

FINAL EXAM

The exam consists of two parts. There are 100 points total. Part I has 25 points and Part II has 75 points.

You have 2 hours.

Some parts of the exam are harder than others. If you get stuck on one part, do the best you can without spending too much time, and then work on other parts of the exam.

## PART I. Multiple choice (25 points)

In your blue book, give the best answer to 5 of the following 6 questions. Note:

– If you wish, you may add a BRIEF explanation of your answer to AT MOST ONE question. In that case, your grade on that question will be based on your answer and explanation together. This means that an explanation can either raise or lower a grade.

– If you answer all 6 questions, your overall score will be based on your average, not on your 5 best scores.

1. Consider a consumer whose behavior is described by the permanent-income hypothesis. In response to an unexpected, permanent fall in his or her labor income, the consumer will:

- A. Borrow against future income to keep his or her consumption unchanged.
- B. Immediately reduce consumption by the amount of the fall in labor income.
- C. Gradually reduce consumption by the amount of the fall in labor income.
- D. Gradually reduce consumption by more than the amount of the fall in labor income.

2. Consider a regression of the form,  $\Delta C_{it} = a + b\Delta Y_{it} + e_{it}$ , where  $C$  is consumption,  $Y$  is income,  $i$  indexes individuals,  $t$  indexes periods, and  $\Delta$  denotes the change from period  $t - 1$  to period  $t$ . The baseline version of the permanent-income hypothesis under uncertainty (quadratic utility,  $\rho = r$ , etc.) predicts  $b = 0$  if:

- A. The equation is estimated by ordinary least squares.
- B. The equation is estimated by instrumental variables, using variables known at time  $t$  as instruments.
- C. The equation is estimated by instrumental variables, using variables known at time  $t - 1$  as instruments.
- D. None of the above – because the permanent income hypothesis implies that consumption depends on permanent income, it predicts  $b = 1$ .

3. Consider the consumption CAPM model in a 2-period economy. The representative agent is an expected-utility maximizer, with utility given by  $U(C_1) + \rho U(C_2)$ ,  $\rho > 0$ . Consider an asset with potentially random period 2 payoff  $D$ . Letting  $E[\cdot]$  denote expectations conditional on period 1 information, the price of the asset in period 1 is:

- A.  $\rho E[U'(C_{t+1})D]/U'(C_t)$ .
- B.  $U'(C_t)/\{\rho E[U'(C_{t+1})D]\}$ .
- C.  $\rho E[U(C_{t+1})D]/U(C_t)$ .
- D.  $U(C_t)/\{\rho E[U(C_{t+1})D]\}$ .
- E.  $\rho\{E[U'(C_{t+1})]E[D] + Cov(U'(C_{t+1}), U'(C_t))\}$ .
- F.  $\rho\{E[U'(C_{t+1})]E[D] - Cov(U'(C_{t+1}), U'(C_t))\}$ .

4. “Performance-based risk” refers to the idea that:

A. Sophisticated investors who make trades that act to correct mispricings are likely to earn high profits, and so cash in their positions and no longer act to correct mispricings.

B. Successful sophisticated investors are reluctant to trade to correct mispricings because they know their any trades they make are likely to be imitated by others.

C. Sophisticated investors are likely to be judged based on their short-run performance, which may be poor even though the expected returns to their investments are high.

D. Sophisticated investors are reluctant to fund entrepreneurs’ projects because they know that the entrepreneurs may abscond with the payoff to the projects if they are successful.

5. Consider the q-theory model of investment with kinked adjustment costs, and suppose that  $K(0) = K_2$  and  $q(0) = 1 - c^-$ . (Here  $c^-$  is the adjustment cost for the first unit of negative investment, and  $K_2$  is defined by  $\pi(K_2)/r = 1 - c^-$ .  $c^-$  is assumed to be strictly positive.) At time 0, there is an unexpected, permanent upward shift of the  $\pi(\bullet)$  function. At time 0:

- A. The  $\dot{q} = 0$  locus shifts down.
- B. The  $\dot{q} = 0$  locus shifts up.
- C. The  $\dot{q} = 0$  locus is unaffected.
- D. The  $\dot{q} = 0$  locus is unaffected if the upward shift of the  $\pi(\bullet)$  is sufficiently small, but shifts if the upward shift of the  $\pi(\bullet)$  is sufficiently large.

6. In the Bernanke, Gertler, and Gilchrist's model (as presented in class):

A. Financial market imperfections make the effect of an upward shift in the productivity of investment projects on the level of investment larger than it would otherwise be.

B. In the regime where the borrowing constraint is binding, an upward shift in the productivity of investment projects has no effect on the level of investment.

C. The fact that an entrepreneur can default on a loan makes the level of investment larger than it would be if financial markets were perfect.

D. The fact that an entrepreneur can default on a loan makes asset prices more volatile than warranted by fundamentals.

## **PART II. 75 points**

### **DO ALL 3 PROBLEMS.**

**(25 points) 1.** Consider an infinitely-lived household. The household's initial wealth,  $A(0)$  is zero; its labor income is constant and equal to  $\bar{Y}$ ,  $\bar{Y} > 0$ ; and the real interest rate is constant and equal to  $\bar{r} > 0$ . The household's flow budget constraint is therefore  $\dot{A}(t) = \bar{r}A(t) + \bar{Y} - C(t)$ , and, as usual, the present discounted value of the household's consumption cannot exceed the present discounted value of its lifetime resources.

In contrast to our usual model, however, the household obtains utility not only from consumption, but also from holding wealth. Specifically, its objective function is

$$\int_{t=0}^{\infty} e^{-\rho t} [u(C(t)) + v(A(t))] dt,$$

where  $u'(\bullet) > 0$ ,  $u''(\bullet) < 0$ ,  $v'(\bullet) > 0$ ,  $v''(\bullet) < 0$ , and  $\rho > 0$ .

a. For this part only, assume  $\rho = \bar{r}$ . Without doing any math, explain whether  $C(0)$  will be less than, equal to, or greater than  $\bar{Y}$ , or whether it is not possible to tell.

b. What is the present value Hamiltonian?

c. Find the conditions that characterize the solution to the household's maximization problem.

**(25 points) 2.** This problem asks you to use the q-theory model to analyze the impact of “forward guidance” about interest rates on investment. Specifically, consider the q-theory model. Assume that initially the economy is in steady state.

At some time, which we will call time 0 for simplicity, there is news: the interest rate,  $r$ , will be temporarily low. Specifically, let  $r^1$  denote its usual value. The news at  $t = 0$  is that  $r$  will equal  $r^2$ ,  $r^2 < r^1$ , from  $t = 0$  to  $t = T$ ,  $T > 0$ . At  $t = T$ ,  $r$  will return permanently to  $r^1$ .

In what follows, be sure to explain your answers.

- How, if at all, do the  $\dot{q} = 0$  and  $\dot{K} = 0$  loci change at  $t = 0$ ?
- How, if at all, do  $q$  and  $K$  change at  $t = 0$ ?
- Describe the behavior of  $q$  and  $K$  after  $t = 0$ .
- Is the impact of the news on investment at  $t = 0$  an increasing function of  $T$ , a decreasing function of  $T$ , unaffected by  $T$ , or is it not possible to tell? Explain.

**(25 points) 3.** Consider the Diamond-Dybvig model as described in lecture. The utility of a Type 1 agent (which occurs with probability  $\theta$ ,  $0 < \theta < 1$ ) is  $\ln C_1^1$ , and the utility of a Type 2 agent (which occurs with probability  $1 - \theta$ ) is  $\rho \ln(C_1^2 + C_2^2)$ , where  $0 < \rho < 1$  and where  $C_t^j$  denotes the consumption of a Type  $j$  agent in period  $t$ .

As usual, 1 unit invested in period 0 yields 1 unit if it is liquidated in period 1, and  $R > 1$  units if it is held until period 2.

We saw in lecture that a social planner who can observe agents’ types and who maximizes the representative agent’s ex ante expected utility will choose  $C_1^{2*} = C_2^{1*} = 0$ ,  $C_1^{1*} = \frac{1}{\theta + (1-\theta)\rho}$ ,  $C_2^{2*} = \frac{\rho R}{\theta + (1-\theta)\rho}$ .

One of the assumptions of the model is that  $\rho R > 1$ . This problem asks you to consider what happens if that assumption fails. That is, assume that  $\rho R < 1$ .

a. Suppose that, as in the usual version of the model, a bank offers an arrangement where anyone who deposits 1 unit in period 0 can withdraw  $C_1^{1*}$  units in period 1 (subject to the availability of funds), with any assets remaining in period 2 divided equally among the depositors who did not withdraw in period 1.

i. Show that if everyone deposits in the bank, and the Type 1’s withdraw in period 1 and the Type 2’s withdraw in period 2, this will yield the social planner outcome described above.

ii. Explain why if the bank offers this arrangement (anyone who deposits 1 unit in period 0 can withdraw  $C_1^{1*}$  units in period 1 (subject to the availability of funds), with any assets remaining in period 2 divided equally among the depositors who did not withdraw in period 1), it is not an equilibrium for the Type 1’s to withdraw in period 1 and the Type 2’s to withdraw in period 2.

b. Is there some other arrangement the bank can offer that improves on the autarchy outcome?