UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2014
P. Gourinchas/D. Romer

Problem Set 1
Due in lecture Tuesday, September 9

1. (Hicks meets Solow.) Consider the Solow model with one change: the production function is Y = AF(K,L). All other assumptions of the model are unchanged. (The type of technical change assumed in the Solow model is known as labor-augmenting or Harrod-neutral technical change; the form of technical change assumed in this problem is called Hicks-neutral.)

a. Show what happens if we try to derive a balanced growth path like the one derived in class.

b. What can you say in the special case $F(K,L) = K^{\alpha}L^{1-\alpha}$, $0 < \alpha < 1$?

2. In the Solow model, the long-run effect of an increase in the rate of population growth (relative to what would have happened without the change in population growth) is to:

A. Lower income per person.
B. Have no impact on the growth rate of income per person.
C. Increase the growth rate of total income.
D. (A) and (B).
E. (A) and (C).
F. (B) and (C).
G. (A), (B), and (C).

3. Romer, Problem 1.9.

4. This question asks you to use a Solow-style model to investigate some ideas that have been discussed in the context of Thomas Piketty's recent work.

Consider an economy described by the assumptions of the Solow model, except that factors are paid their marginal products (as in the previous problem), and all labor income is consumed and all other income is saved. Thus, $C(t) = L(t)\left[\frac{\partial Y(t)}{\partial L(t)}\right]$.

a. Show that the properties of the production function imply that the capital-output ratio, K/Y, is rising if and only if the growth rate of K is greater than $n + g$ – that is, if and only if k is rising.

b. Assume that the initial conditions are such that $\partial Y/\partial K$ at $t = 0$ is strictly greater than $n + g + \delta$. Describe the qualitative behavior of the capital-output ratio over time. (For example, does it grow or fall without bound? Gradually approach some constant level from above or below? Something else?) Explain your reasoning.

c. Many popular summaries of Piketty's work describe his thesis as: Since the return to capital exceeds the growth rate of the economy, the capital-output ratio tends to grow without bound. By assumption, this economy starts in a situation where the return to capital exceeds the economy's growth rate. If you found in (b) that *K/Y* grows without bound, explain intuitively whether the driving force of this unbounded growth is that the return to capital exceeds the economy's growth rate. Alternatively, if you found in (b) that *K/Y* does not grow without bound, explain intuitively what is wrong with the statement that the return to capital exceeding the economy's growth rate tends to cause *K/Y* to grow without bound.

d. Suppose $F(\cdot)$ is Cobb-Douglas. Describe the qualitative behavior over time of the share of *net* capital income (that is, $K(t)\left[\frac{\partial Y(t)}{\partial K(t)} - \delta\right]$) in *net* output (that is, $Y(t) - \delta K(t)$). Explain your reasoning.

Is the common statement that an excess of the return to capital over the economy's growth rate causes capital's share to rise over time correct in this case?

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

5. Describe how, if at all, each of the following developments affects the break-even and actual investment lines in our basic diagram for the Solow model:

a. The rate of population growth falls.

b. The rate of technological progress rises.

c. The production function is Cobb-Douglas, $F(K,AL) = K^{\alpha}(AL)^{1-\alpha}$, and capital's share, α, rises.

d. Workers exert more effort, so that output per unit of effective labor for a given value of capital per unit of effective labor is higher than before.

6. Consider an economy described by the Solow model that is on its balanced growth path. Now suppose that, because of acid rain, the depreciation rate rises permanently.

Sketch the resulting path of log output per worker and what that path would have been if the depreciation rate had not changed. Explain your answer.

7. In the Solow model, a discontinuous rise in s causes $\dot{k}$ (that is, dk/dt) to:

A. Increase discontinuously.

B. Start rising at a faster rate than before, but not to increase discontinuously.

C. Increase discontinuously if the initial level of k is below its new balanced-growth-path value; decrease discontinuously if the initial level of k is above its new balanced-growth-path value; and not change if the initial level of k is equal to its new balanced-growth-path value.

D. Decrease discontinuously, but then rise at a faster rate than before.

8. On the balanced growth path of the Solow model:

A. Investment and break-even investment are equal.

B. Investment per unit of effective labor and break-even investment per unit of effective labor are equal.

C. Investment and saving are equal.

D. All of the above.

9. Consider an economy described by the Solow model. Assume that initially capital and output per unit of effective labor are *less* than their balanced-growth-path values. Now suppose that in this situation, the saving rate rises permanently.

Sketch the resulting path of the log of output per worker and what that path would have been if the saving rate had not changed. Explain your answer.

10. (Problem 4, cont.) Consider a marginal increase in K.

a. Derive an expression (in terms of: K/Y, δ, the marginal product of capital F_K, and the elasticity of substitution between capital and effective labor in the gross production function $F(\bullet)$) that determines whether a marginal increase in K increases, reduces, or has no effect on the share of net capital income in net output.

b. Suppose the capital-output ratio is 3, $\delta = 3\%$, and the rate of return on capital $(F_K - \delta) = 5\%$. How large must the elasticity of substitution in the gross production function be for the share of net capital income in net output to rise when K rises?