University of California
Department of Economics

Economics 202A
Fall 2014
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Problem Set 9
Due in Lecture Thursday Nov. 6

1 Romer 8.14

2 Romer 8.15

3 Consumption with State-Contingent Goods

Consider a consumer whose labor income (which he or she takes as exogenous) is uncertain. Specifically, the consumers labor income in state s in period t is y_{st} where s denotes the state of the world and t the period. The probability that the state in period t is s is π_{st}. Thus, for each t, $\sum_s \pi_{st} = 1$. The realization of the state each period is independent of the realization in all other periods.

The consumer seeks to maximize

$$E\left[\sum_t \beta^t u(c_t)\right]$$

where $0 < \beta < 1$ is the discount factor and u satisfies the usual conditions. The consumer can purchase and sell state-contingent goods. Denote p_{st} for the price of the consumption good at time t in state s. Thus, we can write the consumers objective function as

$$\sum_t \sum_s p_{st} c_{st} \leq \sum_t \sum_s p_{st} y_{st}$$

1. Set up the consumers maximization problem, and find the first-order condition for c_{st}.

2. Consider two states in some period t, s' and s". Under what conditions is consumption the same in the two states? (That is, under what conditions is $c_{s't} = c_{s''t}$?)

3. Consider state s' in period t' and state s" in period t". Under what conditions is $c_{s't'} = c_{s''t''}$?

4. Consider 2 consumers who differ only in their y_{st}'. Show or provide a counterexample to the following claim: If Consumer 1s consumption in one period is greater than Consumer 2s consumption in that period, Consumer 1s consumption in each period is greater than Consumer 2s consumption in the same period.

5. Suppose that both consumers have constant relative risk aversion utility, with the same coefficient of relative risk aversion $\theta > 0$. What, if anything, can one say about how the ratio of Consumer 1s consumption to Consumer 2s consumption behaves over time?

6. In practice, we often see consumption reversals (that is, one consumer initially having consumption higher than another, but later having lower consumption). List 2 or 3 ways the assumptions of this problem could fail that could make such reversals possible; explain each possibility in no more than a sentence.

7. Suppose that in some period, the realization of s is the one that has the highest value of $p_{st}y_{st}$ for that period for the consumer. How, if at all, will that affect the consumers consumption in later periods?

4 Consumption and Liquidity Constraints

You have a feeling that liquidity constraints may matter for intertemporal consumption. However, you also know that that certain specializations of the conventional unconstrained model of consumption may perhaps equally well explain the facts. You'd like to investigate the empirical importance of these liquidity constraints. To make your research more precise you write down the following alternative hypotheses on what is important in the way people go about choosing their consumption paths:

-H0: Individuals are expected lifetime utility maximizers (i.e. choose consumption paths according to the canonical model) without credit market constraints.
versus
-H1: Individuals maximize lifetime utility subject to a constraint on borrowing

To enable you to reject one of these hypotheses (usually the null), you must know some *testable* implications of H0 or H1. They way to find such testable implication is to write down a model which, when solved, implies some restrictions on how data should behave. To keep things simple and parsimonious, you decide to model H1 using the simplest possible extension of the canonical model:

$$\max \sum_{t=0}^{T-1} E_0 \left[\beta^t u(c_t)\right]$$

subject to:

$$a_{t+1} = R(a_t + \tilde{y}_t - c_t)$$

and

$$a_t \geq 0$$

This is identical to the standard problem except for the constraint $a_t \geq 0$ which says that the household may never run into debt. The only source of uncertainty is future income and we assume that $y_{\min} > 0$.

1. Defining 'cash on hand' $x_t = a_t + \tilde{y}_t$, express the liquidity constraint in t in terms of c_t and x_t.

2. Argue that, if the liquidity constraint is binding at time t, the standard Euler equation is violated. Explain the direction of the inequality. Now assume that the Euler condition holds between t and $t+1$. Could liquidity constraints still affect the consumption decision between the two periods? Finally, suppose that, say, the next 5 period liquidity constraints are binding. What is the effect on consumption at time t (today) of an increase in income in period $t+6$?

3. Now let us see how the Euler equation is modified in the presence of a liquidity constraint. Define $v_t(x_t)$ the value function at time t. Why is x_t rather than a_t the state variable of the problem? Write down the Bellman equation, defining μ as the (nonnegative) Lagrange multiplier on the liquidity constraint. Find the first order necessary condition for optimality. Use the envelope condition to show that the Euler equation becomes an inequality involving marginal utility of consumption today $u'(c_t)$, expected marginal utility of consumption tomorrow $E_t[u'(c_{t+1})]$, the discount factor β, the interest rate R and the value of cash on hand x_t. Explain why you obtain an inequality

4. From the above, argue informally that there exists some cash-on-hand level $\underline{x}_t$ such that $c_t(x) = x$ if $x < \underline{x}_t$ and $c_t(x) < x$ if $x \geq \underline{x}_t$.

5. Explain why the relationship between consumption and current income implied in the preceding question cannot be used to test H0 against H1. Following Zeldes, now consider the following way of testing H0 against H1. Suppose we had a panel of consumers which we could split into two groups: G1 is a group of people who are likely to be liquidity constrained whereas G2 is a group of people who the theory says should not be liquidity constrained. Then, estimating and testing the significance of μ should, if the theory is right, give us a significant μ for G1 but not for G2.

6. What criterium does the theory suggest for splitting the sample? To what extent would you trust this criterium to, say, get you only liquidity constrained people into G1? On what features of preferences might this depend?

7. Now work your way from the first order condition to an equation we can use to estimate and test μ for each group

   (a) rewrite the first order condition as

   $$E_t[R\beta u'_{i,t}(c_{t+1})/u'_{it}(c_t)](1+\lambda_{it}) = 1$$

   where you will define λ in terms of μ.

   (b) assume the following utility function: $u_{it}(c) = c^{1-\theta}/(1-\theta)\exp(\gamma_i t)$ where γ_{it} is a taste shifter that depends on time and demographic variables (family size, age) and some other influences that may not be observable

   (c) From the previous two equations, derive an empirical specification with log change of consumption as the left hand side, fixed effects, gross interest rate, observable family variables and an error term on the right hand side. [Hint: you may want to define $\gamma_{it} = z_{it} + \omega_i + \eta_t + u_{it}$ where z_{it} are observable demographics, ω_i is a household fixed effect, η_t is a time effect and u_{it} captures any unobservable determinants.]

8. Using the equation derived above, show that everything else a higher λ corresponds to a higher rise in expected consumption between t and $t+1$.

9. Now consider the right hand side of the regression you specificed. With which variables might you have an endogeneity problem? Why? How would you correct for this?

10. Indicate qualitatively how you would estimate λ_{it}