Currency Crises: Whom to attack or defend?
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Abstract

We develop a model of speculative attacks where two emerging markets fix their currency to the dollar and investors face strategic uncertainty regarding whom to attack or defend out of two pegging regimes. The model predicts a rank-order result where the weakest peg is always more likely to suffer from speculative attacks and where tiny differences in the level of fundamentals are sufficient to sustain currency equilibriums where the weak peg "may" or "would have to" collapse while the strong one remains for sure. Our results suggest the existence of a “tournament competition” among neighbor pegs (where each peg would want to be stronger than the other one in order to avoid being susceptible to speculative attacks) and provide a plausible explanation for why emerging markets may want to be overcautious regarding the strength of their fundamentals.

1. Introduction

During the last two decades we have observed numerous episodes of currency crises, contagion and general financial turmoil. The Mexican crisis in 1994 was accompanied by disruptions in Brazil and Argentina. The Thai crisis in 1998 crashed Malaysia, Indonesia and the Philippines, and the Russian crises in 1999 immediately engulfed the Brazilian economy.

Economists have at least two set of models available to explain these turbulent episodes. On the one hand, we have models of currency crisis based on fundamentals weakness [Krugman (1979)], self-fulfilling expectations [Obstfeld (1996)] and strategic uncertainty [Morris and Shin (1998) and Guimaraes and Morris (2004)]. On the other hand, we have models of contagion based on common fundamentals [Gerlach and Smets (1995)], herding behavior [Calvo and Mendoza (2000)], asymmetric information [Calvo (1999)], hedging macroeconomic risk [Kodres and Pritsker (2001)] and frictions in the financial markets [Pavlova and Rigobon (2005)]. The two sets of models are complementary. The first one explains why we may observe a currency crisis in the first place, while the second one explains why such crisis may propagate to other economies (contagion).

In this paper we provide a first step in an unexplored topic: Where shall we expect the initial crisis to be located? Or in other words where shall we expect the dominoes to start falling? Models of financial crises with complete information provide little guidance in this regard. They predict that the self-fulfilling nature on the investors’ expectations may generate a currency crisis in any of the existent pegs, unless its level of fundamentals is so high that it would not be profitable for investors to attack it in the first place. The argument is the following. If investors believe some peg, either weak or strong, is going to collapse they would attack.
it, inducing the government to abandon the peg and vindicating the initial belief. However, this answer seems somehow incomplete. It begs the question of why shall we expect in the first place to have investors coordinating their actions against a strong peg when a weaker peg would be more likely to collapse in the first place. In this paper, we will argue that a weak peg is always more likely to suffer from speculative attacks than a relatively stronger one, even in those scenarios where the fundamentals of both pegs are almost equally weak or strong.

Building on the work of Morris and Shin (1998) and Guimaraes and Morris (2004) we develop a model where investors face strategic uncertainty regarding whom to attack or defend out of two pegging regimes. We show that when investors are uncertain about the action of other investors there is a rank-order result in equilibrium where the weak peg is always in disadvantage relative to the strong one. The underlying idea is the following. When investors are certain on the actions of other investors they may agree on attacking a strong peg since each investor knows other investors would also be attacking the same peg, inducing the collapse and vindicating the initial belief. However, when investors are uncertain of the action of other investors they would necessarily find optimal either to attack the weak peg or defend the strong one, so that in equilibrium there must be a rank-order result where the weakest peg is always in disadvantage.

In our model there will be two emerging markets fixing their currency to the dollar and a continuum of risk neutral investors. The government of each emerging markets will have a linear cost function from preserving the fixed parity and it will abandon the peg if and only if there is a speculative attack larger in size than the level of fundamentals (i.e. the strength of the economy). The net rate of depreciation will be weakly decreasing in the level of fundamentals, so that it will be profitable to attack a collapsing peg only if the level of fundamentals is sufficiently low. Investors will know the world level of fundamentals, defined as the average level of fundamentals, but will be uncertain about the difference of the fundamentals across pegs. One way to think about this is as a world where investors have a very good idea about the level of fundamentals of both pegs but are a little uncertain on how weak or strong a pegging regime is relative to the other.

Through the paper, we will focus on threshold equilibria, where for each possible realization of the world level of fundamentals there will be a collapse of some peg if and only if its level of fundamentals is below its threshold level. We will show that a threshold equilibrium exists and that in any possible threshold equilibrium the weak peg is always in disadvantage relative to the strong one. Furthermore, we will show that when private information is accurate there is a rank-order result, in the sense that a tiny difference in the level of fundamentals will be sufficient to sustain the equilibrium where the strong peg remains while the weak one collapses.

The method of solution will be the following. First, we will show how to map
signals about the differences on fundamentals into probabilities of collapse for each regime assuming a threshold equilibrium exists. Then, using these inferred probabilities, we will derive the optimal strategies (i.e. the investors’ optimal positions) as a function of the signals and the threshold levels. Next, using these two mappings we will show that if a threshold equilibrium exists then it must be the case that the threshold levels are always equal across pegs. The underlying idea is simple. If the threshold of peg 1 were larger than that of peg 2 then the level of fundamentals at the thresholds would be larger for peg 1, while the size of the attack at the thresholds would be larger for peg 2 (since the size of a depreciation in the event of a collapse is decreasing in the level of fundamentals) and an equilibrium can not exist (since at each threshold the level of fundamentals must equal the size of the attack).

Using the result that the thresholds must be equal across pegs we then show that any threshold equilibrium can be characterized by a mapping of the difference of the fundamentals into three possible states of the world: One intermediate state where either both pegs collapse or remain and two extreme states where only the weakest peg collapses. A large positive or negative difference in the fundamentals results in one of the two extreme states, while moderate differences in fundamentals (i.e. differences close to zero) results in one of the two possible intermediate states.

We use this result to parametrize and solve the model in terms of the size of the two possible intermediate states of the world. We find that for low (high) levels of world fundamentals there is a unique threshold equilibrium where both pegs collapse (remain) in the intermediate state, while for moderate levels of world fundamentals there is multiple equilibria (i.e. there is more than one possible intermediate state). These results show that in equilibrium the weakest peg is always in disadvantage relative to the strong one.

The accuracy of private information will play a crucial role in our model. Having more accurate information will mean that investors would react more strongly to differences in the level of fundamentals and in the limit, we will find that the size of the intermediate states will tend to zero for moderate levels of world fundamentals (while it remains bounded for high and low levels of world fundamentals). These are our key results and our main contribution. It is not only true that the weak peg is always in disadvantage but also that when the world level of fundamentals is moderate and private information accurate tiny differences in fundamentals are sufficient to sustain an equilibrium where the weak peg "may" or "would have to" collapse while the strong one remains for sure. These results suggest the existence of a tournament competition among neighbor pegs and provide a plausible explanation for why emerging markets may want to be overcautious regarding the strength of their fundamentals.

A second contribution of our work is methodological. In the paper, we provide a set of simplifying assumptions that put together allow us to easily solve for a two
country model of speculative attacks where investors face strategic uncertainty. To
our knowledge, models with strategic uncertainty had been restricted to contexts
where only a single peg is considered due to tractability reasons. In the paper we
will emphasize the role of each assumption and we will highlight how the mutual
interaction among them facilitates the computation of the equilibrium solution.

The document is organized as follows. In the following section we will present
the model. In section 3 we will solve the model under complete information. In
sections 4 we will introduce strategic uncertainty and solve a simplified version
of the model where we impose the restrictive assumption that it is always profitable
to attack a collapsing peg (even when the fundamentals are very strong). It will
be convenient to start with this simplified version of the model, since it will be
easier to solve and it will preserve most of the results we would like to emphasize.
In section 5 we will relax the restrictive assumption we imposed in section 4 and
will show how to solve the model with strategic uncertainty.

2. The Model

There are two emerging markets fixing their currency to the dollar. There is a
continuum of risk neutral investors in the unit interval, each one endowed with a
stock of wealth \( W \). There are two periods and each investor wants to maximize the
end of period utility. Investors can buy and/or sell large amounts of each currency
in the market. We will denote \( y_i(j) \) to the dollar amount of a self-financing position
where investor \( j \) borrows in the local currency of peg \( i \) in order to invest that money
in dollars. The dollar amount \( y_i(j) \) can be positive or negative. Whenever \( y_i(j) \)
is positive we will say that the investor is "attacking peg \( i \)" or having a "short
position" in the sense that she would be borrowing (going short) \( y_i(j) \) dollars in
the local currency of peg \( i \) in order to invest that money in dollars. Whenever
\( y_i(j) \) is negative we will say that the investor is "defending peg \( i \)" or having a
"long position" in the sense that she would be borrowing \( y_i(j) \) dollars in dollars
in order to invest that money (go long) in the local currency of peg \( i \).

It is assumed that the interest rates are exogenous while the decision to aban-
don or not each peg remains endogenous. The interest rate on the dollar is normal-
ized to zero while that of the two identical pegging economies is fixed to a scalar
called \( r \). The governments of the emerging markets are assumed to face a linear
cost function from keeping the peg and a normalized gain equal to zero. The peg
of each country is abandoned iff,

\[
\overline{y}_i \equiv \int_{0}^{1} y_i(j) dj > \theta_i
\]  

(1)
where $\bar{y}_i$ is the total demand of currency $i$ measured in dollars and $\theta_i$ is the level of fundamentals of peg $i$. The stronger the fundamentals, the less likely a peg would collapse from a speculative attack.

The net rate of depreciation $g(\theta)$ in the event of a collapse of the peg will be weakly decreasing in the strength of fundamentals $\theta$,

$$g(\theta) = \begin{cases} g_H & \text{if } \theta < \theta^p \\ g_L & \text{otherwise} \end{cases}$$

(2)

where $g_H$, $g_L$ and $\theta^p$ are such that following restrictions hold,

$$\theta^p > 0$$

(3)

$$v_a \equiv (1 - \frac{(1 + r)}{(1 + g_H)}) > r$$

(4)

$$g_L = 0$$

(5)

Equation 3 assumes that if a peg depends on positive flows for its survival then its likely to generate capital gains in the event of a collapse. Equation 4 is called the one way bet assumption. It states that the payoff from attacking a weak collapsing peg is larger than the payoff from defending a peg that remains in place. In order to see this note that $v_A$ is the end of period payoff in dollars of a position $y_i(j)$ equal to 1 when there is a collapse of peg $i$ with $\theta_i < \theta^p$. On the other hand, $r$ is just the payoff in dollars of a position $y_i(j)$ equal to -1 when peg $i$ remains in place. Equation 5 is a highly convenient simplifying assumption. It implies that the state of the world where the peg is abandoned, but fundamentals are strong, is observationally equivalent to the state of the world where the peg remains. With this assumption there are only two states of the world in the investor problem: (i) The peg collapses with $\theta < \theta^p$ (i.e. there is a profitable collapse) and (ii) any other state (which includes a peg that remains in place or a peg that collapses with $\theta \geq \theta^p$).

The investor problem is given by,

$$y^* = \text{Argmax}_{y_1, y_2} E_0 \left[ W + y_1 \left( 1 - \frac{1 + r}{1 + g(\theta_1)} \right) + y_2 \left( 1 - \frac{1 + r}{1 + g(\theta_2)} \right) \right]$$

(6)

s.t. the creditor constrain,

$$|y_1| + |y_2| \leq \frac{W}{c}$$

(7)

where $c \geq v_a$. 


Equation 7 ensures that wealth is positive in every state of the world while imposes the simplifying assumption that the size of an optimal attack equals the size of an optimal defense. Equation 7 could be replaced by the Inada conditions and all the results of the paper would remain in place. That is, there is no loss of substance by imposing this simplifying assumption. In the other hand, equation 7 is very easy to interpret. Investors can build large short or long positions across markets but the size of those positions can not exceed $\frac{W}{c}$ dollars. This specification is analogous to the one used by Morris and Shin (1998). In the single country model of Morris and Shin (1998) investors could either short or not one dollar. Here, investors can either short or long $\frac{W}{c}$ dollars. The main difference is that now investors may be able to split their positions across markets.

The solution of the investor problem is simple to derive because the objective function is linear in $y_1$ and $y_2$. Letting $p_i$ denote the probability of a profitable collapse for peg $i$ we can rewrite the objective function like,

$$y^* = \text{Argmax}_{(y_1,y_2)} [W + y_1 (p_1 v_a - (1 - p_1)r) + y_2 (p_2 v_a - (1 - p_2)r)]$$  \hspace{1cm} (8)

so that a risk neutral investor would just compare the four levels of expected wealth he can get from setting $(y_1,y_2)$ equal to $(\frac{W}{c},0)$, $(-\frac{W}{c},0)$, $(0,\frac{W}{c})$ or $(0,-\frac{W}{c})$ and choose the option with the highest expected payoff. That is, there will be a bang-bang solution.

**Claim 1** The solution of the investor problem is given by,

$$y^*(p_i,p_j,v_a/r,W) = (y^*_1,y^*_2) = \begin{cases} 
(W/c,0) & \text{if and only if } p_i + p_j > \frac{2r}{\alpha v_a + 2} \text{ and } p_i > p_j \\
(\alpha W/c,(1-\alpha)W/c) & \text{if and only if } p_i + p_j > \frac{2r}{\alpha v_a + 2} \text{ and } p_i = p_j \\
(-W/c,0) & \text{if and only if } p_i + p_j \leq \frac{2r}{\alpha v_a + 2} \text{ and } p_i < p_j \\
(-\alpha W/c,-(1-\alpha)W/c) & \text{if and only if } p_i + p_j \leq \frac{2r}{\alpha v_a + 2} \text{ and } p_i = p_j
\end{cases}$$  \hspace{1cm} (9)


where $\alpha$ is a random variable with support in the interval $[0,1]$.

The solution of the investor problem is illustrated in Figure 1. The reader is invited to note how the solution depends only on the sum of probabilities and the $45^0$ line. Attack i at the dollar amount $\frac{W}{c}$ is the solution of the investor problem (i.e. $(y^*_i,y^*_j) = (\frac{W}{c},0)$) if and only if the sum of probabilities is larger than $\frac{2r}{\alpha v_a + 2}$ and $p_i > p_j$. Similarly, defend i at the dollar amount $\frac{W}{c}$ is the solution of the investor problem (i.e. $(y^*_i,y^*_j) = (-\frac{W}{c},0)$) if and only if the sum of probabilities is less than or equal to $\frac{2r}{\alpha v_a + 2}$ and $p_i < p_j$. Whenever $p_i = p_j$ the investor is indifferent between taking positions across markets and either pure or mixed strategies may result from optimal behavior as shown in claim 1.
Figure 1: Solution of the Investor Problem

Throughout the paper we will assume that $\alpha$ is independent across individuals with finite first and second moments given by $\mu_\alpha$ and $\sigma_\alpha^2$. We will also assume that $\mu_\alpha$ is a random variable in the population with first moment equal to $\frac{1}{2}$ so that there is no particular bias against any of the two pegs. The implication of this "no bias assumption" is that whenever a fraction $z$ of the population sets their positions equal to $(y_1, y_2) = (\alpha W/c, (1-\alpha)W/c)$ we will have an aggregate position that will converge in probability to $(zW/2c, zW/2c)$ by the law of large numbers. So that aggregate positions would equal the ones we would obtain if all these investors were perfectly hedging across markets (i.e. $(y_1, y_2) = (W/2c, W/2c)$) rather than randomizing their position across markets. An identical argument applies when a fraction of the investors are randomizing with long positions across markets by setting $(y_1, y_2) = (-\alpha W/c, -(1-\alpha)W/c)$. Taking into account these results, throughout the paper we may refer occasionally to the optimal solutions $(\alpha W/c, (1-\alpha)W/c)$ and $(-\alpha W/c, -(1-\alpha)W/c)$ as the ones where investors are either attacking or defending both pegs equally, in the sense that on average the statement will always be true.
3. Complete information.

In this section of the paper we solve the model under the assumption of complete information. Here, all investors will know the realization of the fundamentals and there will be common knowledge on this information.

As it is the case in the single country models of currency crisis with common knowledge, we will find that in general there is multiple currency equilibria for each possible realization of the fundamentals. The underlying idea behind this result is the same one that in the single country models. If investors were to believe one of the pegs will remain in place then it would be optimal to defend it, inducing the government to maintain the peg and vindicating the initial belief. If instead investors were to believe that some of the pegs would collapse then it would be optimal to attack it, inducing the government to abandon the peg and vindicating the initial investors’ expectations. The common knowledge assumption on the information structure generates a self-fulfilling nature on investors’ initial beliefs that opens the door for a wide variety of possible outcomes for most realization of fundamentals.

In the following discussion we will provide an exact mapping from the realization of the fundamentals into the set of possible currency equilibria. We will do this deriving first the set of probabilities that could be sustained in equilibrium (the set of possible self-fulfilling beliefs) and then computing the set of fundamentals that would be consistent with each possible set of probabilities.

Deriving the set of probabilities that could be sustained in equilibrium is trivial. Casual inspection about the possible realizations of the fundamentals and the net demand reveals that there are only four sets of expected probabilities that could be self-fulfilling.

- **The first set corresponds to the case where investors know \( f_i > \theta_i \) and \( \theta_i < \theta^p \) for \( i=1,2 \); which implies \( p_1 = p_2 = 1 \).** In this case the optimal investment strategy for all investors is to randomize short position across markets \((y^*(p_1, p_2, v_a/r, W) = (\alpha W/c, (1-\alpha)W/c))\) generating a net aggregate demand equal to \( W/2c \) for both pegs. Then, there could be an equilibria where both pegs collapse as long as the fundamentals in both economies are smaller than \( W/2c \) and \( \theta^p \).

- **The second and third set of probabilities correspond to the case where \( f_i > \theta_i \) and \( \theta_i < \theta^p \) hold only for country \( i \), which implies \( p_i = 1 \) and \( p_{-i} = 0 \).** In this scenario, the optimal investor strategy would be to attack country \( i \) at \( W/c \) without taking positions in the other country. These expectations would be self-fulfilling as long as the fundamentals in country \( i \) are smaller than both \( W/c \) and \( \theta^p \) and the fundamentals in the other country are larger than zero (if the fundamentals of the other peg were below zero there would
also be a collapse of that peg and it would also be profitable since $\theta^p > 0$).

• The last set of probabilities that could be sustained in equilibrium correspond to the case where $\theta_i > \theta_1$ and $\theta_i < \theta^p$ does not hold for $i=1,2$, which implies $p_1 = p_2 = 0$. Here, it is optimal for investors to set $y^*(p_i, p_j, v_a/r, W) = (-\alpha W/c, -(1 - \alpha)W/c)$ and there would be a net aggregate demand equal to $-W/2c$ for both pegs. There would be an equilibrium where both pegs remain if the fundamentals of both pegs are larger than $-W/2c$.

In Figures 2 and 3 we present a visual mapping from the realization of the fundamentals into the set of possible currency equilibria. Figure 2 covers the case where $\theta_p > W/2c$ while Figure 3 presents the case where $\theta_p \leq W/2c$. In both figures we include lines denominated CC, RR, RC and CR, where the letters C and R indicates if each one of the pegs "collapses" or "remains" in equilibrium. The set of fundamentals $(\theta_1, \theta_2)$ located to the left and below the lines CC correspond to the equilibrium where both pegs collapse. The set located above and to the right of RR correspond to the equilibrium where both pegs remain. The set located above and to the left of CR correspond to the equilibrium where only peg 1 collapses, while the set below and to the right of RC correspond to the set where only peg 2 collapses.

We now have a full characterization of the currency equilibria in the bidimensional space of fundamentals. Let us focus first in Figure 2 where we assume $\theta_p > W/2c$. Here, there are four sets where only one equilibrium is possible. When the fundamentals are both larger than $\text{Min}[W/c, \theta^p]$ the only possible equilibrium is that both pegs remain. When the fundamentals in country i are smaller than $-W/2c$ and those on country j are larger than $W/2c$ then the only possible equilibrium is the one where only regime i collapses. Finally, when both fundamentals are negative and, at least one of them is smaller than $-W/2c$, then the only possible equilibrium is a collapse of both pegs.

In addition to these four sets, there are other 9 sets where multiple currency equilibria is possible. In all sets close to the origin we could have a successful double attack or defense of the regimes. In sets located to the right of the origin we could also see an equilibrium where only regime 2 collapses, while in those sets located above the origin we could also observe an equilibrium where only regime one collapses. The result is a wide range of currency equilibria around the origin. The extreme case is the set located in the upper right corner of the origin, where all four currency equilibria are possible.

There are two results we would like to emphasize. First, there is a range of fundamentals where the strong regime may collapses while the weak one remains in place. This set is given by the square formed by the intersection of lines CR and RC. Letting $\Delta$ denote such set we have,

$$\Delta = \{ (\theta_1, \theta_2) \in R^2 | \theta_i \in [0, \text{Min}[W/c, \theta^p]] \text{ for } i = 1, 2 \}$$

(10)
Second, the range of multiple equilibria for a given regime depends significantly on the strength of the other peg. The stronger the other regime the larger the range of multiple equilibria. In order to see this, note that if the level of fundamentals of peg 2 were larger than zero then peg 1 could suffer in equilibrium a total attack of $\text{Min}[W/c, \theta p]$ or benefit from a total defense of $-W/2c$. These amounts define the range of multiple equilibria for peg 1. If instead the level of fundamentals of peg 2 were below zero then the peg 1 can no longer be attacked in equilibrium in the amount $\text{Min}[W/c, \theta p]$. The maximum attack decreases to $W/2c$ which is the case of the equilibrium where both pegs collapse. If the level of fundamentals of peg 2 were even lower, say below $-W/2c$, then peg 1 can no longer benefit from a total defense of $-W/2c$. Instead, the maximum defense would be zero, which would be the case of the equilibrium where only the other regime collapses.

It follows that a high realization of fundamentals in the other economy is accompanied by a range of multiple equilibria in the peg of $[-W/2c, \text{Min}[W/c, \theta p]]$, while a low realization of fundamentals is accompanied by the smaller range $[0, W/2r]$. That is, at moments of weakness in the other economy there are fewer resources available to attack or defend the peg. A relatively weak regime would have to collapse, while a relatively strong one would no longer be susceptible to speculative...
attacks. We summarize this result below,

\[ \begin{align*}
\theta_2 > 0 & \rightarrow \text{Range of Multiple Eq. } \theta_1 \in [-W/2c, \min\{W/c, \theta_p\}] \\
\theta_2 \in [-W/2c, 0] & \rightarrow \text{Range of Multiple Eq. } \theta_1 \in [-W/2c, W/2c] \\
\theta_2 < -W/2c & \rightarrow \text{Range of Multiple Eq. } \theta_1 \in [0, W/2c]
\end{align*} \tag{11} \]

The results in the case where \( \theta_p \leq W/2c \), illustrated in Figure 3, are very similar. Here, an attack on a peg with \( \theta_1 > W/2c \) is no longer profitable and the maximum level of fundamentals under which the peg may collapse is the same among the two equilibriums where either both pegs collapse or only one peg collapses. As before, there is a range of fundamentals where only the strong regime may collapse (given also by equation 10) and there is a positive relation between the range of multiple equilibria and the level of fundamentals in the other peg. However, the magnitude of this relation is smaller here. The ranges of multiple equilibria are given by,

\[ \begin{align*}
\theta_2 \geq -W/2c & \rightarrow \text{Range of Multiple Eq. } \theta_1 \in [-W/2c, \theta_p] \\
\theta_2 < -W/2c & \rightarrow \text{Range of Multiple Eq. } \theta_1 \in [0, \theta_p]
\end{align*} \tag{12} \]

Figure 3: Currency equilibria under complete information: \( \theta_p \leq W/2c \)

We will show now how the existence of an equilibrium where the strong peg collapses while the weak one remains in place, is a direct result of the assumption
of complete information and common knowledge. When we assume perfect information exist it is possible for investors to agree on attacking a strong peg making the initial belief self-fulfilling in equilibrium. However, this can no longer be true in the presence of strategic uncertainty. When investors are uncertain about the actions of other investors they would either attack the weak peg or defend the strong one so that in equilibrium it must be the case that the weak peg is always in disadvantage relative to the strong one, providing a rank-order result in the limit. We will formalize this result in the following two sections.

4. Strategic uncertainty with constant payoffs: The standard model ($\theta^p \to \infty$).

Let us introduce strategic uncertainty in our model and show how to solve it under the restrictive assumption that it is always profitable to attack a collapsing peg (i.e. $\theta^p \to \infty$). Throughout the paper, we will call this version of the model the "standard model". Later, in the following section, we will relax this assumption and we will show how to solve the more general model where $\theta^p$ is finite. There, the reader will notice how the apparent restrictive assumption we are making here not only simplifies the problem but also provides most of the main results of the more general model, making it a very appealing initial step for the presentation of our results.

We will introduce strategic uncertainty using the following decomposition,

$$\theta_1 = \theta_l + \frac{\theta_d}{2}$$

$$\theta_2 = \theta_l - \frac{\theta_d}{2}$$

where $\theta_l$ refers to the level of world fundamentals, defined as the average of the fundamentals in the two regimes ($\theta_l \equiv \frac{\theta_1 + \theta_2}{2}$), and $\theta_d$ to the difference of the fundamentals ($\theta_d \equiv \theta_1 - \theta_2$). This decomposition will allow us to work in a tractable way with the issue of strategic uncertainty regarding "whom to attack or defend" while abstracting from the issue of strategic uncertainty related to "whether to attack or not" that is studied in detail in the models of Morris and Shin (1998) and Guimaraes and Morris (2004).

In our model, $\theta_l$ is known to investors but there is uncertainty in the size of the differences $\theta_d$. Agents observe $\theta_l$ and a signal $x_d = \theta_d + \epsilon$ on the differences of the fundamentals across countries. One way to think about this is as a world where investors have a very precise idea about the level of fundamentals of both pegs but are a little uncertain on how weak or strong a pegging regime is relative to the other. The noise $\epsilon$ is assumed to be uniform in the interval $[\frac{-S}{\sqrt{2}}, \frac{S}{\sqrt{2}}]$ and the prior of $\theta_d$ is assumed to be the improper uniform over the entire real line.
Throughout the paper, we will focus on threshold equilibria, so that there is a collapse of peg i iff,
\[ \theta_i < \theta_i^T \]  
(15)

We will show that a threshold equilibrium exists and that in any possible threshold equilibrium the weak peg is always in disadvantage relative to the strong one. We will also show that when private information gets highly accurate there is a rank-order result, in the sense that a tiny difference in the level of fundamentals is sufficient to sustain the equilibrium result where the strong peg remains while the weak one collapses. We will prove these results following three steps. First, we show how to derive the mapping from signals \( x_d \) into beliefs \((p_1, p_2)\) and optimal strategies assuming a threshold solution \((\theta_1^T, \theta_2^T)\) exists. Second, we show that if a threshold solution exists then it must be the case that the threshold are identical across countries (i.e. \( \theta_1^T = \theta_2^T \)). Finally, using the previous two steps, we show how to derive the threshold solutions.

4.1. Distribution of beliefs in the population.

In this section we show how to map signals about the difference of fundamentals into the probabilities \( p_1 \) and \( p_2 \) under the assumption that a threshold solution exists. Here, investors know \( \theta_i \) and receive a signal \( x_d \) about the difference on the level of fundamentals that use to infer the probability that the fundamental level of each peg is below its threshold level (in this section the probability of a collapse equals the probability of a profitable collapse since attacking a collapsing peg is always profitable by assumption).

Recognizing that this problem is one-dimensional (since \( \theta_i \) is known) we will transform it into a standard one-dimensional setting where we measure each variable as the distance from each point to the origin (where the origin here will be the middle point in the \( \theta_i \) line, given by \( \theta_1 = \theta_2 = \theta_i \)). In order to do this we will use the fact that there is a one to one relation between the difference of the fundamentals \( \theta_d \) and a signed measure of the distance between the point \( \theta_1 = \theta_2 = \theta_i \) and any point \((\theta_1, \theta_2)\) along the line \( \frac{\theta_1 + \theta_2}{2} = \theta_i \),

\[ \theta_{an}(\theta_i, \theta_1, \theta_2) = \sqrt{2/\left(\theta_1 - \theta_i\right)} \left(1 - 2 * 1_{\theta_d < 0}\right) \]  
(16)

This "adjusted norm" \( \theta_{an}(\theta_i, \theta_1, \theta_2) \) is positive when \( \theta_1 > \theta_2 \) and negative otherwise. The equivalent relation between \( \theta_d \) and \( \theta_{an}(\theta_i) \) is given by,

\[ \theta_{an}(\theta_i, \theta_1, \theta_2) = \left(\sqrt{(\theta_1 - \theta_i)^2 + (\theta_2 - \theta_i)^2}\right) (1 - 2 * 1_{\theta_d < 0}) \]  
(17)

\[ = \sqrt{2}(\theta_1 - \theta_i)/\left(1 - 2 * 1_{\theta_d < 0}\right) = \sqrt{2}(\theta_1 - \theta_i) \]  
(18)
where in the second equation we have used the fact that along the line $\theta_l$ it is true that $(\theta_1 - \theta_l) = -(\theta_2 - \theta_l)$.

Then, the problem where agents know $\theta_l$ and receive a signal $x_d$ about $\theta_d$ is equivalent to the one where agents know $\theta_l$ and receive instead a signal about $\theta_{an}$ with the following properties,

$$x_{an} = \frac{x_d}{\sqrt{2}} = \frac{1}{\sqrt{2}}(\theta_d + \varepsilon) = \theta_{an} + \lambda$$

where $\lambda$ is independent across individuals and has a uniform distribution with support in the interval $[-\frac{S}{2}, \frac{S}{2}]$, while the prior of $\theta_{an}$ is the improper uniform distribution over the entire real line.

The threshold level of pegs 1 and 2 can also be written in terms of the adjusted norm,

$$\theta^T_1(\theta_l) = \theta_l + \frac{\theta_{d,1}^T}{2}$$

$$\theta^T_2(\theta_l) = \theta_l - \frac{\theta_{d,2}^T}{2}$$

and the probability of collapse of each peg is just a function of the location of the thresholds and the signal along the line $\frac{\theta_1 + \theta_2}{2} = \theta_l$. For example, the probability of collapse of peg 1 is given by,

$$p_1(x_{an}, \theta_{an,1}, S) = P[\theta_{an,1} < \theta_{an}]$$

$$= P[\theta_{an} < \theta_{an,1}|x_{an}]$$

$$= P[x_{an} - \lambda < \theta_{an,1}|x_{an}]$$

$$= 1 - F_{\lambda}[T_{-\frac{S}{2}, \frac{S}{2}}(x_{an} - \theta_{an,1})]$$

$$= F_{\lambda}[T_{-\frac{S}{2}, \frac{S}{2}}^{\theta_{an,1}}(\theta_{an,1} - x_{an})]$$

$$= T_{[0,1]} \left( \frac{(\theta_{an,1} - x_{an}) - \left( -\frac{S}{2} \right)}{S} \right)$$

$$= T \left( \frac{(\theta_{an,1} + \frac{S}{2}) - x_{an}}{S} \right)$$

$$= T \left( \frac{(\theta_{an,1} + \frac{S}{2}) - x_{an}}{S} \right)$$
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Figure 4: Distribution of beliefs for arbitrary pair $[\theta_{an,1}^T, \theta_{an,1}^T]$

where the function $T_{[z_l, z_u]}(x)$ truncates the value of $x$ so that it is always located between $z_l$ and $z_u$. That is,

$$T_{[z_l, z_u]}(x) = \begin{cases} z_l & \text{if } x < z_l \\ x & \text{if } x \in [z_l, z_u] \\ z_u & \text{otherwise} \end{cases}$$  \quad (30)

with $T(x) \equiv T_{[0,1]}(x)$.

Following the same procedure, we can show that the probability of collapse of peg 2 is given by,

$$p_2^S(x_{an}, \theta_{an,2}^T, S) = T\left(\frac{x_{an} - (\theta_{an,2}^T - \frac{S}{T})}{S}\right)$$  \quad (31)

In Figure 4 we illustrate the mapping of signals $x_{an}$ to beliefs for an arbitrary pair $[\theta_{an,1}^T, \theta_{an,2}^T]$. In the upper part of the Figure we show the mapping from the signal $x_{an}$ into $p_1$, while in the lower part we show the mapping from $x_{an}$ into $p_2$. The reader can note that for both pegs the probability of collapse goes gradually from 0 to 1 in the interval $x_{an} \in [\theta_{an,i}^T - \frac{S}{T}, \theta_{an,i}^T + \frac{S}{T}]$, providing us for an easy to use rule for the derivation of an optimal investment strategy (the mapping from signals to optimal actions).
An additional interesting result is that the distribution of beliefs on the collapse of peg i is uniform in the population when \( \theta_{an} = \theta_{tan,i} \). In order to see this consider for example the case of peg 1. When \( \theta_{an} = \theta_{tan,1} \) the proportion of the population with \( p_1 < \overline{p} \) is given by the proportion of the population receiving a signal above \( x'_{an,1}(\overline{p}) \), where \( x'_{an,1}(\cdot) \) is just the inverse function of equation 29. But such proportion is given by \( \frac{\theta_{tan,1} + \hat{z} - x'_{an}(\overline{p})}{S} \) which equals \( \overline{p} \) by equation 29. It follows that a \( \overline{p} \) percentage of the population believes that the probability of collapse of regime 1 is \( \overline{p} \) or lower. That is, the distribution of beliefs is uniform in the population at the threshold level.

The properties described above are not a special feature of our model or the uniform distribution. Guimaraes and Morris (2004) have shown that the distribution of beliefs is uniform in the population at the threshold in a single country model with strategic uncertainty. On the other hand, in this model, it can be shown that if \( \lambda \) were not uniform, but instead a symmetric random variable centered around zero with support in the interval \([s, \bar{s}]\), then we could still easily characterize the mapping of signals to beliefs using \( p^1_s(x_{an}, \theta_{tan,1}) = F_\lambda(T_{[s, \bar{s}]})\left[-\lambda = (\theta_{tan,1} - x_{an})\right] \)
and \( p^2_s(x_{an}, \theta_{tan,2}) = F_\lambda(T_{[s, \bar{s}]})\left[\lambda = (x_{an} - \theta_{tan,2})\right] \). Furthermore, the distribution of beliefs would also be uniform in the population at the threshold. Then the convenience of the uniform distribution in this model is not based on the two previous properties but instead on the fact that the mapping of signals to beliefs given by equations 29 and 31 interacts very easily with the optimal investor rule given by claim 1, facilitating the construction of an optimal investment strategy and the characterization of the currency equilibria.

In the following claim we formalize the statement that the distribution of beliefs is uniform at the threshold. The claim will be a little more general that what we need here. This anticipates the fact that beliefs may be centered at a point different to \( \theta_{tan,i} \) as will be the case in others section of the paper, where it may not always be profitable to attack a peg (i.e. \( \theta^p \to \infty \)).

Claim 2 If a \( \theta^*_{an,1} \) exists so that \( p_1(x_{an}, \theta^*_{an,1}, S) \equiv T \left( \frac{\theta_{tan,1} + \hat{z} - x_{an}}{S} \right) \), then the distribution of beliefs on \( p_1 \) is uniform in the population when \( \theta_{an} = \theta^*_{an,1} \). The same property applies on \( p_2 \) if \( p_2(x_{an}, \theta^*_{an,2}, S) \equiv T \left( \frac{x_{an} - \theta_{tan,2} - \hat{z}}{S} \right) \) and \( \theta_{an} = \theta^*_{an,2} \).

4.2. Symmetric intermediate states
We are now ready to show that if a threshold solution exists then it must be true that \( \theta_1^T = \theta_2^T \). The argument is the following. Let \( \mathcal{F}_i(\theta_{an}, \theta_{an,1}^T, \theta_{an,2}^T, S) \) denote the net demand of currency \( i \) when the realization of fundamentals is \( \theta_{an} \). When \( \theta_{an} = \theta_{an,1}^T \) we can show that the distribution of beliefs in the population regarding the probability of collapse of regime \( i \) and \(-i\) is identical for \( i=1,2 \). This implies that the net aggregate demand a regime faces when \( \theta_{an} \) equals its threshold level is equal for both pegs, that is \( \mathcal{F}_1(\theta_{an,1}^T, \theta_{an,1}^T, \theta_{an,2}^T, S) = \mathcal{F}_2(\theta_{an,2}^T, \theta_{an,1}^T, \theta_{an,2}^T, S) \). Since a necessary condition for an equilibrium is that at the threshold net demand equal the level of fundamentals then it must also be true that \( \theta_{an,1}^T = -\theta_{an,2}^T \), which implies \( \theta_1^T = \theta_2^T = \theta^T \).

The reader is invited to note that whenever \( \theta^T > \theta_l \) then there will be an intermediate state where both pegs collapse, while if instead \( \theta^T < \theta_l \) then there is an intermediate state where both pegs remain. Figure 5 provides an illustration. In the upper part of the figure we have the case where \( \theta^T > \theta_l \) so that \( \theta_{an,1}^T = -\theta_{an,2}^T > 0 \). Here we have that only peg 1 collapses when the realization of \( \theta_{an} \) is smaller than or equal to \( \theta_{an,2}^T \), both pegs collapse when \( \theta_{an} \in (\theta_{an,2}^T, \theta_{an,1}^T) \) and only peg 2 collapses when the realization of \( \theta_{an} \) is larger than or equal to \( \theta_{an,1}^T \).

We will call \( B_{CC} \) to the size of the intermediate state where both pegs collapse (i.e. \( B_{CC} = \theta_{an,1}^T - \theta_{an,2}^T \)). In the lower part of the figure we present the case where \( \theta^T < \theta_l \) so that \( \theta_{an,1}^T = -\theta_{an,2}^T < 0 \). Here we have an intermediate state where both pegs remain. We will call \( B_{RR} \) to the size of this intermediate state (i.e. \( B_{RR} = \theta_{an,2}^T - \theta_{an,1}^T \)). We formalize the result that the threshold solutions need to be equal across pegs in the following claim.

**Claim 3** For any average level of fundamentals, if a threshold equilibrium exist, this must be symmetrically centered around the point \( \theta_1 = \theta_2 = \theta_l \), so that \( \theta_{an,1}^T = -\theta_{an,2}^T \) and \( \theta_1^T = \theta_2^T \). Using this result, we can express the functions of probabilities, optimal strategies and total net demands in terms of the size \( B \) of an intermediate state, so that \( \theta_{an,1}^T = -\theta_{an,2}^T = \frac{B_{CC}}{2} \) if there is an intermediate state \( CC \) where both pegs collapse and \( \theta_{an,1}^T = -\theta_{an,2}^T = \frac{B_{RR}}{2} \) if instead there is an intermediate state \( RR \) where both pegs remain.

We now show how to derive the currency equilibria for any world level of fundamentals \( \theta_l \). The method of solution will be the following. First, we derive the set of \( \theta_l \) for which a solution where CC is an intermediate state exists. We do this constructing the net demand at the threshold as a function of \( B_{CC} \), the level of fundamentals at the threshold as a function of \( B_{CC} \) and then deriving the set of \( \theta_l \) for which a \( B_{CC} > 0 \) exists so that the level of fundamentals equal to the net demand at the threshold for both pegs. Second, we derive the set of \( \theta_l \) for which a solution where RR is an intermediate state exists following an
Figure 5: The two possible scenarios for a threshold solution

identical approach. We will find that for low level of world fundamentals there is only one threshold equilibrium where CC is an intermediate state, for intermediate level of world fundamentals there is multiple equilibria, while for high level of world fundamentals there is only one threshold equilibrium where RR is the intermediate state.

4.3. Intermediate state 1: Both regimes collapse (CC)

The first step in the construction of the net demand at the threshold as a function of $B_{CC}$ is the derivation of the optimal strategies as a function of $B_{CC}$. It can be shown that as long as $B_{CC} < S$ investors would find optimal to attack one of the two regimes for any possible signal $x_{an}$. Investors receiving a negative signal would attack peg 1 while investors receiving a positive signal would attack peg 2. That is if $B < S$ then the optimal strategy is given by,

$$y(x_{an}, B_{CC}, S, \alpha W/r, W) = \begin{cases} (W/c, 0) & \text{if } x_{an} < 0 \\ (\alpha W/2c, (1 - \alpha)W/2c) & \text{if } x_{an} = 0 \\ (0, W/c) & \text{otherwise} \end{cases}$$ (32)

It is often easier to derive $y(x_{an}, B_{CC}, S, \alpha W/r, W)$ taking into account the optimal response of a marginal investor, defined as the one receiving a signal $x_{an} = 0$. 
In this case, the marginal investor assigns a probability of collapse to both regimes of
\( p_M \equiv p_1(x_{an} = 0, B_{CC}, S) \equiv p_2(x_{an} = 0, B_{CC}, S) = \frac{1}{2} + \frac{B_{CC}}{2S} \in [\frac{1}{2}, 1) \).
It follows by the one-way bet assumption that the sum of probabilities would
be larger than \( \frac{2}{x_{an} + T} \), and the marginal investor would prefer to randomize short
positions across markets. For investors receiving a signal larger than zero, we
have \( p_2(x_{an}, B_{CC}, S) > p_1(x_{an}, B_{CC}, S) \) and \( p_2(x_{an}, B_{CC}, S) + p_1(x_{an}, B_{CC}, S) \)
bounded below by \( 2p_M > \frac{2}{x_{an} + T} \), so that attack regime 2 is the optimal action.
An identical argument applies to show that attack 1 is the optimal response for
\( x_{an} < 0 \).

When \( B_{CC} \) gets sufficiently large, individuals with signals close to zero would
prefer to randomize short positions across markets while the others would still
prefer to attack only one of the pegs as before. That is if \( B_{CC} \geq S \) then the
optimal strategy is given by,
\[
y(x_{an}, B_{CC}, S, v_a/r, W) = \begin{cases} 
(W/c, 0) & \text{if } x_{an} < -\frac{B_{CC}}{2} + \frac{S}{2} \\
(\alpha W/2c, (1 - \alpha)W/2c) & \text{otherwise} \\
(0, W/c) & \text{if } x_{an} > \frac{B_{CC}}{2} - \frac{S}{2}
\end{cases} \quad (33)
\]

In order to see this note that \( (p_1 = 1, p_2 = 1) \) for \( x_{an} \in [-\frac{B_{CC}}{2} + \frac{S}{2}, \frac{B_{CC}}{2} - \frac{S}{2}] \).
\( (p_1 = 1, p_2 < 1) \) for \( x_{an} < -\frac{B_{CC}}{2} + \frac{S}{2} \) and \( (p_1 < 1, p_2 = 1) \) for \( x_{an} > \frac{B_{CC}}{2} - \frac{S}{2} \).
Figure 6 provides an illustration of the optimal investment strategy for both \( B < S \)
and \( B \geq S \). In the upper part of the Figure we present the case where \( B < S \) while
in the lower part we present the case where \( B \geq S \). In both cases we present the
mapping \( x_{an} \rightarrow p_1 \) above the \( x_{an} \) line and the mapping \( x_{an} \rightarrow p_2 \) below this same
line. We then compute the sum of probabilities for each \( x_{an} \) and derive the optimal
strategy using claim 1. The reader is invited to note how increasing the size of the
intermediate state increases the values of \( p_1 \) and \( p_2 \) for values of \( x_{an} \) around zero.
When the size of the intermediate state gets large enough these probabilities hit
the value of one and it becomes optimal to attack both pegs. Further increases
in the size of the intermediate state increases the range of \( x_{an} \) where it is optimal
to attack both pegs while the optimal responses of attacking only one of the two
pegs remain always nicely located around the threshold levels (i.e in the interval
\( x_{an} > \theta^T_{an} - \frac{S}{2} \) for peg 1 and in the interval \( x_{an} < \theta^T_{an} + \frac{S}{2} \) for peg 2). It is around
these threshold levels that the probability of collapse of the threshold peg starts
decreasing, so that it is optimal to attack the other peg, which has a probability
of collapse equal to one.

It follows from the optimal strategies that the net aggregate demand a regime
faces at the threshold is decreasing in \( B_{CC} \),
\[
\mathbb{I}_1(\theta_{an} = \theta^T_{an,i}, B_{CC}/S, W) = \int_{\theta^T_{an,i} - \frac{S}{2}}^{\theta^T_{an,i} + \frac{S}{2}} y^*(x_{an}, B_{CC}, S, v_a/r, W)dx_{an} \quad (34)
\]
Figure 6: Optimal Strategies for a CC Intermediate State

\[
\begin{align*}
B_{CC} < S & \\
p_1 & \quad 0 \quad 1/2 \quad 1 \\
p_2 & \quad 1/2 \quad 1 \quad 0 \\
\theta_{an,2} & \quad 0 \quad 1/2 \quad 1 \\
\theta_{an,1} & \quad 1 \quad 0 \quad 0
\end{align*}
\]

Attack 1  Attack 2

\[
\begin{align*}
B_{CC} > S & \\
p_1 & \quad 0 \quad 1/2 \quad 1 \\
p_2 & \quad 1/2 \quad 1 \quad 0 \\
\theta_{an,2} & \quad 0 \quad 1/2 \quad 1 \\
\theta_{an,1} & \quad 1 \quad 0 \quad 0
\end{align*}
\]

Attack 1  Attack both  Attack 2

The net aggregate demand can be obtained computing directly the integral on equation 34 or just by using the property that the distribution of beliefs about the collapse of peg i is uniform at its threshold. Individuals attacking peg i would be those with a \( p_i > p_M = \frac{1}{2} + \frac{B_{CC}}{S} \). It follows that the proportion of individuals attacking the peg would be \( 1 - p_M \) with an aggregate net demand equal to \( (1 - p_M)(\frac{W}{c}) \).

We can now show that for any \( \theta_l \) sufficiently small there is an equilibrium where CC is an intermediate state. In Figure 7 we present a graph of the net aggregate demand from equation 35 (which is invariant to \( \theta_l \)) and the fundamentals level line from equation 36 for several possible \( \theta_l \) values. Since the intercept of the fundamentals level line is given by \( \theta_l \) it is easy to see in this figure how a threshold...
solution exists as long as \( \theta_1 < \frac{W}{2c} \). Such solution is given by,

\[
\frac{B_{CC}}{S} = \begin{cases} 
\frac{W - \theta_1}{\frac{W}{2c} + \frac{S}{2\sqrt{2}}} & \text{if } \theta_1 \in \left[ -\frac{S}{2\sqrt{2}}, \frac{W}{2c} \right] \\
\frac{-\theta_1}{\frac{S}{2\sqrt{2}}} & \text{if } \theta_1 < -\frac{S}{2\sqrt{2}}
\end{cases}
\] (37)

It is also straightforward to note that when the private information is very accurate, that is \( S \to 0 \), we get \( \theta_T^{\pi} \to \theta_1 \) for \( \theta_1 > 0 \) and \( \theta_T^{\pi} = 0 \) for \( \theta_1 < 0 \) since the fundamentals level line tends to become flat. When private information is highly accurate, we find that small variations in the realization of fundamentals can switch large aggregate positions against one country or another so that small differences in fundamentals can determine which one of the two regimes would collapse, providing the foundations for our rank-order results.

We now present a claim where it is shown that an equilibrium with a CC intermediate state exist for any \( \theta_1 \) below \( \frac{W}{2c} \). Figure 8 provides an illustration of how that equilibrium looks like in the space of fundamentals \((\theta_1, \theta_2)\). Let us describe how to derive the currency equilibria graphically. First, we construct two "support lines". Both of them parallel and equidistant to the 45° line with a total distance between them equals to \( S \). Then to build the equilibrium for \( \theta_1 \in \left[ -\frac{S}{2\sqrt{2}}, \frac{W}{2c} \right] \) we graph two lines. The first (second) one goes from the intersection of the above (below) "support line" and the \( \theta_1 (\theta_2) \) axis to the point \( \theta_1 = \theta_2 = \theta_1 = \)
The equilibrium for \( \theta_l < -\frac{S}{2\sqrt{2}} \) is just given by \( \theta^T_i = 0 \) for \( i = 1, 2 \). Using this construction method it is easy to see how both thresholds approach to the 45° line when \( S \to 0 \) for \( \theta_l \in [0, \frac{W}{2c}] \), while the thresholds approach to zero for any \( \theta_l < 0 \).

**Claim 4** For any \( \theta_l < \frac{W}{2c} \), there is a threshold equilibria where CC is an intermediate state with \( \theta^T_i \) given by,

\[
\begin{align*}
\left\{ \begin{array}{ll}
\frac{W}{2c + \frac{S}{\sqrt{2}}} \left( \theta_l + \frac{S}{2\sqrt{2}} \right) & \quad \text{if } \theta_l \in \left[ -\frac{S}{2\sqrt{2}}, \frac{W}{2c} \right] \\
0 & \quad \text{if } \theta_l < -\frac{S}{2\sqrt{2}}
\end{array} \right. \\
\end{align*}
\]

(38)

or equivalently,

\[
\begin{align*}
\left\{ \begin{array}{ll}
\frac{W}{2c + \frac{S}{\sqrt{2}}} \left( \theta_{-i} + \frac{S}{2\sqrt{2}} \right) & \quad \text{if } \theta_{-i} \in \left[ -\frac{S}{\sqrt{2}}, \frac{W}{2c} \right] \\
0 & \quad \text{if } \theta_{-i} < -\frac{S}{\sqrt{2}}
\end{array} \right. \\
\end{align*}
\]

(39)

for \( i = 1, 2 \).

### 4.4 Intermediate state 2: Both regimes remain (RR)

We will now derive the set of \( \theta_l \) for which an equilibrium where RR is an intermediate state exist. We will do this following the same approach we applied
in the CC intermediate state. We construct the net demand at the threshold as a function of $B_{RR}$, the level of fundamentals at the threshold as a function of $B_{RR}$ and then we derive the set of $\theta_1$ for which a $B_{RR} > 0$ exists so that the level of fundamentals equal to the net demand at the threshold for both pegs.

The first step in the construction of the net demand at the threshold is the derivation of the optimal strategies as a function of $B_{RR}$. It can be shown that as long as $B_{RR} \leq \frac{(v_a - r)}{(v_a + r)} S$ investors would still find optimal to attack only one of the two pegs for any possible signal $x_{an}$. As before, investors receiving a negative signal would prefer to attack peg 1 while investors receiving a positive signal would prefer to attack peg 2. That is if $B_{RR} \leq \frac{(v_a - r)}{(v_a + r)} S$ then the optimal strategy $y^*(x_{an}, B_{RR}, S, v_a/r, W)$ is given by,

$$y(x_{an}, B_{RR}, S, v_a/r, W) = \begin{cases} (W/c, 0) & \text{if } x_{an} < 0 \\ (\alpha W/2c, (1 - \alpha) W/2c) & \text{otherwise} \\ (0, W/c) & \text{if } x_{an} > 0 \end{cases} \quad (40)$$

The intuition is simple. A low $B_{RR}$ implies that the probabilities of collapse among investors are still sufficiently high to induce them to keep attacking one of the two pegs. Formally, a $B_{RR} \leq \frac{(v_a - r)}{(v_a + r)} S$ implies a $p_M = \frac{1}{2} - \frac{B_{RR}}{2S} \geq \frac{r}{v_a - r}$ and $p_1 + p_2 = 2p_M > \frac{2r}{v_a + r}$ so that the marginal investor attacks both pegs. For investors receiving a signal larger than zero, we have $p_2(x_{an}, B_{RR}, S) > p_1(x_{an}, B_{RR}, S)$ and $p_2(x_{an}, B_{RR}, S) + p_1(x_{an}, B_{RR}, S)$ bounded below by $2p_M > \frac{2r}{v_a + r}$ so that attack regime 2 is the optimal action. An identical argument applies to show that attack 1 is the optimal response for $x_{an} < 0$.

When $B_{RR}$ gets sufficiently large, individuals with signals close to zero would prefer to take long rather than short positions. It happens that when $B_{RR}$ hits the value $\frac{(v_a - r)}{(v_a + r)} S$ there is a discontinuity in the optimal investment strategy. At this point a marginal investor would assign a probability $p_M = \frac{r}{v_a - r}$ and would no longer find optimal to attack both regimes but instead to defend them. The switch in the optimal investment strategy is not restricted to the marginal investor. Investors with a moderately positive signal, that is $x_{an} \in (0, -\frac{B_{RR}}{2} + \frac{S}{2} = -\frac{r}{v_a + r} S)$ would prefer to defend peg 1 instead of attacking peg 2, while those with a moderately negative signal, that is $x_{an} \in (\frac{B_{RR}}{2} - \frac{S}{2} = -\frac{r}{v_a + r} S, 0)$ would prefer to defend peg 2 instead of attacking peg 1. Investors with a signal sufficiently far from zero would not have a switch on the optimal strategy and they would still prefer to attack one of the two regimes as before.

In Figure 9 we illustrate the discontinuous shift in the optimal strategy. In the upper part of the Figure we present the inferred probabilities and the optimal strategies in the case where $B_{RR} \rightarrow \left( \frac{(v_a - r)}{(v_a + r)} S \right)^-$, while in the lower part we present those same variables in the case where $B_{RR} \rightarrow \left( \frac{(v_a - r)}{(v_a + r)} S \right)^+$. The reader is invited
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Figure 9: Discontinuity in the Optimal Strategy when \( B_{RR} = \frac{v_a - r}{v_a + r} S \)

to note how the magnitude of the discontinuity changes with the structure of the payoffs \( \frac{v_a}{r} \). If \( \frac{v_a}{r} \to \infty \) the discontinuity vanishes while if \( \frac{v_a}{r} \to 1 \) the discontinuity reaches the maximum possible range of \( x_{an} \in [-\frac{S}{2}, \frac{S}{2}] \) given the one-way bet assumption \( v_a > r \).

As long as \( B_{RR} \in \left[ \frac{v_a - r}{v_a + r} S, S \right] \) the optimal strategy would be given by investors either defending or attacking one of the two pegs in the following form\(^1\),

\[
y(x_{an}, B_{RR}, S, v_a/r, W) = \begin{cases} 
(W/c, 0) & \text{if } x_{an} < U - \frac{2r}{v_a + r} S \\
(0, -W/c) & \text{if } x_{an} \in [U - \frac{2r}{v_a + r} S, S] \\
(-\alpha W/2c, -(1-\alpha)W/2c) & \text{if } x_{an} = 0 \\
(-W/c, 0) & \text{if } x_{an} \in [0, L + \frac{2r}{v_a + r} S] \\
(0, W/c) & \text{if } x_{an} > L + \frac{2r}{v_a + r} S 
\end{cases} \tag{41}
\]

where \( U \equiv -\frac{B_{RR}}{2} + \frac{S}{2} \) and \( L \equiv \frac{B_{RR}}{2} - \frac{S}{2} \).

When \( B_{RR} \) gets large enough, that is \( B_{RR} > S \), the marginal investor and those receiving a signal close to zero would assign a probability of zero to the event of having a collapse on either one of the two pegs and they would find optimal to defend both pegs. As before, investors receiving moderately positive or negative signals would prefer to defend only one of the two regimes, while those receiving large positive or negative signals would prefer to attack one of the regimes. The
optimal strategy is given by\(^2\),

\[
y(x_{an},B_{RR},S,v_{a}/r,W) = \begin{cases} 
(W/c,0) & \text{if } x_{an} < U - \frac{2r}{v_{a} + r}S \\
(0,-W/c) & \text{if } x_{an} \in [U - \frac{2r}{v_{a} + r}S, U] \\
(-W/2c,-W/2c) & \text{if } x_{an} = [U,L] \\
(-W/c,0) & \text{if } x_{an} \in [L,L + \frac{2r}{v_{a} + r}S] \\
(0,W/c) & \text{if } x_{an} > L + \frac{2r}{v_{a} + r}S
\end{cases}
\]

(42)

In Figure 10 we provide an illustration of the mapping of signals \(x_{an}\) into beliefs \((p_1, p_2)\) and the optimal strategy. In the upper part of the Figure we present the case where \(B_{RR} \in [v_{a} - r \sqrt{2}, S]\), while in the lower part we present the case where \(B_{RR} > S\). In both cases we present the mapping \(x_{an} \rightarrow p_1\) above the \(x_{an}\) line and the mapping \(x_{an} \rightarrow p_2\) below this same line. We then compute the sum of probabilities for each \(x_{an}\) and derive the optimal strategy using claim 1. The reader is invited to note how increasing the size of the intermediate state decreases the values of \(p_1\) and \(p_2\) for values of \(x_{an}\) around zero. When the size of the intermediate state gets large enough these probabilities hit the value of zero and it becomes optimal to defend both pegs. Further increases in the size of the intermediate state increases the range of \(x_{an}\) where it is optimal to defend both pegs while the optimal responses of either attacking or defending only one of the two pegs remain always nicely located close and around the threshold levels (i.e in the interval \(x_{an} \in [\theta_{T}^T + \frac{S}{2}, \theta_{T}^T - \frac{S}{2}]\)). It is around the threshold levels that the probabilities of the threshold peg start increasing, so that it is optimal to attack the threshold peg if the probability is sufficiently high or defend the other peg if instead the probability is sufficiently low.

It follows from the optimal strategies that except for the discontinuity at \(B_{RR} = \left(\frac{v_{a} - r}{v_{a} + r}\right)S\), the net aggregate demand the regime \(i\) faces at the threshold is increasing in \(B_{RR}\),

\[
\bar{y}_i(\theta_{an} = \theta_{an,i}^T, B_{RR}/S,W) = \int_{\theta_{an,i}^T - \frac{S}{2}}^{\theta_{an,i}^T + \frac{S}{2}} y(x_{an}, B_{RR}, S,v_{a}/r,W)dx_{an}
\]

(43)

\[
= \begin{cases} 
\left(\frac{1}{2} + \frac{B_{RR}}{2S}\right)\frac{W}{c} & \text{if } B_{RR} < \left(\frac{v_{a} - r}{v_{a} + r}\right)S \\
\left(\frac{v_{a} - r}{v_{a} + r}\right)\frac{W}{c} - \left(\frac{1}{2} - \frac{B_{RR}}{2S}\right)\frac{W}{c} & \text{if } B_{RR} \in \left[\left(\frac{v_{a} - r}{v_{a} + r}\right)S,S\right] \\
\left(\frac{v_{a} - r}{v_{a} + r}\right)\frac{W}{c} & \text{if } B_{RR} > S
\end{cases}
\]

(44)

while the level of fundamentals is decreasing in \(B_{RR}\),

\[
\theta_{T}^i = \theta_i + \frac{\theta_{an,i}^T}{\sqrt{2}}(1 - 2 \cdot 1_{i=2}) = \theta_i - \frac{B_{RR}}{2\sqrt{2}} = \theta_i - \frac{S}{2\sqrt{2}}(B_{RR}/S)
\]

(45)
Figure 10: Optimal Strategies for an RR Intermediate State

The aggregate demand can be obtained by computing directly the integral on equation 43 using the optimal strategies we have already derived or using instead the property that the distribution of beliefs on the collapse of peg i is uniform at the threshold. When $B_{RR} < \left( \frac{\theta - r}{\theta + r} \right) S$ we know that $p_M = \frac{1}{2} - \frac{B_{RR}}{2S} > \frac{r}{\theta + r}$ and that all investors with $p_i > p_M$ would be attacking peg i. It follows that at the threshold there is a proportion of $(1-p_M)$ attacking i and a proportion $p_M$ attacking j, so that the net aggregate demand equals $(1-p_M)W_c$. When $B_{RR} \in \left[ \left( \frac{\theta - r}{\theta + r} \right) S, S \right]$ we know $p_M = \frac{1}{2} - \frac{B_{RR}}{2S} \leq \frac{r}{\theta + r}$. Investors with a $p_i$ larger than $\frac{2r}{\theta + r}$ would be attacking i, those with $p_i \in (p_M, \frac{2r}{\theta + r})$ would be defending j and those with $p_i < p_M$ would be defending i. It follows that there is a proportion $(1 - \frac{2r}{\theta + r})$ = $\frac{\theta - r}{\theta + r}$ attacking i and a proportion $p_M$ defending it. Then the aggregate demand on peg i is $\left( \frac{\theta - r}{\theta + r} \right) W_c - p_M W_c$. When $B_{RR} > S$, individuals with $p_i$ larger than $\frac{2r}{\theta + r}$ would be attacking i while those with $p_i \in (0, \frac{2r}{\theta + r})$ would be defending regime j. The aggregate demand on peg i is just $\left( \frac{\theta - r}{\theta + r} \right) W_c$.

The aggregate demand is increasing in $B_{RR}$ in the range $B_{RR} \in [0, \left( \frac{\theta - r}{\theta + r} \right) S]$ because the proportion of the population attacking peg i is increasing in $B_{RR}$. Net
aggregate demand reaches the maximum of \( \left( \frac{v_a}{v_a + \tau} \right) \frac{W}{c} \) when \( B_{RR} \rightarrow \left( \frac{v_a - r}{v_a + \tau} \right) S \) as the proportion of the population attacking peg i tends to \( \left( \frac{v_a}{v_a + \tau} \right) \). At the point of the discontinuity \( B_{RR} = \left( \frac{v_a - r}{v_a + \tau} \right) S \), the net demand drops by \( -\frac{2v_a - r}{v_a + \tau} \left( \frac{W}{c} \right) \) since a \( \left( \frac{r}{v_a + \tau} \right) \) percentage of the population switches from attacking i towards defending j and an additional \( \left( \frac{r}{v_a + \tau} \right) \) percentage of the population switches from attacking j towards defending i. For \( B_{RR} > \left( \frac{v_a - r}{v_a + \tau} \right) S \) further increases in \( B_{RR} \) increases net aggregate demand since the percentage of the population defending i decreases while the percentage defending j increases. Once \( B_{RR} > S \) net aggregate demand is constant. At this point, investors are sure the regime j is going to remain while the probability of collapse of i is uniform in the population, so that a fraction \( \frac{2v_a - r}{v_a + \tau} \) of the population would prefer to defend peg 2 while the rest (a fraction \( \frac{v_a - r}{v_a + \tau} \)) would prefer to attack i.

We can show now that for any \( \theta_l \) sufficiently large there will be an equilibrium where RR is an intermediate state. In Figure 11 we graph the net aggregate demand from equation 44 (which is invariant to \( \theta_l \)) and the fundamental line from equation 45 for several \( \theta_l \) values. Using the fact that the intercept of the fundamentals level line is given by \( \theta_l \), it is straightforward to note that an equilibrium exists for any \( \theta_l > \frac{W}{2c} \). Furthermore, the discontinuity in the aggregate demand ensures that multiple equilibria exists in the range \( \theta_l \in \left[ \frac{v_a - 2r}{v_a + \tau + r} \left( \frac{W}{c} \right), \frac{v_a - r}{v_a + \tau + r} \left( \frac{W}{c} \right) \right] \). If \( \frac{v_a - 2r}{v_a + \tau + r} \left( \frac{W}{c} \right) + \frac{v_a - r}{v_a + \tau + r} \left( \frac{S}{2\sqrt{\tau}} \right) > \frac{W}{2c} \) there are two different equilibria in this range where both have RR as an intermediate state. If instead the previous inequality does not hold then there is an RR and a CC equilibrium for \( \theta_l \in \left[ \frac{v_a - 2r}{v_a + \tau + r} \left( \frac{W}{c} \right), \frac{v_a - r}{v_a + \tau + r} \left( \frac{S}{2\sqrt{\tau}} \right) \right] \) and two different RR equilibria in the range \( \theta_l \in \left[ W - \frac{v_a}{v_a + \tau + r} \left( \frac{W}{c} \right), \frac{v_a - r}{v_a + \tau + r} \left( \frac{S}{2\sqrt{\tau}} \right) \right] \).

We now present a claim where it is shown that an equilibrium with a RR intermediate state exists for all \( \theta_l \) sufficiently large, which includes at least all \( \theta_l \) larger than \( \frac{W}{2c} \). Figure 12 shows how these equilibria looks like in the space of fundamentals \((\theta_1, \theta_2)\). Let us describe how to derive the currency equilibria graphically. First, we construct two "large support lines". Both of them parallel and equidistant to the 45° line with a total distance among them equal to S. Next, we construct two "small support lines". Both of them are also parallel and equidistant to the 45° line but the distance between them is only \( \left( \frac{v_a - r}{v_a + \tau + r} \right) S \). Then, to build the equilibrium based on the first equation of the aggregate demand on equation 44, we graph a line from the point \( \left( \frac{W}{2c}, \frac{W}{c} \right) \) to the point of intersection of \( \theta_1 = \frac{v_a}{v_a + \tau + r} \left( \frac{W}{c} \right) \) and the upper small support line. In order to graph the equi-
Currency Crises: Whom to attack or defend?

Figure 11: Deriving the Equilibrium for a RR Intermediate State

librium, based on the second equation of the aggregate demand, we graph a line from the point of intersection of the upper small support line and \( \theta_1 = \frac{v_a - 2r}{v_a + r} \left( \frac{W}{c} \right) \) to the point of intersection of the upper large support line and \( \theta_1 = \frac{v_a - r}{v_a + r} \left( \frac{W}{c} \right) \). The equilibrium based on the third equation can be built graphing a vertical line above this last point of intersection. Finally, we graph the mirror image along the 45\(^0\) line of the three lines described above.

The claim shown below includes some technical conditions that would be needed to have a RR equilibrium for negative levels of \( \theta_1 \). The fact that the optimal strategy for peg 1 (peg 2) of equation 41 is not weakly decreasing (increasing) in the whole range \( x_{an} \) implies that additional requirements are needed to ensure the existence of an equilibria when a threshold solution exists for negative values of \( \theta_1 \). Such technical restrictions are not needed for positive values of \( \theta_1 \) since in the region where \( x_{an} \) is not weakly decreasing (increasing) the net aggregate demand is bounded below by zero, so that a positive \( \theta_1 \) ensures that the peg 1 (peg 2) will remain whenever \( \theta_{an} > \theta_{an,1} \) (\( \theta_{an} < \theta_{an,2} \)).

Claim 5 There is a set of \( \theta_1 \), which includes at least all \( \theta_1 > \frac{W}{2c} \), for which a threshold equilibria exists where RR is an intermediate state. The set and the equilibria can be described by the following equations,
Figure 12: Currency Equilibria with a RR Intermediate State

The thresholds $\theta^I_i$ equal,

$$\frac{W}{2c} + \frac{S}{\sqrt{2}} (\theta_i + \frac{S}{\sqrt{2}}) \text{ if } \theta_i \in \left[ \frac{W}{2c}, \frac{v_a}{v_a + r} \left( \frac{W}{c} \right) + \left( \frac{v_a - r}{v_a + r} \right) \frac{S}{2\sqrt{2}} \right]$$ (46)

$$\text{Min} \left[ \frac{W}{2c} + \frac{S}{\sqrt{2}} (\theta_i + \left( \frac{v_a - 3r}{v_a + r} \right) \frac{S}{\sqrt{2}}), \left( \frac{v_a - r}{v_a + r} \right) \frac{W}{c} \right] \text{ if } \theta_i \in \Gamma_i$$ (47)

or equivalently,

$$\frac{W}{2c} + \frac{S}{\sqrt{2}} (\theta_i - \frac{S}{\sqrt{2}}) \text{ if } \theta_i \in \left[ \frac{W}{2c}, \frac{v_a}{v_a + r} \left( \frac{W}{c} \right) + \left( \frac{v_a - r}{v_a + r} \right) \frac{S}{\sqrt{2}} \right]$$ (48)

$$\text{Min} \left[ \frac{W}{2c} + \frac{S}{\sqrt{2}} (\theta_i - \left( \frac{v_a - 3r}{v_a + r} \right) \frac{S}{\sqrt{2}}), \left( \frac{v_a - r}{v_a + r} \right) \frac{W}{c} \right] \text{ if } \theta_i \in \Gamma_i$$ (49)

for $i=1,2$.

Where $\Gamma_i$ equals,

$$\theta_{l,RR}^I = \begin{cases} 
\left[ \frac{(v_a - 2r)}{v_a + r} \frac{W}{c} + \left( \frac{v_a - r}{v_a + r} \right) \frac{S}{2\sqrt{2}}, \infty \right] & \text{if } S < -\left( \frac{v_a - 2r}{v_a + r} \right) \frac{W}{c} \\
\frac{S}{\sqrt{2}} & \text{otherwise}
\end{cases}$$ (50)
Figure 13: Characterization of the Equilibria when $S$ is Finite

and $\Gamma$ is given by,

$$
\begin{align*}
\begin{cases}
\left( \frac{\nu_a - 2r}{\nu_a + r} \right) \frac{W}{\nu_a + r} + \left( \frac{\nu_a - r}{\nu_a + r} \right) \frac{S}{\sqrt{2}}, & \text{if } S < - \left( \frac{\nu_a - 2r}{\nu_a + r} \right) \frac{W}{\nu_a + r} \\
\phi_{RR} \left( \frac{W}{\nu_a + r} + \frac{s}{\sqrt{2}} \right) + \frac{S}{\sqrt{2}} \left( \frac{\nu_a - 3r}{\nu_a + r} \right) W, & \text{other}\text{wise}
\end{cases}
\end{align*}
$$

(51)

4.5. Characterization of the equilibria.

The set of equilibria where either CC, RR or $\emptyset$ is an intermediate state can be summarized by equations 38, 46, 47 and 50 or equivalently by equations 39, 48, 49 and 51.

The set of states of the world that are possible on equilibria depends on how accurate the private information is. If the support of the noise $S$ is finite (i.e. the variance of the private signal is finite), we could have up to 6 states of the world: (i) Both collapse, (ii) Both may collapse, (iii) The weakest collapses while the strongest may remain, (iv) Only the weakest collapses, (v) The weakest may collapse while the strongest remains and (vi) Both pegs remain. In Figure 13 we provide an illustration on the location of each one of these states by putting together Figures 8 and 12.

If instead $S \to 0$ (i.e. the variance of the private signal tends to zero), then...
there are only 4 states that could be sustainable in equilibria. In order to see
this note that in the limit, the equilibrium can be easily characterized with the
following three equations,
\[
\begin{cases}
\theta_T^i = 0 & \text{if } \theta_l < 0 \\
\theta_T^i \to \theta_l & \text{if } \theta_l \in (0, \left(\frac{\nu_a}{\nu_a + \tau}\right) \frac{W}{c}] \\
\theta_T^i \to \left(\frac{\nu_a - \tau}{\nu_a + \tau}\right) \frac{W}{c} & \text{if } \theta_l > \left[\left(\frac{\nu_a - \tau}{\nu_a + \tau}\right) \frac{W}{c}, \infty\right]
\end{cases}
\]
(52)
for \(i=1,2\).

It follows that only the 4 states (i), (iv), (iv) and (vi) are possible in equilibria.
State (ii) vanishes directly as a result of \(S \to 0\) while state (iii) is ruled out by
the fact that a threshold solution with intermediate state RR is not an equilibria
when \(\theta_l < 0\) and \(S \to 0\) as expressed in equations 47 and 51. In Figure 14 we
provide an illustration on the location of the four possible states of the world
when the private information is accurate. Here it is easy to notice how lack of
common knowledge and accurate private information generate a strong rank-order
result in the sense that an infinitesimal difference in the level of fundamentals
can be sufficient to support an equilibrium where either "the weak peg collapses
while the strong one remains" or "the weak peg may collapse while the strong one
remains for sure". This rank-order result is present in a large region of the world
fundamentals. If \(\theta_l \in (0, \left(\frac{\nu_a - \tau}{\nu_a + \tau}\right) \frac{W}{c})\) then the weakest regime collapses while the
strong one remains, while if instead \(\theta_l \in \left(\left(\frac{\nu_a - \tau}{\nu_a + \tau}\right) \frac{W}{c}, \left(\frac{\nu_a}{\nu_a + \tau}\right) \frac{W}{c}\right)\) then the weakest
regime may collapse while the strong one remains for sure.

The exact characterization of the equilibria, provided in equation 52, depends
on the structure of the payoffs and the level of wealth. It is easy to note how
the region of fundamentals where only the weakest regime collapses is strictly
increasing in wealth, no matter how high the fundamentals of the weakest country
are. This property is a direct result of our initial assumption that it was always
profitable to attack a peg \((\theta^p \to \infty)\). In the following section, where we solve
the model with finite threshold payoffs, we will show that the size of this state is
strictly increasing in wealth only up to the point where is no longer profitable to
attack a collapsing peg.

5. Strategic uncertainty with threshold payoffs

When there is a finite \(\theta^p\) so that it is optimal to attack a collapsing peg only
when \(\theta < \theta^p\), we have that the probability function of a profitable collapse is just
a slight variation of the ones we obtained before,
\[
p_1^*(x_{an}, \theta_{an,1}, S) = P[\theta_1 < \min\{\theta_1^T, \theta^p\} | x_{an}]
\]
Figure 14: Characterization of the Equilibria when S→ 0

\[ = P[\theta_{an,1} < \min\{\theta_{an,1}^T, \theta_{an,1}^p\}|x_{an}] \]
\[ = P[\theta_{an,1} < \theta_{an,1}^*|x_{an}] \]
\[ = T \left( \frac{(\theta_{an,1}^* + \frac{S}{2}) - x_{an}}{S} \right) \] (56)

where \( D \equiv 2\sqrt{2}(\theta_l - \theta^p) \) is a signed distance between the two points \( \theta_{an,1}^p \) and \( \theta_{an,2}^p \), and \( \theta_{an,1}^* = \min\{\theta_{an,1}^T, \theta_{an,1}^p\} = \min\{\theta_{an,1}^T, -\frac{D}{2}\} \). Using a symmetrical argument it can be shown that the probability of a profitable collapse of peg 2 is given by,

\[ p_2^*(x_{an}, \theta_{an,2}^*, S) = T \left( \frac{x_{an} - (\theta_{an,2}^* - \frac{S}{2})}{S} \right) \] (57)

where \( \theta_{an,2}^* = \max\{\theta_{an,2}^T, \theta_{an,2}^p\} = \max\{\theta_{an,2}^T, \frac{D}{2}\} \).

The property that the probability of collapse is uniformly distributed in the population, at the threshold, is also inherited with a slight modification. Now the probability of a profitable collapse in peg 1 and peg 2 are uniformly distributed around the points \( \theta_{an,1}^* = \min\{\theta_{an,1}^T, -\frac{D}{2}\} = \theta_{an,1}^p \) and \( \theta_{an,2}^* = \max\{\theta_{an,2}^T, \frac{D}{2}\} = \theta_{an,2}^p \).
Currency Crises: Whom to attack or defend?

Example 1: $D > 0$

\[ \begin{align*}
\theta^p_{an} &= 1/2 - D/2S \\
\theta^T_{an} &= 1/2 - 2\theta^p_{an} \\
\theta^p_{an} &= 0 \\
\theta^T_{an} &= 1/2 - 2\theta^p_{an} \\
\theta^T_{an} &= 0
\end{align*} \]

Example 2: $D < 0$

\[ \begin{align*}
\theta^p_{an} &= 1/2 - D/2S \\
\theta^T_{an} &= 1/2 - 2\theta^p_{an} \\
\theta^p_{an} &= 0 \\
\theta^T_{an} &= 1/2 - 2\theta^p_{an} \\
\theta^T_{an} &= 0
\end{align*} \]

Figure 15: Distribution of Beliefs when $\theta^p$ is finite

$\theta^p_{an,2}$, respectively, and not necessarily around the points $\theta^*_{an,i} = \theta^T_{an,i}$ as it was before. A result that follows directly from Claim 2. In order to get some intuition on these results, figure 15 provides an illustration of the mapping of signals to beliefs for two possible scenarios, one where $\theta_l < \theta_p$ (a negative $D$) and another one where $\theta_l > \theta_p$ (a positive $D$). As usual, we present the mapping of $x_{an} \rightarrow p_1$ above the $x_{an}$ line and the mapping $x_{an} \rightarrow p_2$ below this same line. The reader can notice how the probability of collapse of peg $i$ goes from zero to one around the point $x_{an} = \theta^*_{an,i}$ in the interval $x_{an} \in [\theta^*_{an,i} + S/2, \theta^*_{an,i} - S/2]$, providing an easy rule for the derivation of the optimal strategy and the threshold solution. The figure also shows how a $\theta_{an}$ equal to $\theta^*$ generates a uniform distribution of beliefs in the population, since the signals would also be uniformly distributed in the population in the interval $x_{an} \in [\theta^* + S/2, \theta^* - S/2]$ by the law of large numbers.

The main insight of the standard model, that only symmetric threshold equilibriums can exist ($\theta^T_1 = \theta^T_2$), is also present here. The intuition is the following. Imagine a scenario where both thresholds are equal. By symmetry, the net demand and the fundamental levels would be equal across thresholds so that in principle an equilibrium could exist. Now imagine we arbitrarily increase the level of fundamentals for peg 1 so that $\theta^T_1 > \theta^T_2$. Now, we would have a larger level of fundamentals for peg 1 at its threshold and a lower net demand for peg 1 at its threshold (since attacking peg 1 at higher levels of fundamentals is less likely to
generate capital gains). These two facts put together imply that any candidate threshold level $\theta_T^1$ larger than $\theta_T^2$ would have for peg 1 a level of fundamentals larger than the net aggregate demand and a threshold solution can not exist. We formalize this result in the following claim.

**Claim 6** In the general model with threshold payoffs if a threshold equilibrium exist, this must be symmetrically centered around the point $\theta_1 = \theta_2 = \theta_1$, so that $\theta_{an,1} = -\theta_{an,2}$ and $\theta_1^T = \theta_2^T$.

We will now solve the characterization of the equilibria in three steps. First, we show that when $D < \frac{\gamma a - \tau}{\gamma a - r} S$ (i.e. $\theta_t < \theta_p + \left(\frac{\gamma a - \tau}{\gamma a - r} S\right) \frac{S}{2\sqrt{2}}$), the profitability condition is "not binding in general" in the sense that the net aggregate demand functions we will find here will be the same that the ones we derived in the standard model. In this region, all the results of the standard model are inherited, including the threshold solutions and all the sets of possible equilibria. Second, we show how to derive the currency equilibria when $D > 2S$ (i.e. $\theta_t > \theta_p + \frac{S}{\sqrt{2}}$). In this region, the profitable level $\theta^p$ is so small relative to the average level $\theta_t$, that it guarantees having all investors defending both pegs when $\theta_{an} = 0$, even when the intermediate state CC is considered. Finally, we characterize the equilibria in the intermediate nuisance region given by $D \in \left(\frac{\gamma a - \tau}{\gamma a - r} S, 2S\right)$. Here we show that only one of the two intermediate states CC and RR is possible for each $\theta_t$; and we derive bounds for the threshold solutions in equilibrium. This last region is denominated "nuisance" in the sense that all $\theta_t$ will be located in the first or the second case when private information is accurate (i.e. $S \to 0$). Figure 16 provides an illustration of the location of the fundamentals in the three possible cases. The set of fundamentals $(\theta_1, \theta_2)$ located to the left of the line called $D = \frac{\gamma a - \tau}{\gamma a - r} S$ correspond to the first case, the set located above the line $D = 2S$ correspond to the second case, while the set between these two lines correspond to the nuisance region.

5.1. Case 1: $D < \frac{\gamma a - \tau}{\gamma a - r} S$ (i.e. $\theta_t < \theta_p + \left(\frac{\gamma a - \tau}{\gamma a - r} S\right) \frac{S}{2\sqrt{2}}$)

In this region the average level of fundamentals $\theta_t$ is low enough relative to $\theta^p$ that a collapse around $\theta_{an} = 0$ would be profitable in general. Here, as was before in the standard model, investor behavior will be sensitive to whether the intermediate state is RR or CC and the multiplicity of equilibria will remain in place. Furthermore, the net aggregate demand functions we derived in the standard model also apply here and the exact characterization of the currency equilibria is given by claims 4 and 5. We formalize this result in the following claim.

**Claim 7** If $\theta_p$ is finite and $D < \frac{\gamma a - \tau}{\gamma a - r} S$, or equivalently $\theta_t < \theta_p + \left(\frac{\gamma a - \tau}{\gamma a - r} S\right) \frac{S}{2\sqrt{2}}$, then the set of possible states of the world where either CC or RR is an intermediate state is given by claims 4 and 5 of the standard model.
5.2. Case 2: \( D \geq 2S \) (i.e. \( \theta_1 > \theta_p + \frac{S}{\sqrt{2}} \))

When \( D \geq 2S \) the profitable level is so small relative to the average level \( \theta_l \) that it guarantees having all investors defending both pegs at \( \theta_{an} = 0 \), even when the intermediate state CC is considered.

In Figure 17, we describe the mapping from the signal \( x_{an} \) into inferred probabilities of a profitable collapse and the optimal strategy. As long as \( B_{RR} < D \) or \( B_{CC} > 0 \) we will have beliefs uniformly distributed around \( \theta_{an}^* = \theta_{an,i}^p \) with an optimal strategy given by,

\[
y(x_{an}, B_{RR} < D, D, S, v_a/r, W) = y(x_{an}, B_{CC}, D, S, v_a/r, W) =
\begin{cases}
(W/c, 0) & \text{if } x_{an} < U - \frac{2r}{v_a/r+S}S \\
(0, -W/c) & \text{if } x_{an} \in [U - \frac{2r}{v_a/r+S}S, U] \\
(-\alpha W/2c, -(1-\alpha)W/2c) & \text{if } x_{an} = [U, L] \\
(-W/c, 0) & \text{if } x_{an} \in [L, L + \frac{2r}{v_a/r+S}S] \\
(0, W/c) & \text{if } x_{an} > L + \frac{2r}{v_a/r+S}S 
\end{cases}
\]  

(58)

where \( U = -\frac{D}{2} + \frac{S}{2} \) and \( L = \frac{D}{2} - \frac{S}{2} \). If instead \( B_{RR} > D \), beliefs are instead uniformly distributed around \( \theta_{an}^* = \theta_{an,i}^p \) and the optimal strategy is still given by equation 58 but with \( U = -\frac{B_{RR}}{2} + \frac{S}{2} \) and \( L = \frac{B_{RR}}{2} - \frac{S}{2} \).
Once the optimal strategies have been derived it is simple to show that an equilibrium with an intermediate state CC does not exist while an equilibrium where RR is an intermediate state must exist. In order to see this note that the net aggregate demand at $\theta_{an} = 0$ is given by $-\frac{W}{2c}$ for any $B_{CC}$ level so that provided $\theta_l > \theta^p > 0$ it is not possible to have a CC equilibrium. On the other hand, an RR equilibrium must exist, since the level of fundamentals is strictly decreasing in $B_{RR}$ while the net aggregate demand is continuous and bounded everywhere.

Let us now derive the closed form solution of the RR equilibria. Using the optimal strategies described above we can easily compute the net aggregate demand for each peg as a function of the size of the intermediate state RR,

$$\gamma_i(\theta_{an} = \theta^T_{an,i}, B_{RR}, D, S, W) = \frac{\theta^T_{an,i} + \frac{W}{2c}}{\theta^T_{an,i} - \frac{W}{2c}} \int y^*(x_{an}, B_{RR}, S, v_a/r, W)dx_{an}$$
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\[
\begin{cases}
-W & \text{if } B_{RR} \in [0, D - 2S] \\
-(D_{RR} - B_{RR}) \frac{W}{2c} & \text{if } B_{RR} \in [D - 2S, D - 2 \left( \frac{v_a - r}{v_a + r} \right) S] \\
\left( \frac{v_a - r}{v_a + r} \right) \frac{W}{c} - \left( \frac{D_{RR} - B_{RR}}{2S} \right) \frac{W}{2c} & \text{otherwise}
\end{cases}
\]

When \( B_{RR} \in [0, D - 2S] \) we have a constant net demand because we are integrating over the area where all investors are defending both pegs. When \( B_{RR} \in [D - 2S, D - 2 \left( \frac{v_a - r}{v_a + r} \right) S] \) the net aggregate demand on peg \( i \) starts decreasing since some investors prefer to defend peg \( j \) rather than to defend both pegs. When \( B_{RR} \in [D - 2 \left( \frac{v_a - r}{v_a + r} \right) S, D) \) the demand on peg \( i \) starts increasing at a higher speed since some investors would prefer to attack peg \( i \) rather than to defend both pegs. When \( B_{RR} \) hits \( D \) or gets larger the net demand becomes constant. At this point, investors are sure that peg -\( i \) will remain while they assign a uniform distribution to the probability that the peg \( i \) collapses. Then a fraction \( \frac{v_a - r}{v_a + r} \) of the population prefers to attack \( i \) while the rest prefers to defend the other peg.

In Figure 18 we illustrate how an equilibrium can be built for an arbitrary \( D > 2S \) (i.e. an arbitrary \( \theta_l \) larger than \( \theta^p + \frac{S}{\sqrt{2}} \)). In this Figure we present together the aggregate demand and the fundamentals level line. The reader is invited to note that for each possible \( D \), that is for each possible \( \theta_l \), there is a different aggregate demand and a different fundamental line. A larger \( D \), implies a larger \( \theta_l \) (given \( \theta_l = \theta^p + \frac{D}{2 \sqrt{2}} \)), a fundamental line shifted above (given \( \theta_l = \theta_l - \frac{B_{RR}}{2 \sqrt{2}} \)) and an aggregate demand shifted to the right. Furthermore, by construction, we must have a fundamental line equal to \( \theta^p \) whenever \( B_{RR} = D \), so that the optimal line for an arbitrary \( D \) level can be constructed using this point and the slope of the optimal line which always equals \( -\frac{1}{\sqrt{2}} \). Casual inspection of these graphs reveals some useful properties of the RR equilibria:

1. The fundamental level and the net aggregate demand must be positive at the threshold solution so that only the third and fourth parts of the aggregate demand in equation 59 are useful in the construction of the equilibria. This follows from the fact that the fundamental line is positive when \( B_{RR} \leq D \) while the net aggregate demand is positive whenever \( B_{RR} \geq D \).

2. Whenever \( \theta^p \geq \left( \frac{v_a - r}{v_a + r} \right) \frac{W}{c} \) we must have a threshold solution equal to

\[
\theta^T = \left( \frac{v_a - r}{v_a + r} \right) \frac{W}{c}
\]

for \( i=1,2 \). A result that follows from the fact that the fundamental line is
Figure 18: Deriving the Equilibrium when $D>2S$

bounded below by $\theta^p$ when $B_{RR} \leq D$ while the net aggregate demand is bounded above by $\left(\frac{v_a - r}{v_a + r}\right) \frac{W}{c}$ everywhere.

3. Whenever $\theta^p < \left(\frac{v_a - r}{v_a + r}\right) \frac{W}{c}$ we must have a threshold solution given by the intersection of the third part of the aggregate demand on equation 59 and the fundamentals line. A result that follows from the fact that when $B_{RR} > D$ we have a fundamental level smaller than $\theta^p$ and a net aggregate demand equal to $\left(\frac{v_a - r}{v_a + r}\right) \frac{W}{c}$. The threshold solution is given by,

$$\theta^T_i = \theta^p - \frac{\sqrt{2}}{3} \left[ \frac{\theta^p - \left(\frac{v_a - r}{v_a + r}\right) \frac{W}{c}}{\frac{W}{c} + \frac{\sqrt{2}S}{3}} \right] S > \theta^p $$  \hspace{1cm} (61)

for $i=1,2$. Where we have $\theta^T_i \to \theta^p$ as $S \to 0$. A result that follows from the fact that the aggregate demand becomes steeper around $D$.

We formalize these results in the following claim.
Claim 8 Whenever $\theta_l > \theta^p + \frac{S}{\sqrt{2}}$, or equivalently $D \geq 2S$, a CC equilibrium does not exist, while an equilibrium where RR is an intermediate state exists with threshold solutions $\theta^*_T$ given by,

$$\begin{align*}
\left\{ \begin{array}{ll}
\left( \frac{v_a-r}{v_a+r} \right) \frac{W}{c} & \text{if } \theta^p \geq \left( \frac{v_a-r}{v_a+r} \right) \frac{W}{c} \\
\theta^p = \frac{\sqrt{2}}{S} \left[ \theta^p - \left( \frac{v_a-r}{v_a+r} \right) \frac{W}{c} \right] S > \theta^p & \text{if } \theta^p < \left( \frac{v_a-r}{v_a+r} \right) \frac{W}{c}
\end{array} \right. \right. 
\end{align*}$$

(62)

for $i=1,2$.

5.3. Case 3: The nuisance region $D \in [\frac{v_a-r}{v_a+r}S, 2S)$.

In the nuisance region we have a $\theta_l$ sufficiently low to accept the possibility of having some of the individuals either attacking or defending one peg when $\theta_{an} = 0$ and sufficiently high to prevent the existence of an optimal strategy where all individuals are either attacking one peg or the other as we had before when $D < \frac{v_a-r}{v_a+r}S$.

There are two types of optimal strategies in this region. The first one is given by an intermediate state CC of arbitrary size $B_{CC}$ or by an intermediate state RR of size $B_{RR} < D$. If any of these two conditions hold we have $\theta^*_{an,1} = -\frac{D}{2}$ with $\theta^*_{an,2} = \frac{D}{2}$ and the distribution of beliefs together with the optimal strategy are independent of the size of the intermediate state. Following the same steps of the previous sections, we could easily show that the optimal strategy is given by equation 41 if $D < S$ and equation 42 if $D \geq S$ where $U = -\frac{D}{2} + \frac{S}{2}$ and $L = \frac{D}{2} - \frac{S}{2}$.

The second possible optimal strategy is given when $B_{RR} \geq D$. In such case we know $\theta^*_{an,1} = -\frac{B_{an}}{2}$ with $\theta^*_{an,2} = \frac{B_{an}}{2}$ and the distribution of beliefs and optimal strategies are again given by the standard model. That is the optimal strategy is given by equation 41 if $B_{RR} < S$ and by equation 42 if instead $B_{RR} \geq S$. The reader is invited to note that the optimal strategy, where investors attack peg 1 (peg 2) if $x_{an} < 0$ ($x_{an} > 0$) is ruled out. The fact that $D \geq \frac{v_a-r}{v_a+r}S$ imposes a maximum to the probability of the marginal investor given by $p_M \geq \frac{1}{2} - \frac{D}{S} = \frac{v_a-r}{v_a+r}$ that applies for any value of $B_{CC}$ or $B_{RR}$. The result is that the discontinuity in the optimal strategy vanishes and now only one equilibrium will be possible.

Even though the derivation of the optimal strategies is straightforward, the derivation of the exact functional form of the net aggregate demand in the nuisance region becomes tedious since having a finite support in the noise implies that we have different step functions for different ranges of values on $\frac{D}{S}$ and $\frac{v_a-r}{v_a+r}$. In the appendix we provide the exact functional form of the aggregate demand for any value $\frac{D}{S}$, $B_{an}$, $\frac{B_{an}}{S}$, $\frac{v_a-r}{v_a+r}$ and $S$. Here, instead, we provide 4 general properties of the net demand that will allow us to understand and characterize the equilibria.

1. When RR is the intermediate state, the net aggregate demand is continuous and weakly decreasing in $D$ everywhere.
2. The net aggregate demand is continuous and weakly increasing in $B_{RR}$ everywhere.  

3. When CC is the intermediate state, the net aggregate demand is continuous in $D$ and is strictly decreasing in $D$ in the "relevant range" where there is a non-empty set of investors attacking the peg.  

4. The net aggregate demand is always continuous in $B_{CC}$; and it is weakly decreasing in $B_{CC}$ in the range where there is a non-empty set of investors attacking the peg. When $B_{CC}$ becomes large enough this set becomes empty and the aggregate demand must be negative. Further increases in $B_{CC}$ may increase the aggregate demand but total positions are bounded above by zero.  

Properties 1-4 are sufficient to show that only one of the two intermediate states CC or RR is an equilibrium. In the case where the net aggregate demand at $B_{RR} = B_{CC} = 0$ is smaller that the average level of fundamentals only the RR intermediate state is an equilibrium. In order to see this, note that the net demand is weakly decreasing in $B_{CC}$, in the relevant range, while the fundamentals are strictly increasing so that a CC equilibrium cannot exist. On the other hand, the fact that the fundamentals are strictly decreasing in $B_{RR}$ while the net demand is continuous, bounded and weakly increasing in $B_{RR}$ ensures that an RR equilibria exist. On the opposite case where the net aggregate demand at $B_{RR} = B_{CC} = 0$ is larger that the average level of fundamentals, we know that a CC equilibrium must exist since the net demand is continuous and weakly decreasing in $B_{CC}$, in the relevant range, up to a negative number while the fundamentals are strictly increasing in $B_{CC}$. On the other hand, an RR equilibria cannot exist since the fundamentals are strictly decreasing while the demand is weakly increasing. In the case that the fundamentals equal the average level of fundamentals at the point $B_{RR} = B_{CC} = 0$ the only possible equilibria is where the empty set is the intermediate state. We can think of this possibility case as a special case of the two previous ones. Figure 19 provides an illustration on the existence of equilibrium when $\theta_l = \frac{v_a - r}{v_a + r}$ and $v_a > 3r$. We graph the exact functional form of the net aggregate demand (which correspond to case 2 of section B.1 in the appendix) and the fundamentals level line for a $\theta_l < \frac{(v_a - r) W}{(v_a + r) c}$ and a $\theta_l > \frac{(v_a - r) W}{(v_a + r) c}$. The reader can notice how for both levels of world fundamentals there is only one threshold solution. In the first case there is a threshold solution where CC is an intermediate state while in the second case the threshold solutions has a RR intermediate state.  

We now know that the net aggregate demand, at the point $B_{RR} = B_{CC} = 0$, will play a key role in the characterization of the equilibria in the nuisance region $D \in \left(\frac{\sqrt{v_a - r}}{v_a + r} S, 2S\right)$. If the demand at $B_{RR} = B_{CC} = 0$ is larger than $\theta_l$ when
Figure 19: Deriving the Equilibrium in the Nuisance Region

\[ D = \frac{v_a - r}{v_a + r} S \] (i.e. larger than \( \theta_l = \theta^p + \frac{v_a - r}{v_a + r} \frac{S}{2\sqrt{2}} \)), then there will be a critical value \( \theta^*_l \) so that when \( \theta_l \in [\theta^p + \frac{v_a - r}{v_a + r} \frac{S}{2\sqrt{2}}, \theta^*_l] \) there is only a CC equilibria while if \( \theta_l \in (\theta^*_l, \theta^p + \frac{S}{\sqrt{2}}) \) there is only a RR equilibria.

The exact computation of the net aggregate demand at the point \( B_{RR} = B_{CC} = 0 \) is trivial since the optimal strategy is always symmetrical, around \( x_{an} = 0 \). That is, if there is a proportion of investors defending only one peg then by symmetry there will also be an equal proportion of investors defending only the other peg. And the same argument applies for those individuals attacking only one peg. The implication of this symmetry is that we do not have to discriminate between those defending one peg and those defending both pegs. That is, for purposes of the computation of the demand at the point \( B_{RR} = B_{CC} = 0 \), we can safely assume that all the individuals that are not attacking one peg are defending both pegs. Using this result and the fact that in any optimal strategy with \( B_{RR} = B_{CC} = 0 \) individuals attack the peg 1 only if \( x_{an} > \frac{D}{2} - \frac{S}{2} + \frac{2r}{v_a + r} S \) we can easily compute the aggregate demand when \( B_{RR} = B_{CC} = 0 \) for any possible \( D/S \),

\[ g_1(\theta_{an} = \theta^T_{an,i}, B_{RR} = B_{CC} = 0, D, S, W) = \]
Figure 20: Deriving $\theta^*_i$ in the Nuisance Region

$$
\begin{cases}
\frac{W}{2c} & \text{if } D/S \in [0, \frac{v_a-r}{v_a+r}] \\
\frac{3v_a-5r}{2(v_a+r)} - W/c & \text{if } D/S \in \left[\frac{v_a-r}{v_a+r}, \frac{2(v_a-r)}{v_a+r}\right] \\
-\frac{W}{2c} & \text{if } D/S \in \left(\frac{2(v_a-r)}{v_a+r}, \infty\right)
\end{cases}
$$

(63)

In Figure 20 we illustrate how to derive for the critical value $\theta^*_i$ using this net aggregate demand and the so called "fundamentals line”,

$$\theta_i = \theta_l = \theta^p + \left(\frac{D}{S}\right) \frac{S}{2\sqrt{2}}$$

(64)

Casual inspection of this graph reveals that the existence of a range of $\theta_i$, where CC is an intermediate state in equilibria, depends on the structure of payoffs $\frac{\theta^p}{2}$, the size of $\theta^p$ and the size of the support $S$. It is easy to see that necessary and sufficient conditions for the existence of such region are given by,

$$
\begin{cases}
\theta^p + \left(\frac{v_a-r}{v_a+r}\right) \frac{S}{2\sqrt{2}} < \left(\frac{v_a-3r}{2(v_a+r)}\right) \frac{W}{c} & \\
\frac{W}{2c} > 0
\end{cases}
$$

(65)

where the first equation is needed to have a positive demand for some values of $D/S \geq \frac{v_a-r}{v_a+r}$ and the second one is needed to have a demand larger than the fundamentals for some range of $\theta_i$. 
We are now ready to fully characterize the equilibria in the nuisance region. We will do this in four stages. First we will consider the case where equation 65 holds. In this case, we know there is a CC equilibrium in the region $[\theta^p + \frac{v_a - r}{v_a + r} \frac{S}{2\sqrt{2}}, \theta_i^T]$ and an RR equilibrium in the region $[\theta_i^T, \theta^p + \frac{S}{2\sqrt{2}}]$. Bounds for the location of the threshold solution for each type of equilibria are easy to build. Regarding the CC equilibrium, since the net aggregate demand is decreasing in $D$ and the level of fundamentals is increasing in $\theta_i$, then the level $B_{CC}$ in equilibrium in the range of $\theta_i \in [\theta^p + \frac{v_a - r}{v_a + r} \frac{S}{2\sqrt{2}}, \theta_i^T]$ cannot be larger than the one obtained in the first part of equation 37 for the point $\theta_i = \theta^p + \frac{v_a - r}{v_a + r} \frac{S}{2\sqrt{2}}$, providing us with an upper bound for the location of the threshold. Regarding the RR equilibria we know that $\theta^p < \left( \frac{v_a - 2r}{v_a + r} \right) \frac{W}{c}$ is true given equation 65. This tells us that there is no solution in the standard model with $B_{RR} > D$ because $\left( \frac{v_a - 2r}{v_a + r} \right) \frac{W}{c}$ is the lowest value a threshold solution $\theta_i^T$ can take in the standard model. Since the net demand in the general threshold model and the standard model are identical when $B_{RR} \geq D$ then the threshold solution must be such that $B_{RR} < D$ and $\theta_i^T > \theta^p$ holds, providing us with a lower bound for the location of the threshold solution. Figure 21 provides an illustration on the location of the bounds. For the intervals $D < \left( \frac{v_a - 2r}{v_a + r} \right) S$ and $D > 2S$ we provide the exact solution of claims 7 and 8.

The second case we consider is when $\theta^p + \left( \frac{v_a - r}{v_a + r} \right) \frac{S}{2\sqrt{2}} \leq \left( \frac{v_a - 3r}{v_a + r} \right) \frac{W}{c}$ or $\theta^p + \left( \frac{v_a - r}{v_a + r} \right) \frac{S}{2\sqrt{2}} \leq \left( \frac{v_a - 2r}{v_a + r} \right) \frac{W}{c}$. In this case we know there is only an RR equilibrium in the nuisance region. Since still is the case that $\theta^p < \left( \frac{v_a - 2r}{v_a + r} \right) \frac{W}{c}$ the arguments offered previously still apply and $\theta^p$ is a lower bound in the RR threshold solution.

The third case is given by $\theta^p \leq \left( \frac{v_a - 2r}{v_a + r} \right) \frac{W}{c}$ where, again, there is only an RR equilibrium. Here we have $\theta^p > \left( \frac{v_a - 2r}{v_a + r} \right) \frac{W}{c}$ and the lower bound $\theta_i^T > \theta^p$ no longer applies for the whole nuisance region. However, it is still true that this bound works well for a set of the nuisance region. Furthermore, it is also true that the threshold in the standard model is also the threshold solution for the rest of the nuisance region where the bound $\theta_i^T > \theta^p$ does not apply, so that the characterization of the equilibria we will provide in this case is even more accurate than the ones we derived in the previous two cases. This result is illustrated in Figure 22. There the reader can notice that for $\theta_i \in [\theta^p + \left( \frac{v_a - r}{v_a + r} \right) \frac{S}{2\sqrt{2}}, \theta_i^B]$ we have an exact solution while for $\theta_i \in [\theta_i^B, \theta^p + \frac{S}{\sqrt{2}}]$ we use the bound $\theta_i^T > \theta^p$, where $\theta_i^B$ denotes the average level of fundamentals for which the RR threshold solution of claim 5 equals $\theta^p$. The first result follows from the fact that in the range $\theta_i \in [\theta^p + \left( \frac{v_a - r}{v_a + r} \right) \frac{S}{2\sqrt{2}}, \theta_i^B]$ the threshold solution of the standard model implies a
Figure 21: Bounds of the Nuisance Region when $\theta^p + \left(\frac{v_a - r}{v_a + r}\right) S < \left(\frac{v_a - 3r}{2(v_a + r)}\right) W/c$.

$B_{RR} > D$, so that the threshold solution is also a solution here. The second result follows from the fact that in the range $\theta^l \in [\theta^p, \theta^p + \frac{v_a - r}{v_a + r} S]$ there is no threshold solution in the standard model with $\theta^T_i$ lower than $\theta^p$, so that a threshold solution with $B_{RR} > D$ and $\theta^T_i \geq \theta^p$ must exist.

The fourth and last interval is given by $\theta^p > \left(\frac{v_a - r}{v_a + r}\right) W/c$. Here, the reader can notice that all the RR solutions of the standard model $\theta^T_i$ are lower than $\theta^p$ (i.e. $B_{RR} > D$) so that the threshold solutions of the standard model are also the threshold solutions in this case. This case is illustrated in Figure 23.

We formalize the previous results in the following claim.

**Claim 9** In the nuisance region $D \subseteq \left[\frac{v_a - r}{v_a + r}, 2S\right]$ there is a continuous threshold function where either CC or RR are an equilibria, but not both, according to the following characterization,

1. If $\theta^p + \left(\frac{v_a - r}{v_a + r}\right) \frac{S}{2\sqrt{2}} < \left(\frac{v_a - 3r}{2(v_a + r)}\right) \frac{W}{c}$ then there is a CC equilibrium in the region $\theta^l \in [\theta^p + \frac{v_a - r}{v_a + r} \frac{S}{2\sqrt{2}}, \theta^*_l]$ and a RR equilibrium in the region $\theta^l \in [\theta^*_l, \theta^p + \frac{S}{\sqrt{2}}]$,
Figure 22: Bounds of the Nuisance Region when $\theta^p \in \left( \left( \frac{v_a - 2r}{v_a + r} \right) \frac{W}{c}, \left( \frac{v_a - r}{v_a + r} \right) \frac{W}{c} \right)$

Figure 23: Bounds of the Nuisance Region when $\theta^p > \left( \frac{v_a - r}{v_a + r} \right) \frac{W}{c}$
where

\[ \theta^*_i = \frac{\theta_p + \frac{S}{2\sqrt{2}} \left( \frac{3v_a - 5r}{2(v_a + r)} \right)}{W_c} \left( \frac{W}{c} \right) \]  

(66)

The thresholds of the CC and RR equilibrium satisfy, respectively, the following bounds,

\[ \theta^T_i \in (\theta_i, \theta_i + B_{CC}^b) \]  

(67)

\[ \theta^T_i \in [\theta^p, \theta_i) \]  

(68)

for \( i = 1, 2 \). Where \( B_{CC}^b \) is given by the first part of equation 37 evaluated at \( \theta_i = \theta^p + \frac{v_a - r}{v_a + r} \frac{S}{2\sqrt{2}} \).  

2. If \( \theta^p + \frac{(v_a - r)}{(v_a + r)} \frac{S}{2\sqrt{2}} \in \left( \left( \frac{v_a - 3r}{(v_a + r)} \frac{W}{c}, \left( \frac{v_a - 3r}{(v_a + r)} \frac{S}{2\sqrt{2}} \right) \right) \right) \) then there is a RR equilibrium, with thresholds that satisfy the bound given by equation 68 for \( i = 1, 2 \).  

3. If \( \theta^p \in \left( \left( \frac{v_a - 3r}{(v_a + r)} \frac{W}{c}, \left( \frac{v_a - 3r}{(v_a + r)} \frac{S}{2\sqrt{2}} \right) \right) \right) \) then there is a RR equilibrium, with thresholds that satisfy the following bounds,

\[
\begin{cases} 
\theta^*_i = \frac{W}{2\sqrt{2}} \left( \theta_i + \left( \frac{v_a - 3r}{v_a + r} \right) \frac{S}{2\sqrt{2}} \right) & \text{if } \theta_i \in [\theta^p + \left( \frac{v_a - r}{v_a + r} \right) \frac{S}{2\sqrt{2}}, \theta^p] \\
\theta^*_i \in [\theta^p, \theta_i) & \text{if } \theta_i \in [\theta^p, \theta^p + \frac{S}{\sqrt{2}}] 
\end{cases}
\]  

(69)

where

\[ \theta^p_{c_i} = \frac{W}{2\sqrt{2}} + \frac{v_a - r}{v_a + r} \frac{S}{2\sqrt{2}} \]  

(70)

for \( i = 1, 2 \).  

4. If \( \theta^p > \left( \frac{v_a - r}{(v_a + r)} \right) \frac{W}{c} \) then there is a RR equilibrium with thresholds given by

\[ \theta^*_i = \text{Min} \left[ \frac{W}{2\sqrt{2}} \left( \theta_i + \left( \frac{v_a - 3r}{v_a + r} \right) \frac{S}{2\sqrt{2}} \right), \left( \frac{v_a - r}{v_a + r} \right) \frac{W}{c} \right] \]  

(71)

for \( i = 1, 2 \).
5.4. **Characterization of the equilibria.**

When private information is accurate, that is $S \rightarrow 0$, there are either 3 or 4 states that can be sustainable in equilibrium. In order to see this, note that in the limit, based on claims 7, 8 and 9, the equilibrium can be easily characterized by the following three equations:

\[
\begin{cases}
\theta^T = 0 & \text{if } \theta_l < 0 \\
\theta^T \rightarrow \theta_l & \text{if } \theta_l \in (0, \text{Min}[\theta_p, \left( \frac{\nu_a}{\nu_a+r} \right) \frac{W}{c}]] \quad (i) \\
\theta^T \rightarrow \text{Min}[\theta_p, \left( \frac{\nu_a-r}{\nu_a+r} \right) \frac{W}{c}] & \text{if } \theta_l > \left[ \text{Min}[\theta_p, \left( \frac{\nu_a-r}{\nu_a+r} \right) \frac{W}{c}], \infty \right] 
\end{cases}
\]

(72)

There are three possible scenarios. The first one is given by $\theta^p > \left( \frac{\nu_a}{\nu_a+r} \right) \frac{W}{c}$. In this case the threshold parameter is high enough to preserve all the results of the standard model. The four states of the world are the same we obtained before: (i) Both collapse, (iv) Only the weakest collapses, (iv) The weakest may collapse while the strongest remains and (vi) Both remain. Figure 24 provides an illustration of the location of each one of these states on the space of fundamentals $(\theta_1, \theta_2)$.
Figure 25: Equilibria when $S \to 0$ and $\theta^p \in \left( \frac{\nu_a - r}{\nu_a + r}, \frac{\nu_a}{\nu_a + r} \right) \frac{W}{c}$.

The second possibility is given by $\theta^p \in \left( \frac{\nu_a - r}{\nu_a + r}, \frac{\nu_a}{\nu_a + r} \right) \frac{W}{c}$. In this range we still have 4 states of the world but the size of state (v) is smaller. There is now a smaller space of fundamentals where the weakest peg may collapse since such collapse would no longer be profitable. Figure 25 provides an illustration on how the size of state (v) can decrease by decreasing the value of $\theta^p$.

The third and last possibility is given by $\theta^p \leq \left( \frac{\nu_a - r}{\nu_a + r} \right) \frac{W}{c}$, where the profitability condition is strong enough to vanish the state of the world where the weak peg may collapse and decrease the size of the state where only the weak peg collapses. In Figure 26 we set a profitable level $\theta^p$ below $\left( \frac{\nu_a - r}{\nu_a + r} \right) \frac{W}{c}$ and illustrate how the state (vi) can increases its size at the expense of vanishing state (v) and decreasing the size of state (iv).

The reader can notice that the only significant difference with the standard model is the result that the state of the world where only the weak peg collapses is no longer strictly but weakly increasing in wealth (since the profitability condition ensures the existence of an upper bound for such state). On the other hand, our main finding, the rank-order result, is present in both versions of the model.
Currency Crises: Whom to attack or defend?

6. Conclusion

In this paper we constructed a simple model of simultaneous speculative attacks where investors face strategic uncertainty related to whom to attack or defend out of two pegging regimes. The model formalizes the intuition that the weakest peg is more likely to suffer from speculative attacks, even in those scenarios where the fundamentals of both regimes are almost equally weak or strong. We found there are four possible states of the world in equilibrium when private information is accurate: both pegs remain, the weakest peg collapses while the other remains, the weakest peg may collapse while the other remains and both regimes collapse. The exact characterization of the equilibria depends on the structure of the payoffs and the level of wealth. The size of the range of fundamentals where only the weakest peg collapses is weakly increasing in wealth and the magnitude of the depreciation in the event of a collapse.

Footnotes

1 The optimal strategy can be derived noting that a \( B_{RR} > \frac{va-r}{va+r}S \) implies a \( p_M < \frac{r}{va+r} \) so that the marginal investor finds optimal to defend both pegs. For moderately positive signals, that is \( x_{an} \in [0, \frac{B_{RR}}{2} - \frac{S}{2} + \frac{2r}{va+r}S] \), we have
$p_1 < p_2$ with $p_1 + p_2 = 2p_M < \frac{2v_0}{\theta + r}$ if $x_{an} < -\frac{BBR}{2} + \frac{S}{2}$ and $p_1 + p_2 = p_1 \leq \frac{2v_0}{\theta + r}$ if instead $x_{an} > -\frac{BBR}{2} + \frac{S}{2}$. Then it is optimal to defend peg 1. For moderately negative signals, that is $x_{an} \in \left[-\frac{BBR}{2} + \frac{S}{2}, \frac{2v_0}{\theta + r}S\right]$, we have $p_1 < p_2$ with $p_1 + p_2 = 2p_M < \frac{2v_0}{\theta + r}$ if $x_{an} > \frac{BBR}{2} - \frac{S}{2}$ and $p_1 + p_2 = p_2 \leq \frac{2v_0}{\theta + r}$ if instead $x_{an} < \frac{BBR}{2} - \frac{S}{2}$. Then it is optimal to defend peg 2. For large positive signals $(x_{an} > \frac{BBR}{2} - \frac{S}{2} + \frac{2v_0}{\theta + r}S)$ we have $p_1 < p_2$ with $p_1 + p_2 = p_2 > \frac{2v_0}{\theta + r}$ and it is optimal to attack peg 2. For large negative signals $(x_{an} < -\frac{BBR}{2} + \frac{S}{2} - \frac{2v_0}{\theta + r}S)$ we have $p_1 > p_2$ with $p_1 + p_2 = p_2 = \frac{2v_0}{\theta + r}$ and it is optimal to attack peg 1.

2 The optimal strategy can be derived noting that $p_1 = p_2 = 0$ for $x_{an} \in \left[-\frac{BBR}{2} + \frac{S}{2}, \frac{BBR}{2} - \frac{S}{2}\right]$, $p_1 = 0$ with $p_2 \in \left[0, \frac{2v_0}{\theta + r}\right]$ for $x_{an} \in \left[\frac{BBR}{2} - \frac{S}{2}, \frac{2BBR}{2} - \frac{S}{2} + \frac{2v_0}{\theta + r}S\right]$ and $p_1 = 0$ with $p_2 > \frac{2v_0}{\theta + r}$ for $x_{an} > \frac{BBR}{2} - \frac{S}{2} + \frac{2v_0}{\theta + r}S$.

3 When $BBR > D$ the optimal strategy is independent of $D$ and therefore the net aggregate demand is also independent of $D$. If $BBR < D$ and $D < S$ a larger $D$ means that investors with low signals are more willing to defend than to attack one of the two pegs and the net position must be weakly decreasing in $D$. If instead $BBR < D$ and $D > S$ then a larger $D$ means that investors with signals close to zero are more willing to defend both pegs and that higher signals would be needed for them to instead defend or attack only one of the two pegs. Given that the aggregate demand results in the integral over the signals in the interval $[\theta_{an,1}^T, \theta_{an,2}^T]$, with $\theta_{an,1}^T < 0$ and $\theta_{an,2}^T > 0$ it happens that if there is a loss of investors defending peg 1 this is always exactly compensated in average by investors defending both pegs. Furthermore, if the integral includes investors attacking peg 1 then the demand necessarily decreases. Continuity of the aggregate demand in $D$ results from the continuity of the optimal strategy in $D$ when $D > \frac{BBR}{2} - \frac{S}{2} + \frac{2v_0}{\theta + r}S$.

4 When $BBR \geq D$ net aggregate demand equals that of the standard case which we know is weakly increasing when $BBR > \frac{v_0}{\theta + r}$. When $BBR < D$ it is easy to understand and utilize the fact that the optimal strategy is independent of $BBR$ to show that the net demand is also weakly increasing in $BBR$. We know that for peg 1 (peg 2) the optimal strategy is always weakly decreasing (increasing) in $x_{an}$ in the range $x_{an} \in (-\infty, \frac{D}{2} - \frac{S}{2} + \frac{2v_0}{\theta + r}S]$ ($x_{an} \in (-\frac{D}{2} + \frac{S}{2} - \frac{2v_0}{\theta + r}S, \infty]$) so that if the integral of the aggregate demand only includes signals in this range then the demand must be weakly increasing (decreasing) in $BBR$. If instead, the integral includes signals where investors are attacking only one peg, that is $x_{an} > \frac{D}{2} - \frac{S}{2} + \frac{2v_0}{\theta + r}S$ ($x_{an} < -\frac{D}{2} + \frac{S}{2} - \frac{2v_0}{\theta + r}S$), then the aggregate demand would also be weakly increasing (decreasing) in $BBR$, since (i) the share of investors attacking the peg would increase, (ii) the share of those attacking the other peg would decrease and (iii) the proportion of those defending one peg or both would remain constant. Continuity results from the continuity of the optimal strategy.
on $B_{RR}$ when $D > \frac{v_a - r}{v_a + r} S$.

5 Since $\theta_1 > \theta^p > 0$ in any CC equilibrium the aggregate demand must be positive and there must be investors attacking the peg. Since $\theta^T_{n,1} < 0$ and $\theta^T_{n,2} > 0$ this also implies that investors attacking the other peg are also included in the integral. It follows that an increase in $D$ decreases the proportion of investors attacking either peg 1 or 2, while it increases the proportion of investors either defending both pegs (if $D \geq S$) or defending one of the two pegs (if $D < S$), so that the net aggregate demand must always decrease. As before, continuity results from the continuity of the optimal strategy on $D$ when $D > \frac{v_a - r}{v_a + r} S$.

6 Since $\theta^T_{n,1} < 0$ and $\theta^T_{n,2} > 0$ the fact that the set of investors attacking the peg is non-empty also implies that the set attacking of investors attacking the other peg is also non-empty. It follows that an increase in $B_{CC}$ decreases the number of investors attacking the peg while increases the proportion of investors attacking the other peg so that the net aggregate demand must decrease. Once $B_{CC}$ gets large enough, the set of investors attacking the peg will be empty and investors will be taking long positions or attacking the other peg so that demand is bounded above by zero. Continuity results from the fact that the optimal strategy is independent of $B_{CC}$.

A Proof of claims.

Proof of Claim 1. The solution to the risk neutral problem can be characterized with the following statements,

(i) The investor prefers to attack peg $i$ rather than defend it if and only if,

$$p_i > \frac{r}{v_a + r}$$  \hspace{1cm} (73)

In what follows we will call attack "the preferred action in the single country problem" if and only if equation 73 holds.

(ii) If attack $i$ and attack $j$ are the preferred actions in the single country problem then attack $i$ will be the solution to the investor problem if and only if,

$$p_i > p_j$$  \hspace{1cm} (74)

(iii) If defend $i$ and defend $j$ are the preferred actions in the single country problem then defend $i$ is the optimal action if and only if,

$$p_i < p_j$$  \hspace{1cm} (75)
(iv) If attack i and defend j are the optimal actions in the single country problem then attack i is the solution to the investor problem if and only if,

\[ p_i > \left( \frac{2r}{v_a + r} \right) - p_j \]

(v) If attack is the preferred action in both pegs and \( p_1 = p_2 \) then the investor is indifferent between attacking one peg or the other. In this case either pure or mixed strategies may result from optimal behavior and the investor’s positions would be given by,

\[ (y_i^*, y_j^*) = \left( \frac{W}{c}, (1 - \alpha) \frac{W}{c} \right) \]

where \( \alpha \) is a random variable with support in the interval \([0,1]\).

If instead defend is the preferred action in both pegs and \( p_1 = p_2 \) then,

\[ (y_i^*, y_j^*) = \left( -\alpha \frac{W}{c}, -(1 - \alpha) \frac{W}{c} \right) \]

It is straightforward to derive each one of these statements. Statement (i) follows from comparing the payoff of going long versus going short in a single regime. Here, we imposed the innocuous assumption that when an investor is indifferent between attacking or defending a peg she would defend it. Statements (ii) and (iii) result from comparing long and short positions across markets. Statement (iv) follows from comparing the payoffs of going short in i versus long in j. Statement (v) results from the fact that the securities are perfect substitutes when the probability of a profitable collapse is equal among countries.

It is now straightforward to show that attack i is the solution if and only if \( p_i + p_j > \frac{2r}{v_a + r} \) and \( p_i > p_j \). In order to see this note that if these two equations hold then either \( p_i > \frac{r}{v_a + r}, p_j > \frac{r}{v_a + r}, p_i > p_j \) or \( p_i > \frac{r}{v_a + r}, p_j \leq \frac{r}{v_a + r}, p_i + p_j > \frac{2r}{v_a + 2} \) must be true, so that in both cases the solution to the investor problem is to attack i. The reverse argument is straightforward since from statements (ii) and (iv) we know that either \( p_i > \frac{r}{v_a + r}, p_j > \frac{r}{v_a + r}, p_i > p_j \) or \( p_i > \frac{r}{v_a + r}, p_j \leq \frac{r}{v_a + r}, p_i + p_j > \frac{2r}{v_a + 2} \) must be true if attack i is the optimal solution, being that both of these group of equations imply \( p_i + p_j > \frac{2r}{v_a + 2} \) and \( p_i > p_j \). Following an identical argument, we can show that defend i will be the solution of the investor problem if and only if \( p_i + p_j \leq \frac{2r}{v_a + 2} \) and \( p_i < p_j \).

**Proof of Claim 2.** Let \( \delta_1(\theta_{an} = \theta_{an,1}^*, \overline{\theta}) \) denote the proportion of the population with \( p_1 < \overline{\theta} \). Then,

\[ \delta_1(\theta_{an} = \theta_{an,1}^*, \overline{\theta}) = \int_0^1 1_{p_1(j) < \overline{\theta}} dj \]
\[
\int_{0}^{1} 1_{F_{\lambda}(-\lambda_j) < \varphi} dj
\]

\[
= \mathbb{E}_{\lambda} \left[ 1_{-\lambda < F_{\lambda}^{-1}(\varphi)} \right]
\]

\[
= \mathbb{E}_{\lambda} \left[ 1_{\lambda < F_{\lambda}^{-1}(\varphi)} \right]
\]

\[
= F_{\lambda}(F_{\lambda}^{-1}(\varphi)) = \varphi
\]

where equation 80 follows from 79 and 25, equation 81 follows from the law of large numbers and equation 82 uses the fact that \(\lambda\) has the same distribution as \(-\lambda\). The proof that \(\alpha_1(\theta_{an} = \theta_{an,2}^*, \varphi) = p_2\) is simpler. A specification like that of equation 79 yields directly equation 82 since \(p_2(x_{an}, \theta_{an,2}^T, S)\) is given by \(P[\theta_{an} = x_{an} - \lambda > \theta_{an,2}^T]\) which equals \(F_{\lambda}(x_{an} - \theta_{an,2}^T)\).

**Proof of Claim 3.** The aggregate demand of currency \(i\) when the realization of the fundamentals is \(\theta_{an}\) is given by,

\[
\bar{y}_{i}(\theta_{an}, \theta_{an,1}^{T}, \theta_{an,2}^{T}, S) = \int_{0}^{1} y_{i}^{*}(j) dj
\]

\[
= \mathbb{E}[y_{i}^{*}(p_{1}(x_{an}, \theta_{an,1}^{T}, S), p_{2}(x_{an}, \theta_{an,2}^{T}, S), v_{a}/r, W)]
\]

\[
= \int \frac{dy}{\bar{y}}\left( p_{1}(\theta_{an} + \lambda, \theta_{an,1}^{T}, S), p_{2}(\theta_{an} + \lambda, \theta_{an,2}^{T}, S), v_{a}/r, W \right) d\lambda
\]

where equation 85 uses the law of large numbers and equation 86 takes the expectations with respect to the random variable \(\lambda\). Using equation 86 we can compute the aggregate demand of each regime at its threshold and show they are equal,

\[
\bar{y}_{i}(\theta_{an,1}^{T}, \theta_{an,2}^{T}, S) = \int_{0}^{1} y_{i}^{*}(j) dj
\]

\[
= \int \frac{dy}{\bar{y}}\left( T \left( \frac{\lambda + \frac{S}{2} - \theta_{an,1}^{T}}{S} \right), T \left( \frac{\lambda + \frac{S}{2} + (\theta_{an,1}^{T} - \theta_{an,2}^{T})}{S} \right), v_{a}/r, W \right) d\lambda
\]
\[ y_i^* = \int_{y_i}^{\hat{y}} y_s(x, B_{CC}, S, v_a/r, W)dx \]  

where \( y_i^* \) uses the symmetry on the solution of the investor problem and the equality between equation 89 and 90 can be shown using a change of variables in the integral such as \( z = -\lambda \). It follows that the net currency demand at the thresholds are equal and if an equilibrium exist it must be true that the threshold levels are also equal, proving our claim.

**Proof of Claim 4.** Equations 36 and 37 deliver equation 38, while the following equation,

\[ \theta_i^T = \theta_l + \frac{\theta_{an}}{\sqrt{2}}(1 - 2\ast 1_{i=2}) = \theta_{-i} + \frac{B_{CC}}{\sqrt{2}} = \theta_{-i} + \frac{S}{\sqrt{2}} \left( \frac{B_{CC}}{S} \right) \]  

and equation 37 deliver the solution in terms of \( \theta_{-i} \) as in equation 39.

In order to show that equations 38 and 39 are indeed an equilibrium we need to show that net aggregate demand is larger than the fundamentals only when \( \theta_{an} < \theta_{an,1} \) (\( \theta_{an} > \theta_{an,2} \)) for country 1 (country 2). This condition follows from the facts that for country 1 (country 2) fundamentals are increasing (decreasing) in \( \theta_{an} \),

\[ \theta_i = \theta_l + \frac{\theta_{an}}{\sqrt{2}}(1 - 2\ast 1_{i=2}) \]  

while net aggregate demand is weakly decreasing (increasing) in \( \theta_{an} \),

\[ y_i^*(\theta_{an}, B_{CC}/S, W) = \int_{\theta_{an}-\frac{S}{\sqrt{2}}}^{\theta_{an}+\frac{S}{\sqrt{2}}} y^*(x_{an}, B_{CC}, S, v_a/r, W)dx_{an} \]  

since the optimal strategy of the individual investor given by equations 32 and 33 is weakly decreasing (increasing) in \( x_{an} \).

**Proof of claim 5.** The threshold solutions of equation 46 follow from solving the first part of the aggregate demand on equation 44 and the fundamentals line of equation 45. These threshold solutions are indeed an equilibrium since the optimal
strategy of equation 40 is weakly decreasing (increasing) everywhere in $x_{an}$ for peg 1 (peg 2). The threshold solution on equation 47 nests the solution of the second and third part of the aggregate demand and the fundamentals line. Since the optimal strategies of equations 41 and 42 are not weakly decreasing (increasing) everywhere for peg 1 (peg 2) we need to corroborate that a peg would collapse only when the fundamentals are weaker than the threshold solution.

In those cases where $\theta_1$ is positive the corroboration exercise is straightforward. If $B_{RR} \leq S$ the net aggregate demand is weakly decreasing (increasing) in the range $\theta_{an} \in [-\infty, \frac{S}{2}]$ ($\theta_{an} \in [-\frac{S}{2}, \infty]$) for peg 1 (peg 2) and the solution will be well defined in this range. When $\theta_{an} = \frac{S}{2}$ ($\theta_{an} = -\frac{S}{2}$) investors are either defending 1 or attacking 2 (defending 2 or attacking 1) so that aggregate demand is negative. Further increments (decrements) in $\theta_{an}$ may increase net aggregate position but this will always be bounded below by zero. It follows that a positive level of fundamentals when $\theta_{an} = \frac{S}{2}$ ($\theta_{an} = -\frac{S}{2}$) is sufficient to guarantee that the threshold solution is indeed an equilibrium. Since $\theta_1 = \theta_1 + \frac{\theta_{an}}{\sqrt{2}}$ ($\theta_2 = \theta_1 - \frac{\theta_{an}}{\sqrt{2}}$) is always positive when $\theta_{an} = \frac{S}{2}$ the result follows. If instead $B_{RR} > S$ we know at the threshold there are only two types of investors. Those attacking the peg and those defending the other peg. For $\theta_{an} < \theta^T_{an,1}$ the demand is weakly decreasing and the regime collapses. For $\theta_{an} > \theta^T_{an,1}$ we can show that the peg remains using an auxiliary strategy. Let us define the auxiliary strategy as the optimal strategy with the only exception that when $p_1 = p_2 = 0$ investors arbitrarily set $y_1 = y_2 = 0$. The net demand at the threshold is equal using either the optimal or the auxiliary strategy so that the threshold solution holds using either one of them. On the other hand the net aggregate demand using the optimal strategy is bounded above by the one you would obtain using the auxiliary strategy so that if the peg remains for $\theta_{an} > \theta^T_{an,1}$ using the auxiliary strategy it must also remain using the optimal strategy. But showing that the peg remains with the auxiliary strategy is trivial since demand would be weakly increasing for peg 1 (peg 2) in the range $\theta_{an} \in [-\infty, \frac{S}{2}]$ ($\theta_{an} = -\frac{S}{2}$) and the fundamentals for peg 1 (peg 2) would be positive when $\theta_{an} = \frac{S}{2}$ ($\theta_{an} = -\frac{S}{2}$) so that the peg must remain by those same arguments we used when $B_{RR} \leq S$.

In what follows it will be useful to note that a threshold solution with $B_{RR} \geq S$ only exists for positive values of $\theta_1$ since the lowest value with such type of solution is given by $\left(\frac{v_a - r}{v_a + r}\right) \frac{W}{c} + \frac{S}{2\sqrt{2}} > 0$. Then when $\theta_1$ is negative and a threshold solution with RR as an intermediate state exists we know that $B_{RR} < S$ and we can easily compute the net aggregate demand when $\theta_{an} > \frac{S}{2}$,

$$\begin{cases} \left[ \frac{2r}{v_a + r} - \left( \frac{\theta_{an}}{2} - \frac{B_{RR}}{2\theta_{an}} \right) \right] \left( -\frac{W}{c} \right) \text{ if } \theta_{an} \in \left[ \frac{S}{2}, \frac{B_{RR}}{2} + \frac{2r}{v_a + r}S \right] \\ 0 \text{ if } \theta_{an} > \frac{B_{RR}}{2} + \frac{2r}{v_a + r}S \end{cases}$$
Since the net aggregate demand is well behaved for peg 1 in the range $x_{an} \in (-\infty, \frac{S}{2}]$, we know that the average level of fundamentals is larger than the net aggregate demand when $\theta_{an} = 0$ if a threshold solution exists. This fact, together with the functional form of the aggregate demand of peg 1 above, implies that if a threshold solution exists a necessary condition for not being an equilibrium is that the level of fundamentals becomes lower than the net aggregate demand at the point $\theta_{an} = 0$ if a threshold solution exists. This fact, together with the functional form of the aggregate demand of peg 1 above, implies that if a threshold solution exists a necessary condition for not being an equilibrium is that the level of fundamentals becomes lower than the net aggregate demand.

Since the solution of $B_{RR}(\theta_1)$ is increasing in $\theta_1$, this solution tells us that if a $\theta_{1,RR}^*$ exists where $\theta_1 = \theta_l + \frac{B_{RR}(\theta_l)}{\sqrt{2}}$, then it must be the case that the threshold solution would be an equilibrium if and only if $\theta_1 > \theta_{1, RR}^*$. It also tells us that such condition would be binding if and only if the lowest level of $\theta_1$ for which a threshold solution exists is not an equilibrium. That is if

$$\theta_1 = \theta_l + \frac{B_{RR}(\theta_l)}{\sqrt{2}} + \left(\frac{2r}{v_a + r}\right) \frac{S}{\sqrt{2}} \theta_{l,RR}^* = \theta_l + \frac{\theta_{l,RR}^*}{\sqrt{2}} \left(\frac{v_a - 2r}{v_a + r}\right) + \left(\frac{v_a - r}{v_a + r}\right) \frac{S}{\sqrt{2}}$$

as expressed in equation 50. A symmetrical argument applies for peg 2. The proof for equations 48, 49 and 51 follow identical steps, using the equation,

$$\theta_{T_i} = \theta_{-i} \frac{2\theta_{an,i}^*}{\sqrt{2}} (1 - 2 \ast 1_{i=2}) = \theta_{-i} - \frac{B_{RR}}{\sqrt{2}} = \theta_{-i} - \frac{S}{\sqrt{2}} (B_{RR})$$

instead of the fundamental line of equation 45.

**Proof of Claim 6.** We will show that a threshold equilibrium with $\theta_{1}^{T} > \theta_{2}^{T}$ does not exist. The reader should keep in mind that identical arguments can be used to show that $\theta_{2}^{T} > \theta_{1}^{T}$ is also not possible. The proof will follow two steps. First, we show there are three scenarios for $\theta_{an,1}^{T}, \theta_{an,1}^{P}, \theta_{an,2}^{T}$ and $\theta_{an,2}^{P}$ that can not characterize a threshold equilibrium with $\theta_{1}^{T} > \theta_{2}^{T}$. Then we show that $\theta_{1}^{T} > \theta_{2}^{T}$ necessarily implies the existence of one of these three scenarios, completing our proof. The three scenarios are the following,

(a) $\theta_{an,1}^{*} = \theta_{an,1}^{T}, \theta_{an,2}^{*} = \theta_{an,2}^{T}$.

(b) $\theta_{an,1}^{*} = \theta_{an,1}^{P}, \theta_{an,2}^{*} = \theta_{an,2}^{T}$.

(c) $\theta_{an,1}^{T} > \theta_{an,1}^{P}, \theta_{an,2}^{*} < \theta_{an,2}^{P}$. 


The result that scenario (a) is not possible follows directly from claim 3. The net demands are identical across thresholds which imply that the level of fundamentals must also be identical across thresholds if a threshold equilibrium exist. Scenario (b) is also not possible since it implies \(\theta_{\text{an}} = \theta_{\text{an}}^* > \theta_{\text{n}} \) and \(\overline{g}(\theta_1^*, \theta_{\text{an},1}^*, \theta_{\text{an},2}^*, \mathbf{S}) = \overline{g}(\theta_1^*, \theta_{\text{an},1}^*, \theta_{\text{an},2}^*, \mathbf{S})\), where the last equality follows from equations 87-91 replacing the variables \(\theta_{\text{an},1}^*\) and \(\theta_{\text{an},2}^*\) with their equivalents \(\theta_{\text{an},1}^*\) and \(\theta_{\text{an},2}^*\). These two results together imply a contradiction since it would not be possible to have a peg 1 that collapses at \(\theta_{\text{an}} = \theta_{\text{an},1}^*\) and a peg 2 that remains in place when \(\theta_{\text{an}} = \theta_{\text{an},2}^*\).

Showing that scenario (c) is also not possible is a little more involved. Since \(\theta_{\text{an},1}^* > \theta_{\text{an},1}^*, \theta_{\text{an},2}^* < \theta_{\text{an},2}^*\) and \(\theta_{\text{p}} > 0\) we have that the threshold levels are necessarily positive and if an equilibrium exist this implies that a positive fraction of the population must be attacking each peg at its threshold. We will show later that in scenario (c) when there is a positive set of investors attacking the peg at its threshold then the net demand is weakly decreasing in its threshold for peg 1 and weakly increasing in its threshold for peg 2. Since the level of fundamentals are strictly increasing in its threshold for peg 1 and strictly decreasing in its threshold for peg 2 it follows that it is not possible to have asymmetric intermediate states in this scenario, so that an equilibrium with \(\theta_{\text{an},1}^* > \theta_{\text{an},2}^*\) is also not possible.

It remains to show that when there is a positive set of investors attacking the pegs at its thresholds, then the net demand is weakly decreasing in its threshold for peg 1 and weakly increasing in its threshold for peg 2. These results follow directly from the set of possible optimal strategies. It can be easily shown that the set of feasible optimal strategies in scenario (c) can be characterized by either (Attack 1 if \(x_{\text{an}} < \alpha_1\), Attack both if \(x_{\text{an}} \in [\alpha_1, \alpha_2]\), Attack 2 if \(x_{\text{an}} > \alpha_2\)) or (Attack 1 if \(x_{\text{an}} < \beta_1\), Defend 2 \(x_{\text{an}} \in [\beta_1, \beta_2]\), Defend both if \(x_{\text{an}} \in [\beta_2, \beta_3]\), Defend 1 if \(x_{\text{an}} \in [\beta_3, \beta_4]\), Attack 2 if \(x_{\text{an}} > \beta_4\)) where \(\alpha_2 \geq \alpha_1\) and \(\beta_4 > \beta_3 > \beta_2 > \beta_1\). Since \(\theta_{\text{an},1}^* > \theta_{\text{an},1}^*, \theta_{\text{an},2}^* < \theta_{\text{an},2}^*\) we have that \(\theta_{\text{an},1}^* = \theta_{\text{an},1}^*\) and \(\theta_{\text{an},2}^* < \theta_{\text{an},2}^*\) so that the probability functions and the optimal strategies are independent of the threshold levels. It follows that whenever there is a set of investors attacking peg 1 at its threshold the net demand can not be decreasing in the threshold level, while the opposite statement for peg 2 is also not possible. This completes the proof that scenarios (a)-(c) are not possible.

We now show that \(\theta_{\text{an},1}^* > \theta_{\text{an},2}^*\) necessarily implies one of the three previous scenarios. Let us start with the case where the average level of fundamentals is relatively high, that is \(\theta_{\text{f}} \geq \theta_{\text{p}} > 0\). Here we have that if \(\theta_{\text{an},2}^* \geq 0\) then we have scenario (a) if \(\theta_{\text{an},1}^* \geq \theta_{\text{an},1}^*,\) scenario (b) if \(\theta_{\text{an},1}^* \in [-\theta_{\text{an},2}^*, \theta_{\text{an},1}^*]\) and scenario (c) if \(\theta_{\text{an},1}^* < -\theta_{\text{an},2}^*\). If instead \(\theta_{\text{an},2}^* < 0\) then scenario (c) results. Let us now focus in the case where the average level of fundamentals is relatively low, that is \(\theta_{\text{f}} < \theta_{\text{p}}\). If \(\theta_{\text{an},2}^* \geq 0\) we have that scenario (a) applies when either
\[ \theta_{an,2} \in [\theta_{an,1}, 0] \text{ or } (\theta_{an,2} \in [-\theta_{an,2}^T, \theta_{an,1}^T] \land \theta_{an,1}^T < \theta_{an,1}^T) \] hold, while scenario (b) applies if either \((\theta_{an,2} \in [-\theta_{an,2}^T, \theta_{an,1}^T] \land \theta_{an,1}^T \geq \theta_{an,1}^T)\) or \((\theta_{an,2} < -\theta_{an,2}^T)\) hold. If instead \(\theta_{an,2}^T < 0\) then scenario (c) applies when \(\theta_{an,2}^T \in [\theta_{an,2}^T, 0]\), scenario (b) when \(\theta_{an,2}^T \in [-\theta_{an,2}^T, \theta_{an,2}^T]\) and scenario (a) when \(\theta_{an,2}^T < -\theta_{an,2}^T\). It follows that \(\theta_1^T > \theta_2^T\) implies the existence of scenarios (a), (b) or (c), completing the proof that a threshold equilibrium with \(\theta_1^T > \theta_2^T\) is not possible.

**Proof of Claim 7.**

Let us consider first an equilibrium with intermediate state RR. If \(D < 0\), we would have a positive \(\theta_{an,1}^P = -\frac{D}{T} > 0\) and a negative \(\theta_{an,2}^P = \frac{D}{T} < 0\). Since \(\theta_{an,1}^T = -\frac{B_{an}}{2} < 0\) and \(\theta_{an,2}^T = \frac{B_{an}}{2} > 0\) it follows that \(\theta_{an,i}^* = \theta_{an,i}^P\) for both pegs. Then from equations 56 and 57 we know that the probability functions would always equal those we derived in the standard model, so that the optimal strategies and the net aggregate demand would also be the same. If instead \(D \in [0, \frac{v - r}{v + r}]\), we would have a negative \(\theta_{an,1}^P = -\frac{D}{T} < 0\) and a positive \(\theta_{an,2}^P = \frac{D}{T} > 0\), so that the probability functions would be different to the standard case i.e. \(B_{RR} < D\). But even in this case the optimal strategy would still be the same since \(B_{RR} < D < \frac{v - r}{v + r}S\) implies a \(\pi_M = \frac{r}{2} - \frac{D}{2S}, q_1 + q_2 = 2p_M > 2\frac{v}{v + r}\) if \(\theta_{an} \in \left(\frac{D}{2} - \frac{S}{v + r}, \frac{D}{2} + \frac{S}{v + r}\right)\) and \(p_1 + p_2 > 2p_M\) otherwise, so that attack 1 (2) is the optimal strategy if \(x_{an} < 0 (x_{an} > 0)\) as it was in the standard case. Since the optimal strategies are identical, the net aggregate are also the same.

Let us consider now an equilibrium where CC is an intermediate state. If \(D \leq 0\), we would have a weakly positive \(\theta_{an,1}^P = -\frac{D}{T} \geq 0\) and a weakly negative \(\theta_{an,2}^P = \frac{D}{T} \leq 0\). Since \(\theta_{an,1}^T = \frac{B_{CC}}{2} > 0\) and \(\theta_{an,2}^T = -\frac{B_{CC}}{2} > 0\) it would be true that the probability function equals the standard probability function if \(B_{CC} < D\). Once \(B_{CC}\) reaches the size \(D\), the beliefs are no longer distributed uniformly around the point \(\theta_{an,i}^* = \theta_{an,i}^P\) but instead around \(\theta_{an,i}^* = \theta_{an,i}^P\). The optimal strategy is given by equation 32 if either \((B_{CC} < S), (B_{CC} > S, B_{CC} < -D)\) or \((B_{CC} > S, B_{CC} \geq -D, -D < S)\). If instead \((B_{CC} > S, B_{CC} \geq -D, -D \geq S)\) then the optimal strategy is given by

\[
\begin{cases} 
(W/c, 0) & \text{if } x_{an} < \frac{D}{T} + \frac{S}{T} \\
(\alpha W/c, (1 - \alpha)W/c) & \text{otherwise} \\
(0, W/c) & \text{if } x_{an} > -\frac{D}{T} - \frac{S}{T} 
\end{cases}
\]  

(95)

In order to see this note that as long as \(B_{CC} \leq S\) it is true that the probabilities are distributed uniformly around \(\theta_{an,1}^*, \theta_{an,2}^* = \min\{\theta_{an,1}^T = -\frac{D}{T}, \theta_{an,1}^P = \frac{B_{CC}}{2}\} \leq \frac{S}{T} \) and \(\theta_{an,2}^* = \max\{\theta_{an,2}^T = \frac{D}{T}, \theta_{an,1}^P = -\frac{B_{CC}}{2}\} \geq -\frac{S}{T}\). It follows that the marginal investors has \(\pi_M \in (\frac{1}{2}, 1)\) with \(p_1 + p_2 = p_M\) on the range \(x_{an} \in [\theta_{an,1}^T - \frac{S}{T}, \theta_{an,2}^T + \frac{S}{T}]\) and \(p_1 + p_2 > p_M\) otherwise. Since \(p_1 > p_2\) if and only if \(x_{an} < 0\) it follows that equation 32 is the optimal strategy, no matter the value of \(D\). When \(B_{CC} > S\) and
$B_{CC} < D$ the optimal strategy equals the standard strategy $B_{CC} < D$ since the probability functions are also equal. When $B_{CC} > S$ and $B_{CC} > -D$ the beliefs will be distributed uniformly around $\theta^*_{an,1} = -\frac{D}{2}$ and $\theta^*_{an,2} = \frac{D}{2}$ so that equation 32 is the optimal strategy as long as $p_M < 1$, that is if $-D < S$. If instead $-D > S$ we have a set of investors with signal close to zero that would prefer to attack both pegs as expressed in equation 95.

We will show now that the aggregate demand function is identical to the one we got in the standard case. If either $B_{CC} > S$ or $B_{CC} > -D$ does not hold we know the optimal strategies are equal and therefore the demand would also be equal. If instead, $B_{CC} > S$ and $B_{CC} > -D$ we know that all investors would be attacking the peg 2 when $\theta_{an} = \theta^T_{an,1} = \frac{B_{CC}}{-2}$ or attacking peg 1 when $\theta_{an} = \theta^T_{an,2} = -\frac{B_{CC}}{2}$, so that the aggregate demand at the threshold would equal zero as it does in the standard model.

When $D \in (0, \frac{v_a - r}{v_a + r} S)$ the proof that the net aggregate demand is the same that the one we derived in the standard case is much simpler since the optimal strategy here would be independent of $B_{CC}$. That is, since $D > 0$ we know that $\theta^p_{an,1} = \frac{-D}{2} < 0 < \theta^p_{an,1} = \frac{B_{CC}}{-2}$ and $\theta^p_{an,2} = \frac{D}{-2} > 0 > \theta^p_{an,2} = -\frac{B_{CC}}{2}$ so that the distribution of beliefs would always be uniformly distributed around $\theta^*_{an,i} = \theta^p_{an,i}$ for both pegs. The probability functions would never be equal to those of the standard case but it can be shown that the optimal strategies will be identical as long as $B_{CC} < S$. In order to see this note that attacking 1 (2) is always the optimal strategy when $x_{an} < 0$ ($x_{an} > 0$) for any level $B_{CC}$. This result follows from the fact that $p_M = \frac{1}{2} - \frac{D}{2} > \frac{v_a - r}{v_a + r}$ and $p_1 + p_2 \geq \frac{2v_a - r}{v_a + r}$ which is independent of the level $B_{CC}$. Since the optimal strategies are identical when $B_{CC} < S$ then the aggregate demand are also equal in this case. If instead $B_{CC} \geq S$ the aggregate demand would still be equal since aggregate demand equals zero as it does in the standard model.

We have shown in all possible cases that the net aggregate demand equals that of the standard model so that the threshold solutions are also the same. It only remains to show here that the threshold solutions are also an equilibrium when $\theta_p$ is finite. In the case of an intermediate CC equilibria we know the optimal strategy must be given by equations 32 or 95. Since in both cases the optimal strategies are weakly decreasing (increasing) $x_{an}$ for peg 1 (peg 2) it follows that the threshold solutions are indeed an equilibrium as in the standard model. In the case of an RR equilibrium we have shown that the optimal strategies are always identical to those of the standard model. It follows that the equilibrium proofs provided in Claim 5 also apply here.

**Proof of Claim 8.** The inexistence of a CC equilibrium has already been shown. The threshold solutions of the RR equilibrium on equation 62 are derived in equation 60 and 61. It only remains to show that such threshold solutions are indeed an equilibria. This results follows from the structural forms of the optimal
strategies given in equation 58 with \( U = -\frac{D}{2} + \frac{S}{2} \) and \( L = \frac{D}{2} - \frac{S}{2} \) if \( B_{RR} \leq D \) and \( U = -\frac{B_{RR}}{2} + \frac{S}{2} \) and \( L = \frac{B_{RR}}{2} - \frac{S}{2} \) if \( B_{RR} > D \). Note that the positions on the peg 1 are weakly decreasing in the range \( x_{an} \in [-\infty, \frac{Min[D, B_{RR}]}{2} - \frac{S}{2} + \frac{v_{a-r}}{|v_{a+r}|}S] \) and are bounded below by zero in the range \( x_{an} > \frac{Min[D, B_{RR}]}{2} - \frac{S}{2} + \frac{v_{a-r}}{|v_{a+r}|}S \). Since \( \frac{Min[D, B_{RR}]}{2} + \frac{v_{a-r}}{|v_{a+r}|}S > \theta_{1}^{P} = -\frac{B_{RR}}{2} \) it follows that the threshold solutions for peg 1 are truly an equilibrium. A symmetrical argument applies for peg 2.

**Proof of Claim 9.** The continuity of the threshold function follows from the continuity in the aggregate demand and the continuity of the fundamentals line. The statement that only one of the possible intermediate states is an equilibrium and the existence of bounds have already been shown in the text. Equation 66 follows from solving the system given by \( \theta_{1} = \theta^{p} + \left( \frac{S}{2\sqrt{v}} \right) \frac{D}{S} \) and the second part of equation 63. The first part of equation 69 and equation 71 are given by equation 47. Equation 70 follows from solving 69 and \( \theta_{2}^{p} = \theta^{p} \). It only remains to show that the threshold solutions are indeed an equilibrium. In the case of a CC equilibrium all threshold solutions are an equilibrium since the optimal strategies are weakly decreasing (increasing) in \( x_{an} \) for peg 1 (peg 2). In the case of a RR intermediate state all the threshold solutions are also an equilibria since for any possible threshold solution we can always find a scalar \( A > 0 \) such that the demand would be weakly decreasing (increasing) for peg 1 (peg 2) in a range \( \theta_{an} \in (-\infty, A) \) \( (\theta_{an} \in (-A, \infty)) \) while it would be bounded above by zero in the complementary range \( \theta_{an} \in (A, \infty) \) \( (\theta_{an} \in (-\infty, -A)) \). Since \( \theta_{1} \) \( (\theta_{2}) \) is positive when \( A > 0 \) provided \( \theta_{1} > \theta^{p} > 0 \) the result follows. One possible characterization is the following. In the case where \( B_{RR} \) or \( D \) were less than \( S \) then \( A = \frac{S}{2} \) since for \( \theta_{an} > \frac{S}{2} \) we start increasing the proportion of investors attacking the other peg at the expense of investors defending the peg. If both \( B_{RR} \) and \( D \) are larger than \( S \) and \( \frac{S}{2} < \text{Max}[\frac{B_{RR}}{2}, \frac{D}{2}] - \frac{S}{2} + \frac{2r}{v_{a+r}}S \) then \( A \) could be given by \( \text{Max}[\frac{B_{RR}}{2}, \frac{D}{2}] - \frac{S}{2} + \frac{2r}{v_{a+r}}S \) since for \( \theta_{an} > A \) we start having investors attacking the other peg. If instead \( B_{RR} \) and \( D \) are larger than \( S \) but \( \frac{S}{2} \geq \text{Max}[\frac{B_{RR}}{2}, \frac{D}{2}] - \frac{S}{2} + \frac{2r}{v_{a+r}}S \) then \( A \) could be given by \( S - \text{Max}[\frac{B_{RR}}{2}, \frac{D}{2}] - \frac{2r}{v_{a+r}}S \) since for \( \theta_{an} > A \) we would no longer have investors attacking the peg.

**B** Functional form of the aggregate demand in the nuisance region where \( D \in [\frac{v_{a-r}}{v_{a+r}}S, 2S) \).

**B1.** Case 1: \( v_{a} > 3r \)

1. \( \frac{D}{S} < \frac{v_{a-r}}{v_{a+r}} \)

\[
\begin{align*}
\left\{ \begin{array}{ll}
\left( \frac{1}{2} - \frac{B_{CC}}{2S} \right) \frac{W}{v} & \text{if } B_{CC} \leq \frac{1}{2} \\
0 & \text{otherwise}
\end{array} \right.
\end{align*}
\]
Currency Crises: Whom to attack or defend?

2. \( \frac{D}{S} \in \left[ \frac{v_a - r}{v_a + r}, 1 \right) \)

\[
\left\{ \begin{align*}
&\left( \frac{v_a - r}{v_a + r} - \frac{D}{2S} - \frac{B_{CC}}{2S} \right) \frac{W}{c} - \left( \frac{3r - v_a}{2(v_a + r)} + \frac{D}{2S} \right) \frac{W}{c} \quad \text{if } B_{CC} \in \left[ 0, \frac{2(v_a - r)}{v_a + r} - \frac{D}{S} \right] \\
&\quad - \left( \frac{3r - v_a}{2(v_a + r)} + \frac{D}{2S} \right) \frac{W}{c} \quad \text{if } B_{CC} \in \left( \frac{2(v_a - r)}{v_a + r} - \frac{D}{S}, 1 \right] \\
&\quad - \left( \frac{2r}{v_a + r} + \frac{D}{2S} - \frac{B_{CC}}{2S} \right) \frac{W}{c} \quad \text{if } B_{CC} \in (1, \frac{D}{S} + \frac{4r}{v_a + r}) \\
&\quad 0 \quad \text{if } B_{CC} > \frac{D}{S} + \frac{4r}{v_a + r}
\end{align*} \right. 
\]

3. \( \frac{D}{S} \in [1, \frac{2(v_a - r)}{v_a + r}) \)

\[
\left\{ \begin{align*}
&\left( \frac{v_a - r}{v_a + r} - \frac{D}{2S} - \frac{B_{CC}}{2S} \right) \frac{W}{c} - \left( \frac{D}{S} - 1 \right) \frac{W}{c} \quad \text{if } B_{CC} \in \left( \frac{2(v_a - r)}{v_a + r} - \frac{D}{S}, 2 - \frac{D}{S} \right] \\
&\quad - \left( \frac{D}{2S} - \frac{B_{CC}}{2S} \right) \frac{W}{c} - \left( \frac{2r}{v_a + r} \right) \frac{W}{c} \quad \text{if } B_{CC} \in \left( 2 - \frac{D}{S}, \frac{D}{S} + \frac{4r}{v_a + r} \right] \\
&\quad - \left( \frac{2r}{v_a + r} + \frac{D}{2S} - \frac{B_{CC}}{2S} \right) \frac{W}{c} \quad \text{if } B_{CC} \in (\frac{D}{S}, \frac{D}{S} + \frac{4r}{v_a + r}) \\
&\quad 0 \quad \text{if } B_{CC} > \frac{D}{S} + \frac{4r}{v_a + r}
\end{align*} \right. 
\]
4. \( D \in \left[ \frac{2(v_a - r)}{v_a + r}, \frac{2v_a}{v_a + r} \right] \)

\[
\begin{align*}
- \left( \frac{D}{S} - 1 \right) \frac{W}{c} - \left( 1 - \frac{D}{2S} - \frac{BRB}{2S} \right) \left( \frac{W}{c} \right) & \text{ if } \frac{BRB}{S} \in \left[ 0, \frac{2(v_a - r)}{v_a + r} \right] \\
\left( \frac{v_a - r}{v_a + r} - \frac{D}{2S} \right) + \frac{BRB}{2S} \left( \frac{W}{c} \right) & \text{ if } \frac{BRB}{S} \in \left( \frac{D}{S} - \frac{2(v_a - r)}{v_a + r} \right), S \]
\end{align*}
\]

5. \( D \in \left[ \frac{2v_a}{v_a + r}, 2 \right] \)

\[
\begin{align*}
- \left( \frac{D}{S} - 1 \right) \frac{W}{c} - \left( 1 - \frac{D}{2S} - \frac{BRB}{2S} \right) \left( \frac{W}{c} \right) & \text{ if } \frac{BRB}{S} \in \left[ 0, 2 - \frac{D}{S} \right] \\
- \left( \frac{D}{2S} - \frac{BRB}{2S} \right) \frac{W}{c} & \text{ if } \frac{BRB}{S} \in \left( 2 - \frac{D}{S}, \frac{2(v_a - r)}{v_a + r} \right) \\
\left( \frac{v_a - r}{v_a + r} - \frac{D}{S} \right) + \frac{BRB}{2S} \left( \frac{W}{c} \right) & \text{ if } \frac{BRB}{S} \in \left( \frac{D}{S} - \frac{2(v_a - r)}{v_a + r}, \frac{2v_a}{v_a + r} \right) \text{, S} \]
\end{align*}
\]

6. \( D \in \left[ 2, \infty \right) \)

\[
\begin{align*}
- \frac{W}{c} \text{ if } \frac{BRB}{S} \in \left[ 0, \frac{D}{S} - 2 \right] \\
- \left( \frac{D}{2S} - \frac{BRB}{2S} \right) \frac{W}{c} & \text{ if } \frac{BRB}{S} \in \left( \frac{D}{S} - 2, \frac{2(v_a - r)}{v_a + r} \right) \\
\left( \frac{v_a - r}{v_a + r} - \frac{D}{S} \right) + \frac{BRB}{2S} \left( \frac{W}{c} \right) & \text{ if } \frac{BRB}{S} \in \left( \frac{D}{S} - \frac{2(v_a - r)}{v_a + r}, \frac{2v_a}{v_a + r} \right) \text{, S} \]
\end{align*}
\]

B2. Case 2: \( v_a \leq 3r \)

The net aggregate demand when \( v_a < 3r \) is exactly the same as the ones we obtained before for intervals 1, 5 and 6. Interval 2 and 4 have the same functional form but the ranges over \( D/S \) change. Interval 3 has a different functional form and the range over \( D/S \) also changes.

The differences between the previous case where \( v_a \geq 3r \) and this one where the opposite holds relies on the fact that when \( B_{RR} = B_{RR} = 0 \) there is an empty set of individuals attacking a peg if \( D/S > 2(v_a - r)/v_a + r \). When \( v_a \geq 3r \) this happens at \( D/S = 2(v_a - r)/v_a + r > 1 \) while if \( v_a < 3r \) this happens at a point \( D/S = 2(v_a - r)/v_a + r < 1 \).
3. \( \frac{D}{S} \in \left( \frac{2(v_a - r)}{v_a + r}, 1 \right) \)

\[
\left\{ \begin{array}{ll}
-(\frac{1}{2} - \frac{BBR}{2S}) \frac{W}{c} & \text{if } \frac{BBR}{S} \in \left[ 0, \frac{2(v_a - r)}{v_a + r} \right] \\
\left( \frac{v_a - r}{v_a + r} \right) \frac{W}{c} - (\frac{1}{2} - \frac{BBR}{2S}) \frac{W}{c} & \text{if } \frac{BBR}{S} \in \left( \frac{D}{S} - \frac{2(v_a - r)}{v_a + r}, \frac{D}{S} \right] \\
\left( \frac{v_a - r}{v_a + r} \right) \frac{W}{c} & \text{if } \frac{BBR}{S} > 1
\end{array} \right.
\]

4. \( \frac{D}{S} \in [1, \frac{2v_a}{v_a + r}) \)

References

Calvo, Guillermo (1999), "Contagion in emerging markets: When Wall Street is a carrier", Preliminary.


