expectations about the future, rather than attitudes towards the status quo. To this end, Koszegi and Rabin provide a model of endogenous reference-point formation, in which the reference point is determined by the rational expectations the individual has about the environment she will face.

Some empirical papers have suggested the existence of an endowment effect without actual possession. In this case, under a loss-aversion framework, an individual's reference point is likely this anticipated sense of ownership in the future in the Koszegi-Rabin sense rather than the present lack of ownership. Two papers find evidence of this endowment effect in auctions.

Ariely, Heyman, and Orhun (2004) conduct two different experiments that suggest bidders develop an attachment to the auctioned object, despite the lack of actual ownership. In the first experiment, subjects are told to imagine a hypothetical five-day auction at a store in which bidders can come by at any time to submit bids. The highest bidder wins the object and pays the second-highest amount. Subjects are told one of two scenarios. In scenario one, bidders know about the auction but only come by the store on the last day. In scenario two, bidders submit a bid on the first day and become the highest bidder but return the last day and discover that they are no longer the highest bidder. Subjects are then asked to state the amount they would bid on that last day in both scenarios. Ariely et al find that for a $75 item, scenario-one subjects on average rebid $68.31 and scenario-two subjects rebid $72.77, and the difference is statistically significant. This suggests that being the leading bidder for a longer amount of time leads to "overbidding" behavior in the sense that a factor that should be irrelevant for a rational bidder positively influences bidding amounts.

In the second experiment, they conduct actual auctions in an experimental setting for various items such as gift certificates. Bidders participate in the same auction format as experiment one, except there are nine rounds of bidding. Bidders either participate in all nine rounds, or they observe eight rounds and only participate in the final round. The results are that average final bids in the former group are higher than the latter, which leads to a slightly different conclusion than the first experiment, namely that longer participation in an auction induces overbidding.

Arkes, Muhanna, and Wolf find similar evidence in the field, using data from eBay Motors. In their study, length of auction participation and the total length of time a bidder is listed in the lead are both positively correlated with the probability of rebidding.

Both these papers provide evidence of a quasi-endowment effect. More significantly, the results suggest a theory of reference-point formation for bidders in a dynamic auction, namely that the reference point is some dynamic function of future anticipated ownership, which evolves according to two factors: the length of the auction and the more frequently the individual is the leading bidder. Modeling bidders with dynamic preferences is no small technical feat, so we first present simpler but instructive results for sealed-bid auctions with loss-averse bidders and then turn to possible ways to tackle the problem of dynamic auctions.

3. Equilibria in First- and Second-Price Auctions

We model bidders subject to the quasi-endowment effect (in the form of loss aversion) in sealed-bid auctions. As before, we assume there are *N* bidders with types $X_i \subseteq \Re_+$ independently and identically distributed on [0, ω] with density *f*. Let Y_1 be the first order statistic of the values for the *N*-1 bidders besides bidder *i,* and let G be its CDF. Let *p* be an exogenously given reference point universal to all bidders. In a first-price auction, a bidder's payoff will be given by

$$\Pi^I(x_i, x_{-i}) = \begin{cases} x_i - b_i & \text{if} \quad b_i > \max_{j \neq i} b_j \\ 0 & \phantom{\text{if}} \quad b_i < \max_{j \neq i} b_j \end{cases} \qquad (2)$$

where b_i is player *i*'s bid. Rather than maximizing payoffs, bidders instead maximize utility according to the function

$$U^A(x_i, x_{-i}) = \begin{cases} \Pi^A(x_i, x_{-i}) + \varepsilon(\Pi^A(x_i, x_{-i}) - p) & \text{if} \quad b_i > \max_{j \neq i} b_j \\ -\lambda\varepsilon p & \phantom{\text{if}} \quad b_i < \max_{j \neq i} b_j \end{cases} \qquad (3)$$

where $\varepsilon \in \Re_+$ controls the degree of reference-dependence and $\lambda > 1$ is the loss-aversion parameter. In a first-price auction, the $A = I$, meaning the payoff rule is (2). In a second-price auction, $A = II$, so we use the payoff rule given by (1) in section one.

Since there is no time component or leading bidder in such auctions, we assume bidders have some fixed reference utility *p* for the auction, and the extent to which a bidder's actual payoff in the auction (value minus payment) deviates from this reference utility determines gain/loss utility. The quasi-endowment effect results from the fact that a bidder incurs a psychic loss of $-\lambda\varepsilon p$ if she loses because she anticipates having utility *p*, her effective "endowment." Thus, one would expect that she will bid higher than a rational bidder to avoid this loss, which matches the intuitive idea of quasi-endowment posed by Ariely et al. This is the substance of the next two propositions.

Proposition 1: The following strategy is a symmetric equilibrium for the auction defined above:

$$\beta(x_i, x_{-i}) = E(Y_1 \mid Y_1 < x_i) + \frac{\varepsilon(\lambda - 1)}{1+\varepsilon} p \quad (4).$$

PROOF: Suppose all but bidder *i* bid according to (4) and that bidder *i* instead bids some value *b*. Let $z = \beta^{-1}(b)$, and G be the CDF of Y_1. Then bidder *i*'s expected utility from bidding $b = \beta(z)$ is

$$E[U^I(x_i, x_{-i})] = P(\beta(Y_1) < \beta(z)) E[U^I(x_i, x_{-i}) \mid \beta(Y_1) < \beta(z)] +$$
$$(1 - P(\beta(Y_1) < \beta(z))) E[U^I(x_i, x_{-i}) \mid \beta(Y_1) > \beta(z)]$$
$$= P(Y_1 < z)[x_i - E(Y_1 \mid Y_1 < z) - \frac{\varepsilon(\lambda-1)}{1+\varepsilon} p + \varepsilon(x_i - E(Y_1 \mid Y_1 < z) - \frac{\varepsilon(\lambda-1)}{1+\varepsilon} p - p)]$$
$$- (1 - P(Y_1 < z))\lambda\varepsilon p \qquad \textbf{(*)}$$
$$= (1+\varepsilon)[G(z)x_i - G(z)E(Y_1 \mid Y_1 < z) - G(z)\frac{\varepsilon(\lambda-1)}{1+\varepsilon} p] - G(z)\varepsilon p - \lambda\varepsilon p + G(z)\lambda\varepsilon p$$
$$= (1+\varepsilon)G(z)x_i - (1+\varepsilon)G(z)z + (1+\varepsilon)\int_0^z G(t)dt - \lambda\varepsilon p$$

Taking the first order condition of this expression with respect to *z*, we get

$$(1+\varepsilon)g(z)x_i - (1+\varepsilon)g(z)z - (1+\varepsilon)G(z) + (1+\varepsilon)G(z) = 0$$

which implies $x_i = z$. Thus, it is optimal for bidder *i* to bid according to (3). □

Since $\lambda > 1$ it is clear that the equilibrium bid of bidders with quasi-endowment (QE) in a first-price auction exceeds the rational equilibrium by $\frac{\varepsilon(\lambda-1)}{1+\varepsilon} p$, since the rational equilibrium bid is $E(Y_1 \mid Y_1 < x_i)$. This reflects the intuition that bidders with QE compensate for a potential psychic loss of $\lambda\varepsilon p$ by overbidding just enough to offset it in expectation. Notice that in the case of no QE ($\varepsilon = 0$ or $p = 0$) this reduces to the rational equilibrium. A similar result holds for the second-price auction.

Proposition 2: The following strategy is a symmetric equilibrium for the auction defined above under a second-price payoff rule given by (1): $\beta(x_i) = x_i + \frac{\varepsilon(\lambda-1)}{1+\varepsilon} p \quad (5).$

PROOF: Suppose all but bidder i bid according to (5) and that bidder i instead bids some value b. Let $z = \beta^{-1}(b)$, and G be the CDF of Y_1. Then bidder i's expected utility from bidding $b = \beta(z)$ is

$$E[U^{II}(x_i, x_{-i})] = P(\beta(Y_1) < \beta(z))E[U^{II}(x_i, x_{-i}) \mid \beta(Y_1) < \beta(z)] +$$

$$(1 - P(\beta(Y_1) < \beta(z)))E[U^{II}(x_i, x_{-i}) \mid \beta(Y_1) > \beta(z)]$$

$$= P(Y_1 < z)[x_i - E(Y_1 - \frac{\varepsilon(\lambda-1)}{1+\varepsilon} p \mid Y_1 < z) + \varepsilon(x_i - E(Y_1 - \frac{\varepsilon(\lambda-1)}{1+\varepsilon} p \mid Y_1 < z) - p)]$$

$$- (1 - P(Y_1 < z))\lambda\varepsilon p$$

= **(*)** in the proof of Proposition 1, so the rest of the proof follows similarly. □

Again, we find that loss-averse bidders overbid by an amount $\frac{\varepsilon(\lambda-1)}{1+\varepsilon} p$.

We can also extend the revenue equivalence principle to compare expected payments of bidders with QE against rational bidders.

Proposition 3 (Revenue Equivalence with Loss Aversion): Suppose bidders have i.i.d. values with utility function given by (3), and suppose the expected payment of a bidder with value zero is zero. Then for any symmetric equilibrium of a standard auction, the expected payment of a bidder is given by $m^{LA}(x) = G(x)E(Y_1 \mid Y_1 < x) + G(x)\frac{\varepsilon(\lambda-1)}{1+\varepsilon} p$.

PROOF: Let $\beta(x_i, x_{-i})$ denote the bid that bidder i must pay if she wins, and let $m^{LA}(z) = G(z)\beta(z, x_{-i})$ be her expected payment if her value were z. Then her expected utility from reporting a value of z when her true value is x would be:

$$E[U(z, x_{-i})] = G(z)[(x - m(z)) + \varepsilon(x - m(z) - p)] - (1 - G(z))\varepsilon\lambda p$$

$$= G(z)(1+\varepsilon)x - (1+\varepsilon)m(z) + G(z)(\lambda-1)\varepsilon p + \varepsilon\lambda p$$

The first order condition is then:

$$\frac{\partial}{\partial z} E[U(z, x_{-i})] = g(z)(1+\varepsilon)x - 1 + \varepsilon)m'(z) + g(z)(\lambda-1)\varepsilon p = 0$$

Because we have an equilibrium, $z = x$ is optimal. So we get

$$m'(x) = g(x)x + g(x)\frac{(\lambda - 1)\varepsilon}{1+\varepsilon}p$$

By the fundamental theorem of calculus and using the assumption that $m(0) = 0$,

$$m(x) = \int_0^x tg(t)dt + \frac{(\lambda - 1)\varepsilon}{1+\varepsilon}p\int_0^x g(t)dt = G(x)E(Y_1 \mid Y_1 < x) + G(x)\frac{(\lambda - 1)\varepsilon}{1+\varepsilon}p \quad \square$$

Notice that the expected payment of bidders with QE is greater than that of rational bidders by $G(x)\frac{(\lambda - 1)\varepsilon}{1+\varepsilon}p$, which is the consequence of overbidding.

A few comments are in order. These results are presented only for illustrative purposes, since there has been no empirical research on quasi-endowment in first- and second-price sealed-bid auctions. The assumption is that bidders already enter the auction with some sense of attachment to the outcome, its determination unknown. Moreover the precise meaning of this attachment is unknown—hence the rather nebulous specification of *p*. Because there is no a priori reason for having a particular reference point in this auction, we skirt the issue of what constitutes reference utility by simply letting it be exogenously given. This is not to say that *p* is meaningless or does not exist. Bidders might consider an auction's reserve price or perhaps its "buy-it-now" price in the case of online auctions with this option, like eBay, to inform her reference point.[4] Certainly *p* might conceivably also depend on the value of the object itself, whether it be some resale price, or the bidder's actual valuation.

As we have previously mentioned, the determination of reference points is still an open research problem. Koszegi and Rabin make the assumption that the reference point is determined by the rational expectations the individual has about the environment she will face. This assumption, while extreme, at least addresses the influence of expectations in reference-point formation. However, an application of this theory to, say, a first-price auction would mean that *p* is the expected payoff – the expectation of (2) – to the bidder, which substantially increases the complexity of the problem. Our attempts to find a closed-form solution have not been met with much success. These issues are doubly problematic for dynamic auctions with quasi-endowment in which not only is there the challenge of reference-point determination, but this reference point

[4] Shunda (2007) models the effect of a buy-it-now price by using a model of reference-dependent payoffs, though without loss aversion, which we do.

changes during the course of the auction in response to auction length and the status of being the lead-bidder. In light of these problems, we will propose a model that captures the quasi-endowment effect in a more simple way than a loss-aversion framework.

4. Quasi-Endowment in English Auctions

We first consider a model of English auctions due to Milgrom and Weber (1981). This model is common in the literature on interdependent-value[5] auctions because of its analytical simplicity, and we employ it here because of the lack of a model for private-value English auctions, this presumably due to its strategic equivalence to the second-price auction. In the Milgrom-Weber (MW) model, the price for an object rises continuously, and bidders signal their willingness to pay the posted price by, say, constantly depressing a button. As soon as a bidder releases the button, she must exit the auction permanently. In an interdependent-value environment, a strategy in such a game is an *n*-tuple. This is because the price at which a bidder exits conveys information to the remaining bidders about the value of the object, which thus changes their strategy. When values are private, this information is of no use, since bidders already know the full value of the object—it is their private value.

Our purpose in employing this model is to allow bidder utility to change with the bidding process. Specifically, we posit that a bidder's valuation of the object rises as other bidders exit the auction, so when L bidders have exited the auction, bidder i's perceived value of the good is $h(L)\ x_i$, where $h(0) = 1$, $h' > 0$, and $h'' > 0$. This model has the advantage of having some semblance of analytical tractability, while still reflecting some empirical insights. L in some sense proxies the lead bidder effect, since the more bidders leave, the more one might expect remaining bidders to develop an attachment to the object. Such bidders are essentially those who have persevered in the bidding race, and thus are likely to develop a greater sense of ownership, which is precisely the quasi-endowment effect. An increasing valuation captures this effect. Moreover, our assumption that h is convex captures the idea that bidder attachment to the object becomes greater the closer bidders are to the finish line. L also captures a "competition effect"[6]

[5] Unlike private-value auctions, values are not independent, and the actual value of the object is a function of the bidder types.

[6] This was also studied in Ariely et al (2004).

in that the more bidders have exited, tighter the competition among remaining bidders, and one might expect valuations to increase, since the act of winning a perceived competition may carry psychic value.

Analytically, this auction is an extensive form game with simultaneous-move "stages," one stage for every L. Every time a bidder leaves, the remaining bidders' valuations update, so the game progresses to the next stage of the auction, and bidders update their strategies accordingly. Since it is not the case that one bidder moves before another, the strategies are determined simultaneously at each stage, as they are in the previous models of sealed-bid auctions. Hence the stages are simultaneous-move. If we consider each stage as an isolated game, it is clear that every bidder's equilibrium strategy in that stage is to stay in the auction until the price reaches $h(L)\, x_i$, which is just the equilibrium in the rational model. Of course, each stage cannot be analyzed in isolation, since bidder i at L = some k would anticipate that her future selves would have different preferences, and this might lead her to choose a strategy to counteract future overbidding. After all, since bidders exit only when the price reaches $h(L)\, x_i > x_i$, they may well bid higher than x_i, yielding a net utility loss for i at $L = 0$. An equilibrium solution would have to take into consideration these strategic interactions between the different selves of each bidder, something which the Nash equilibrium solution lacks.[7]

Our alternate model of the English auction with quasi-endowment uses a more complicated English auction model in order to have a better analytical concept of a lead bidder. In the "alternating recognition" (AR) model due to Harstad and Rothkopf (2000), when bidders signal their willingness to compete, an auctioneer selects or recognizes two bidders at random to compete one-on-one in an MW English auction. When a bidder exits such an "AR duel," she does not exit the auction for good, and thus has the option of re-entering an AR duel in the future. Then, bidders besides the winner of the duel signal their willingness to compete, and the auctioneer randomly chooses a replacement bidder to compete against the winner of the duel. The price rises continuously but only during duels, of course, and the price does not reset with each duel.

In this model, it should be natural to call the winner of an AR duel the lead bidder. We thus let bidder i's value be $h(W)\, x_i$, where W is the number of times i has won an AR duel. Again,

[7] The concept of credible equilibrium due to Ferreira, Gilboa, and Maschler (1992) may hold promise for this task. It lays out a complicated solution for games with changing preferences, which, admittedly, we do not quite fully comprehend at this point in time.

$h(0) = 1$, $h' > 0$, and $h'' > 0$. W essentially proxies the length of time a bidder is the lead bidder in a manner arguably more realistic than L, and as the empirical results surveyed earlier show, the quasi-endowment effect should strengthen with this variable. This model has the same analytical complexity as the first model with one addition: bidders may find it advantageous to purposefully lose AR duels or to not enter duels even if the price remains below their valuation. This is because if they anticipate overbidding due to quasi-endowment, they may not want to end up as the lead bidder repeatedly, since being such a bidder is precisely what produces quasi-endowment. Strategies under this setup are likely to be more complex than under the first setup.

It should be noted that any equilibrium solution will likely fall short of being good predictions for behavior in the real world. This is because such solutions determine what is optimal for each bidder, given the actions of other bidders, which yield strategies in which bidders *rationally* cope with their irrational preferences. In reality, it is unlikely that bidders do a good job of rationally accounting for their irrational attachments to the object being auctioned. Any model of irrationality in auctions under a game-theoretic framework will suffer from this sort of predictive handicap, which suggests that a more realistic theory of auctions may have to abandon its roots in game theory.

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