Exchange Rate Manipulation and Constructive Ambiguity: The Meaning of Transparency

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Abstract

I explore the implications of central bank transparency during foreign exchange interventions and develop dynamic models in which investors have imperfect common knowledge about both interventions and fundamentals. The benchmark two-period model is simple enough so that most of it can be solved analytically, and it presents the main result that transparency can often exacerbate any misalignment between the exchange rate and fundamentals. This is a consequence of two distinct effects of transparency. First, transparency reveals some information about fundamentals to investors (the truth-telling effect). Second, transparency increases the precision of the exchange rate as a signal of those fundamentals that remain unknown (the signal-precision effect). If a central bank announcement does not reveal much information about fundamentals, then this second effect dominates and transparency magnifies exchange rate misalignment. An important implication of this result is that during times of crisis, when risky assets are often oversold for reasons of agency, liquidity, and psychology, a policy of ambiguity can in fact increase the effectiveness of intervention to support a declining currency. The benchmark model is both extended to an infinite horizon (with and without common knowledge of the past) and also expanded into a Bayesian signalling game, and I demonstrate that the principal results do not change.

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1 Introduction

Over the past decade, a growing body of evidence has demonstrated that all but a few countries exert some control over the value of their exchange rates. According to Calvo and Reinhart (2002), this “fear of floating” is common not only among countries that openly admit it, but also among those that claim not to let currency prices affect policy. Just as there is broad agreement about the prevalence of exchange rate interventions, there is broad disagreement about the policies that should accompany these interventions, especially with regard to central bank transparency. In this paper, I develop dynamic models of foreign exchange intervention that address these questions.

I focus on the issue of central bank transparency, specifically on the implications of credible and truthful public announcements about the size and timing of foreign exchange interventions as opposed to deliberate attempts to be secretive and create uncertainty about those interventions. While there are other important aspects of central bank intervention policy, the question of transparency is among both the most important and the most disputed. Indeed, there is extensive evidence that central banks from around the world hold opposing views about the effectiveness of predictability relative to unpredictability, and that they implement different policies for different reasons (Bank for International Settlements 2005, Canales-Kriljenko 2003, Chiu 2003).

Two examples from the financial crisis highlight this lack of policy consensus. Both Mexico and Russia faced intense capital outflows and speculative pressure as the price of risky assets throughout the world declined in the months after the collapse of Lehman Brothers in September 2008.1 The Bank of Mexico has a longtime commitment to transparent foreign exchange intervention, but at the height of this crisis in early February 2009, the bank became convinced that transparency was hurting its efforts to stabilize the peso and abruptly switched to a secretive and purposely ambiguous policy. In that month alone, the bank spent nearly two billion dollars of its reserves in unannounced interventions.2 In this same period, the Bank of Russia fought a protracted battle with the markets over the falling ruble. Their well-publicized attempts to initially guide the currency to an orderly and predictable depreciation eventually gave way to a looser, more ambiguous policy in which the target band for the ruble was substantially widened and made more flexible.3 Ultimately, the Bank...

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1Between August 2008 and March 2009, both the Mexican peso and the Russian ruble lost more than one third of their values against the US dollar before eventually stabilizing at slightly higher levels.  
2Although these interventions were intentionally kept secret, the Bank of Mexico did reveal their size publicly afterwards. For a discussion of the bank’s normally transparent policy, see Sidaoui (2005).  
3In the second half of 2008, the Bank of Russia widened the target band for the ruble to 16.9% (top to bottom) via a series of small adjustments. It then widened the band further to 28.9% in a little over one week in January 2009. Two examples of some of the press coverage surrounding this episode are the articles...
of Russia’s extensive interventions contributed to a loss of more than 200 billion dollars in foreign exchange reserves (nearly 40% of the bank’s total reserves) in a period of only six months. In both of these cases, policymakers appear to have been uncertain about the best way to complement their interventions and to help effectively stabilize and defend their currencies. In this era of enormous foreign exchange reserves and large-scale interventions, a better understanding of the implications of these different policies is important.

The main prediction of my analysis is that central bank transparency can in fact magnify any existing misalignment between the exchange rate and fundamentals. This follows because a transparent intervention policy improves the precision of the exchange rate as a signal of fundamentals (the signal-precision effect of transparency), and thus compels rational Bayesian investors to weigh that public signal more heavily in their expectations. Although transparency reveals some information about fundamentals (the truth-telling effect of transparency) and thus also diminishes the signal value of the exchange rate, this extra information can be outweighed by the extra precision provided by a public announcement. It is precisely in situations such as these, when the signal-precision effect is larger than the truth-telling effect, that exchange rate misalignment worsens.

This conclusion has many implications. Arguably the most important is that a policy of ambiguity will often increase the effectiveness of central bank intervention during periods of crisis and large capital outflows. In these episodes, asymmetric information, pro-cyclical liquidity provision, and psychology often lead to excessive sales of risky assets, as shown by Brunnermeier and Pedersen (2009) and Shleifer and Vishny (1997). My model predicts that it is precisely in situations like these, when risky countries’ currencies are undervalued, that transparent interventions to support a currency are less effective than more opaque and secretive interventions. In the case of Mexico and Russia, the model argues that both countries would have likely benefited from more secrecy and ambiguity (as they eventually chose) to go along with their extensive foreign exchange interventions.

I build on a simple model of a cashless economy in which investors are heterogeneously informed about both central bank interventions and fundamentals. The first model I present, the benchmark two-period model, posits that the future value of the exchange rate is a linear function of interventions and fundamentals. In the style of Grossman and Stiglitz (1976), information about future fundamentals is embedded in the current exchange rate so that, by observing the price of currency, investors learn about these fundamentals and update their beliefs. This learning is imperfect, however, as noise traders push the exchange rate away from its fundamental value. Since the price of foreign currency is a publicly observable

“The Flight from the Rouble” and “Down in the Dumps” from The Economist, November 20, 2008 and February 5, 2009, respectively.

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signal, any time that the exchange rate differs from its fundamental value average beliefs about fundamentals will differ from the true value of fundamentals. Within this framework, I characterize the conditions under which transparency worsens exchange rate misalignment.

The benchmark model assumes that either central bank interventions are independent of the underlying state of the economy or investors are naive and unable to infer anything from the bank’s chosen policy of transparency. While these assumptions simplify the analysis, they are not realistic. Indeed, there is both theoretical (Angeletos, Hellwig, and Pavan 2006, Mussa 1981) and empirical (Bank for International Settlements 2005, Chiu 2003) evidence that transparency policy is an important signal to investors. To explore this question, I expand the two-period benchmark model into a Bayesian signalling game in which the central bank has a clearly defined objective function and investors are not naive. Given a set of assumptions for the model’s primitives, I prove the existence of a partially-separating Bayesian equilibrium that preserves the intuition and analysis from the benchmark model.

The two-period benchmark model is also extended to an infinite horizon. This exercise examines the robustness of the results in a more complete setting and provides the central motivation for the setup of the two-period model by generating an endogenous expression for the future value of the exchange rate. The first of these infinite-horizon models assumes that investors have common knowledge of the past. This causes higher-order expectations to disappear and keeps the analysis relatively tractable, so that even though a full analytic solution is not possible, an analytic characterization of the equilibrium conditions can be obtained. In this richer setup, I describe some cases in which transparency magnifies exchange rate misalignment and provide exact numerical values for all of the model’s endogenous parameters. The results match the benchmark model’s predictions. The second infinite-horizon model assumes that investors are perpetually disparately informed. In this setup, higher-order expectations are part of the steady-state equilibrium as in similarly structured dynamic macroeconomic models with information heterogeneity such as Bacchetta and van Wincoop (2006), Lorenzoni (2009), and Nimark (2010a). Like these authors’ models, my model is analytically intractable and must instead be approximated numerically.

Throughout this paper, I consider the implications of a policy of publicly and truthfully announcing the size of interventions versus a policy of secrecy. One advantage of focusing on these two policies is that they have a clear economic interpretation in terms of the information sets of investors, making rigorous theoretical analysis easier. In practice, however, a

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4There is a class of models in which the equilibrium is fully revealing even though agents are heterogeneously informed about fundamentals. The most famous example of this is given by Townsend (1983). As shown by Kasa (2000), Pearlman and Sargent (2005), and Sargent (1991), the agents in Townsend’s model can actually infer the information of others so that higher-order expectations are not part of equilibrium. The investors in my dynamic model, in contrast, cannot infer other agents’ information and higher-order expectations do not disappear.
central bank wishing to be transparent will often announce not only the size of a current intervention, but also the size of past interventions, the size and timing of interventions planned for the future, and the likely stance of other policies in the future. Furthermore, foreign exchange interventions are usually correlated with future changes in macroeconomic policy that affect exchange rates and asset returns. If this correlation has the correct sign, then a transparent intervention is an important source of information about these future policies, a point emphasized by Dominguez and Frankel (1993a), Mussa (1981), and the whole literature about the signalling hypothesis. These considerations have a natural interpretation in my models, since all of the results about central bank transparency are statements about the extent of information that is revealed to investors. Indeed, the more information that is credibly communicated to investors through a public announcement, the less likely it is that transparency will exacerbate exchange rate misalignment.

Another factor to consider is that central banks are often unable to make credible public announcements. This is especially true about their actions in the future, since any promise to engage in large-scale interventions or to alter monetary policy in ways that might significantly disrupt the domestic economy inherently lack credibility. Given this reality, central banks often choose to reveal only information that can be easily verified, such as information about current or past interventions. If a central bank can credibly reveal all information that is relevant to the pricing of its currency, then the effects of transparency are obvious and uninteresting as well as unrealistic. It is only when information revelation is partial that meaningful predictions are possible.

A truthful central bank announcement affects investors’ beliefs in two different ways in my model. First, and more apparently, any parameters the central bank reveals to investors eliminate the role of the exchange rate as a signal of those parameters. This is the truth-telling effect of transparency. Second, and less apparently, any parameters the central bank reveals to investors increase the precision of the exchange rate as a signal of other, still-unknown parameters, and hence increase the weight that investors place on the exchange rate signal when forming their beliefs about those unknown parameters. This is the signal-precision effect of transparency. These two effects push in opposite directions. The truth-telling effect directly raises expectations of parameters for which average beliefs are too low. This tends to reduce misalignment and appreciate an exchange rate that, because of sales by noise traders, is undervalued relative to fundamentals. Conversely, the signal-precision effect indirectly

\[5\] Dominguez and Panthaki (2007) and Gnabo, Laurent, and Lecourt (2009) provide detailed empirical analyses of the effects of various kinds of central bank statements related to foreign exchange interventions.

\[6\] This correlation is usually positive (purchasing domestic currency implies better future domestic fundamentals), but there are examples in which it is actually negative, as shown by Kaminsky and Lewis (1996). Sarno and Taylor (2001) and Vitale (2007) both provide excellent surveys of the signalling-hypothesis literature (and the intervention literature, more broadly).
lowers expectations of parameters for which average beliefs are too low and tends to increase misalignment and further depreciate an already undervalued exchange rate. The main results of this paper characterize the conditions for which this second, indirect effect dominates.

There are several important conditions that imply that transparency will magnify exchange rate misalignment. The most essential of these conditions is that the central bank reveal only partial information. This is necessary because the truth-telling effect of transparency increases with the completeness of information revelation, and thus partial revelation is crucial in order to limit the size of this effect relative to the size of the signal-precision effect. To illustrate this point, consider the extreme case in which information revelation is totally complete. As described earlier, in this extreme case, the central bank credibly announces the value of all exchange rate fundamentals so that the fundamental value of its currency is fully revealed. The exchange rate, then, is no longer a relevant signal since the investors know the fundamental value with certainty, and therefore transparency has only a truth-telling effect and lacks a signal-precision effect. It is only as the information revealed by the central bank’s announcement becomes less complete that the signal-precision effect begins to grow relative to the truth-telling effect.

The mechanism I describe matches well with the justification that central banks often provide for their ambiguous policies. In particular, survey evidence from Bank for International Settlements (2005) and Chiu (2003) indicates that central banks worry that unsuccessful transparent interventions might undermine both a bank’s credibility and the market’s confidence in its currency. Central banks are concerned that highly visible and extensive interventions coupled with continued undesirable movements in the exchange rate will intensify doubts about a bank’s ability to achieve its goals. Indeed, a transparent failure of this nature publicly reveals the market’s true sentiment about exchange rate fundamentals and magnifies pessimism among market participants with different beliefs. This paper gives these intuitive but vague ideas a precise meaning within a clearly specified economic model.

1.1 Related Literature

My models assume that domestic and foreign assets are imperfect substitutes, which ensures that foreign exchange interventions alter the currency risk premium and have a permanent effect on the exchange rate. There remains, however, a considerable amount of both theoretical and empirical uncertainty about the relative impact of interventions that leave interest rates and the money supply unchanged. Indeed, as described by Edison (1993), some of the earliest literature on this topic concluded that interventions only affect the exchange rate by enhancing the credibility of future policy. I emphasize that this paper’s main results do not require that interventions have a persistent impact on the exchange rate. In fact, even if I
assume that interventions have no predictable effect on the exchange rate at any horizon, the results remain intact as long as interventions have an effect on the volatility of the exchange rate.7

Recently, a growing empirical literature has shown that foreign exchange interventions do have an immediate and statistically significant impact on exchange rates regardless of whether or not a central bank publicly announces the size and timing of its interventions. This literature includes Chaboud and Humpage (2005), Dominguez and Frankel (1993b), Dominguez and Panthaki (2007), Fatum and Hutchison (2003), Ghosh (1992), Ito (2002), Kearns and Rigobon (2005), and Payne and Vitale (2003), among others. No consensus has been reached, however, about how much of this impact is due to direct, portfolio-balance effects versus indirect, signalling effects, and how persistent these effects are.

Much recent research has emphasized the interaction between market expectations and central bank interventions. On the theoretical side, both Bhattacharya and Weller (1997) and Vitale (1999) incorporate ideas from the literature on microstructure and order flow in asset pricing and develop models in which interventions have large effects on market expectations. The market participants in their models observe order flow and rationally infer what an intervention reveals about fundamentals, so that an intervention that has only a temporary effect on order-flow can still have a meaningful impact on the exchange rate by affecting the foreign exchange market’s information. Both models predict that central banks cannot make credible public announcements and benefit from secrecy if their objectives are not consistent with exchange rate fundamentals. On the empirical side, Dominguez and Panthaki (2007) show that both falsely reported interventions and unrequited interventions—interventions that the market expects but do not materialize—have statistically significant effects on the exchange rate. This observation leads inexorably to the conclusion that interventions influence the beliefs of currency traders in important ways.

There is a vast and insightful literature on managed exchange rates. Its focus, however, is almost entirely on fixed currency pegs, in which no movement in the exchange rate is allowed, and target zones, in which the exchange rate is allowed to float freely only within some specified range. Among the most notable contributions are those of Flood and Garber (1984), Hellwig, Mukherji, and Tsyvinski (2006), Jeanne and Rose (2002), Krugman (1991), Morris and Shin (1998), and Obstfeld (1996). In general, the fixed exchange rate literature focuses on the causes and consequences of speculative attacks and currency crises, while the target zone literature focuses on the effects of policy on expectations of the future and hence on the value of today’s exchange rate. I consider foreign exchange interventions as part

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7Beine, Lahaye, Laurent, Neely, and Palm (2007) provide recent evidence that interventions increase exchange rate volatility, while Vitale (2007) presents a survey of some of the past literature on this topic.
of a managed floating exchange rate, although the logic behind my results applies to fixed currency pegs and target zones as well.\footnote{In the case of a fixed currency peg, interest rates are an important price signal that can be directly manipulated by central banks.}

The structure of my models shares much in common with other models of imperfect information in asset-pricing and crises. Indeed, the benchmark two-period model operates in an environment that is similar to the asset-pricing and crisis hybrid model of Angeletos and Werning (2006). It is no surprise, then, that my model replicates one of their main insights—the positive relationship between the precision of agents’ private signals of fundamentals and the precision of the exchange rate as an endogenous public signal of those fundamentals. Angeletos and Werning (2006) examine this relationship’s implications for the equilibrium outcome in global coordination games but do not consider the possibility of price manipulation as I do. Given the similarity between the two models, an extension of this paper’s main results about transparency and currency mispricing to a global-games setting is likely a promising direction for future research.

The extension of the benchmark model to an infinite horizon with perpetually disparately informed traders adopts assumptions that are similar to the assumptions in the asset-pricing models of Allen, Morris, and Shin (2006), Bacchetta and van Wincoop (2008), and Kasa, Walker, and Whiteman (2007), Nimark (2010b). Each of these papers shows that persistent gaps between prices and fundamentals are common in such an environment, as is the case in my model. These papers emphasize this gap and offer a compelling explanation for several important empirical puzzles in finance, but they do not examine price manipulation as I do.

The idea that transparency might have counterintuitive implications and lead to bad outcomes is also explored by Angeletos and Pavan (2007), Cornand and Heinemann (2004), and Morris and Shin (2002). These papers consider environments in which high levels of coordination among agents can be socially suboptimal and examine how public information facilitates this coordination and can lead to undesirable effects. These environments are static and highly stylized so that actions, information, and payoffs may be interpreted to represent many different things. My model avoids any analysis of total welfare and is instead a positive exercise in the interaction of asset-price manipulation and central bank transparency.

Bannier and Heinemann (2005) examine the effects of central bank transparency in the context of currency crises and global games. Their main conclusion is that transparency helps prevent a crisis when prior beliefs about fundamentals are pessimistic since transparency causes agents to place greater weight on their private information when forming expectations. While this paper’s results do share some of this same logic, my emphasis is primarily on the interaction between partial information revelation and deviations of asset prices from their
fundamental values.

Chamley (2003) develops a model in which speculators learn about fundamentals by observing the exchange rate move within a target band. He examines how speculators’ ability to coordinate an attack against this band is affected by the information present in the exchange rate, and concludes that any central bank policy that reduces exchange rate volatility facilitates such coordination. Once again, my results do share some of this same logic, but my emphasis is on partial information revelation and asset mispricing rather than coordination.

The paper is organized as follows. Section 2 presents the benchmark two-period model and the main results about central bank transparency. Section 3 expands the benchmark two-period model into a Bayesian signalling game. Section 4 extends the benchmark two-period model to an infinite horizon, with Section 4.1 considering the case in which investors have common knowledge of the past and Section 4.2 considering the case in which investors are perpetually disparately informed. Section 5 concludes. The proofs for all the results from Sections 2 and 3 are provided in Appendix A, and the proofs for all the results from Section 4 are provided in Appendix B.
2 Benchmark Two-Period Model

There are two periods, \( t \in \{1, 2\} \), and two countries, home and foreign. I shall refer to the home country’s currency as the dollar and the foreign country’s currency as the peso. There is only one good and its price in each country is linked by the law of one price, so that \( e_t + p_t^* = p_t \) in each period \( t \), where \( p_t \) is the log of the price of the good in the home country, \( p_t^* \) is the log of the price of the good in the foreign country, and \( e_t \) is the log of the nominal exchange rate, which is defined as the dollar price of one peso.

Three assets are traded in this economy: a nominal one-period bond issued by the domestic central bank with return \( i_1 \), a nominal one-period bond issued by the foreign central bank with return \( i_1^* \), and a risk-free technology with real return \( r \). The payoffs of all assets are realized in period two. I assume that the domestic central bank credibly commits to a constant domestic price level in all periods so that the interest rate on dollar bonds \( i_1 \) is equal to \( r \). Without loss of generality, this constant price level is normalized so that \( p_1 = p_2 = 0 \), which implies that the log-linearized real return on foreign bonds is equal to \( -p_2^* - e_1 + i_1^* = e_2 - e_1 + i_1^* \). In the foreign country, the interest rate in period one is equal to \( i_1^* = \mu_1 + r \), where \( \mu_1 \in \mathbb{R} \). All investors observe \( \mu_1 \) publicly in period one.

The economy is populated by a continuum of investors indexed by \( i \in [0, 1] \). Each investor is endowed with real wealth \( w_{i1} \in \mathbb{R} \) at the beginning of period one and has negative exponential utility over her consumption in period two. Because the log-linearized excess return of peso bonds is equal to \( e_2 - e_1 + i_1^* - i_1 = e_2 - e_1 + \mu_1 \), the maximization problem solved by each investor \( i \) is given by

\[
\max_{b_{i1} \in \mathbb{R}} -E_{i1} \left[ e^{-\gamma c_{i2}} \right], \quad \text{subject to} \quad c_{i2} = (1 + i_1)w_{i1} + (e_2 - e_1 + \mu_1)b_{i1}, \quad (2.1)
\]

where \( b_{i1} \) is the dollar amount of investor \( i \)'s purchases of peso bonds in period one, \( c_{i2} \) is the quantity of the economy’s only good consumed by investor \( i \) in period two, \( \gamma > 0 \) is the coefficient of absolute risk aversion, and \( E_{i1}[\cdot] \) denotes the conditional expectation with respect to the information set of investor \( i \) in period one. In addition to the investors, the economy also consists of a mass of noise traders that purchases \( \xi_1 \) dollars worth of peso bonds in period one, where \( \xi_1 \sim N(0, \sigma_{\xi_1}^2) \). The net supply of peso bonds is equal to zero.

The foreign central bank complements its interest rate policy in period one with a foreign exchange intervention in which it purchases \( \nu_1 \in \mathbb{R} \) dollars worth of peso bonds. Clearly, this intervention affects the exchange rate in period one since it changes the total demand for peso bonds in that period, but I also assume that this intervention affects the exchange rate...
rate in period two. In particular, I assume that

$$e_2 = \theta \mu_2 + \theta \nu_1 + \kappa_2,$$  \hspace{1cm} (2.2)

where $\theta \mu_2 \in \mathbb{R}$ represents exchange rate fundamentals in period two, $\kappa_2 \sim N(0, \sigma^2_\kappa)$ is a shock to the exchange rate in period two, and $\theta \nu > 0$ is a constant that measures the effect of the central bank intervention $\nu_1$ on the exchange rate in period two. The form of this expression for $e_2$ is common knowledge among all investors.

It is important to emphasize that an expression for the future value of the exchange rate similar to equation (2.2) appears endogenously in equation (4.15) from the infinite-horizon extension of this model presented in Section 4. In that setup, $\theta \mu_2$ is equal to the discounted sum of future interest rate spreads, with the discount factor determined by the structure of the foreign central bank’s interest rate rule, and the constant $\theta \nu$ is proportional to both the risk-aversion of investors and the degree of persistence of central bank interventions over time. \footnote{In a standard dynamic monetary model, $\mu_2$ is equal to the sum of future values of the foreign money supply (relative to the domestic, constant money supply) discounted by some function of the semi-elasticity of money demand with respect to the interest rate.} The fact that equation (2.2) appears endogenously in the infinite-horizon setup of Section 4 is the primary motivation for the two-period model presented in this section.

Although the constant $\theta \nu$ is interpreted in the infinite-horizon model as a measure of only the direct effect of interventions through persistent changes in the available supply of peso bonds, this constant is intended to capture more than just this direct effect. In particular, a higher value of $\theta \nu$ may also represent a partial correlation between fundamentals in period two $\mu_2$ and the foreign central bank’s intervention in period one $\nu_1$. This would be the case if, for example, $\mu_2 = \mu + \beta \mu \nu_1$, where $\mu \in \mathbb{R}$ represents the part of fundamentals that is not correlated with interventions and $\beta \mu \in \mathbb{R}$ measures the correlation between fundamentals and interventions. \footnote{I allow for $\beta \mu < 0$ since it is sometimes the case that interventions are negatively correlated with future fundamentals or policy, as shown by Kaminsky and Lewis (1996).} Under this assumption, the exchange rate in period two is given by

$$e_2 = \theta \mu_2 + \theta \nu_1 + \kappa_2 = \theta \mu_2 + (\theta \nu + \theta \mu \beta \mu) \nu_1 + \kappa_2,$$  \hspace{1cm} (2.3)

so the setup is equivalent to a version of this benchmark model in which $\mu_2$ represents the part of fundamentals that is not correlated with interventions and $\theta \nu$ is either smaller or larger depending on the value of $\theta \mu \beta \mu$. A second possibility is that $\nu_1 = \nu + \beta \nu \mu_2$, where $\nu \in \mathbb{R}$ represents the part of interventions that is not correlated with fundamentals. Under this assumption, $\mu_2 = -\beta \nu^{-1} \nu + \beta \nu^{-1} \nu_1$ so that $-\beta \nu^{-1} \nu$ represents the part of fundamentals that is not correlated with interventions and the setup is equivalent to the previous example’s
setup and hence also equivalent to this benchmark model.

Throughout this paper, I make the restrictive assumption that interventions are independent of future fundamentals in order to keep the analysis tractable. As the previous examples demonstrate, this assumption is implicitly relaxed by adjusting the value of the constant \( \theta \nu \) in equation (2.2). This constant represents not just the persistent effects of central bank interventions on the exchange rate but also the information about fundamentals that is embedded in those interventions. In this manner, the model captures the reality that foreign exchange interventions are important sources of information about future fundamentals and policies as emphasized by Bhattacharya and Weller (1997), Dominguez and Frankel (1993a), Mussa (1981), Vitale (1999), and the whole literature about the signalling hypothesis.

I assume in this benchmark model that investors have uninformative priors for \( \mu_2 \) and \( \nu_1 \). Each investor \( i \) receives private signals \( x_i = \mu_2 + \epsilon_i \) and \( y_i = \nu_1 + \eta_i \) in period one, where \( \epsilon_i \sim N(0, \sigma_\epsilon^2) \), \( \eta_i \sim N(0, \sigma_\eta^2) \), \( \epsilon_i \) and \( \eta_i \) are independent, and all noise terms are independent across investors. In equilibrium, investors rationally combine their private signals with the information about both \( \mu_2 \) and \( \nu_1 \) that is present in the exchange rate in period one.

Let \( F \) denote the information set consisting of all common public information together with \( \mu_2, \nu_1, \) and \( e_1 \). The aggregate demand for peso bonds by the investors is equal to the average demand of the investors and is denoted by \( B_1 = E[b_{i1} \mid F] \). It follows that the total demand for peso bonds in period one is equal to \( B_1 + \xi_1 + \nu_1 \). Let \( \overline{E}_1[\cdot] = E[E_{i1}[\cdot] \mid F] \) denote the average expectation of investors in period one, and let \( \overline{\text{Var}}_{i1}[\cdot] \) denote the conditional variance with respect to the information set of investor \( i \) in period one and \( \overline{\text{Var}}_{1}[\cdot] = E[\text{Var}_{i1}[\cdot] \mid F] \) the average conditional variance of investors in period one. Finally, let \( \sigma_1^2 = \overline{\text{Var}}_{1}[e_2] \) denote the average conditional variance of the exchange rate in period two.

All proofs from this section are in Appendix A.

**Definition 2.1.** An equilibrium of this economy is a function for the exchange rate in period one \( e_1 \), such that (i) the demand for peso bonds by each investor \( b_{i1} \) solves the maximization problem (2.1), where investor \( i \)'s information set consists of all common public information together with \( x_i, y_i, \epsilon_i, \) and, if the foreign central bank announces its intervention, \( \nu_1 \) as well; (ii) the peso bond market clears: \( B_1 + \xi_1 + \nu_1 = 0 \); (iii) the exchange rate is a linear function of the demand for peso bonds by noise traders \( \xi_1 \), the foreign central bank's intervention \( \nu_1 \), and the interest rate parameters \( \mu_1 \) and \( \mu_2 \).

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11 An alternative but equivalent assumption is that investors’ priors for \( \mu_2 \) and \( \nu_1 \) are uniform over \( \mathbb{R} \).

12 This notation is commonly written \( B_1 = \int_0^1 b_{i1} \, di \), with the understanding that this integral is equal to the average across investors. As detailed by Judd (1985), however, the law of large numbers often does not hold for a continuum of random variables. I avoid this technical issue by explicitly defining continuums of this kind as the expected value of an individual investor’s demand conditional on observing the parameters of the model.
In this definition of equilibrium, the foreign central bank’s transparency policy does not convey any information about the parameters of the model, an assumption that is essential in order to keep the analysis in this model tractable. I relax this assumption in Section 3 and investigate how signalling affects the equilibrium predictions of this model.

**Theorem 2.2.** The unique equilibrium exchange rate in period one is given by

\[ e_1 = \mu_1 + \theta_\mu \mu_2 + (\theta_\nu + \gamma \sigma^2_1) \nu_1 + \lambda \xi_1, \]  

where \( \lambda \) and \( \sigma^2_1 \) are given by the solution to

\[ \lambda = \frac{\lambda \theta_\mu^2 \sigma^2_\epsilon + \lambda \theta_\nu (\theta_\nu + \gamma \sigma^2_1) \sigma^2_\eta + \gamma \sigma^2_2}{\theta_\mu^2 \sigma^2_\epsilon + (\theta_\nu + \gamma \sigma^2_1)^2 \sigma^2_\eta + \lambda^2 \sigma^2_\xi}, \]  

\[ \sigma^2_1 = \theta_\mu^2 \sigma^2_\epsilon + \theta_\nu^2 \sigma^2_\eta + \sigma^2_\kappa - \frac{(\theta_\mu^2 \sigma^2_\epsilon + \theta_\nu (\theta_\nu + \gamma \sigma^2_1) \sigma^2_\eta)^2}{\theta_\mu^2 \sigma^2_\epsilon + (\theta_\nu + \gamma \sigma^2_1)^2 \sigma^2_\eta + \lambda^2 \sigma^2_\xi}. \]  

As detailed in the appendix, the system of equations that determines \( \lambda \) and \( \sigma^2_1 \) jointly is nonlinear and of too high an order to solve analytically. It is not difficult to show, however, that there exists a unique real solution. A number of important properties of this unique equilibrium stand out. First, the effects of noise traders on the exchange rate extend beyond the standard demand channel since \( \lambda > \gamma \sigma^2_1 \). In models with rational expectations and heterogeneously informed investors such as this, the equilibrium exchange rate is a publicly observed signal of both future exchange rate fundamentals \( \mu_2 \) and the foreign central bank’s intervention \( \nu_1 \). Noise traders drive the exchange rate away from its fundamental value by altering the total demand for peso bonds, which then biases the average expectations of investors about both \( \mu_2 \) and \( \nu_1 \). The difference between \( \lambda \) and \( \gamma \sigma^2_1 \) captures this extra effect and is exactly equal to the bias in investors’ expectations.

A sketch of the proof of Theorem 2.2 illustrates this point. Market clearing implies that the exchange rate in period one is of the form

\[ e_1 = \mu_1 + \theta_\mu \mu_2 + (\theta_\nu + \gamma \sigma^2_1) \nu_1 + \xi_1. \]  

Solving for the equilibrium requires evaluating the average expectations \( E_1[\mu_2] \) and \( E_1[\nu_1] \) and determining how much weight each one places on the noise term \( \xi_1 \). The sum of these weights makes up the bias of investors’ average expectations of \( \mu_2 \) and \( \nu_1 \). Evaluating these expectations is accomplished using standard Bayesian formulas. In particular, these formulas imply that for each investor \( i \),

\[ E_{i1}[\mu_2] = x_i + \frac{\text{Cov}_{i}[\mu_2,e_2]}{\text{Var}[e_1]} (e_1 - E_i[e_1]), \]  

13
and

\[ E_{i1}[\nu_1] = y_i + \frac{\text{Cov}_i[\nu_1, e_2]}{\text{Var}_i[e_1]} (e_1 - E_i[e_1]), \]  

(2.8)

where \( E_i[\cdot], \text{Var}_i[\cdot], \) and \( \text{Cov}_i[\cdot, \cdot] \) denote, respectively, the expected value, variance, and covariance with respect to the information set consisting only of \( \mu_1 \) and the private signals \( x_i \) and \( y_i \). The exchange rate in period one is of the form \( e_1 = \mu_1 + \theta_\mu \mu_2 + (\theta_\nu + \gamma \sigma_1^2) \nu_1 + \lambda \xi_1 \), so it follows that \( e_1 - E_i[e_1] = \theta_\mu (\mu_2 - x_i) + (\theta_\nu + \gamma \sigma_1^2)(\nu_1 - y_i) + \lambda \xi_1 \) and hence that \( e_1 - E_i[e_1] = \lambda \xi_1 \) as well. This last equality implies that

\[ \bar{E}_1[\mu_2] = \mu_2 + \frac{\text{Cov}_i[\mu_2, e_2]}{\text{Var}_i[e_1]} \lambda \xi_1, \]  

(2.9)

\[ \bar{E}_1[\nu_1] = \nu_1 + \frac{\text{Cov}_i[\nu_1, e_2]}{\text{Var}_i[e_1]} \lambda \xi_1, \]  

(2.10)

so that the bias of investors’ average expectations is equal to \( \theta_\mu \) times the last term in equation (2.9) plus \( \theta_\nu \) times the last term in equation (2.10). These terms reflect the fact that the exchange rate in period one contains information about \( \mu_2 \) and \( \nu_1 \) (since both \( \text{Cov}_i[\mu_2, e_2] \) and \( \text{Cov}_i[\nu_1, e_2] \) are nonzero) and thus its value contributes to equilibrium expectations.

For most parameterizations of this model, \( \lambda \) is increasing in both the variance of investors’ private signals about future fundamentals \( \sigma_\epsilon \) and the effect of the central bank’s intervention (both directly and indirectly) on the future exchange rate \( \theta_\nu \). An increase in the unpredictability of noise traders \( \sigma_\xi \) can either increase or decrease the value of \( \lambda \), although the magnitude of this response tends to be significantly smaller than the response to an increase in \( \sigma_\epsilon \) or \( \theta_\nu \).

The fact that \( \lambda \) is decreasing in the precision of investors’ private signals about \( \mu_2 \) implies that the precision of the exchange rate as a public signal of \( \mu_2 \) is increasing in this quantity. This follows because the term \( \lambda \) multiplies \( \xi_1 \) in equation (2.5), so a decrease in \( \lambda \) implies a decrease in the variance of the exchange rate assuming that \( \sigma_\xi \) remains unchanged. Intuitively, this increase in precision is a consequence of investors with better private information trading more aggressively and thus moving the value of the exchange rate closer to its fundamental value. This property is examined by Angeletos and Werning (2006) in a model of asset-pricing with both rational expectations and heterogeneous private information similar to this one.

The effect of an increase in the variance of investors’ private signals about the foreign central bank’s intervention in period one \( \sigma_\eta \) are the most interesting. As I shall prove in Theorem 2.4 below, if the parameter \( \lambda \) is greater than the corresponding parameter when the central bank makes a public announcement about \( \nu_1 \) (denoted by \( \tilde{\lambda} \)), then this must be the case for all \( \sigma_\eta^2 > 0 \). In other words, if the bias in investors’ average expectations is
larger (smaller) with transparency than without transparency, then this bias must be larger (smaller) regardless of the precision of investors’ signals about central bank interventions. I also find that \( \lambda \) is increasing in \( \sigma_\eta^2 \) whenever \( \lambda > \tilde{\lambda} \) and decreasing in \( \sigma_\eta^2 \) whenever \( \lambda < \tilde{\lambda} \). This implies that decreases in the precision of investors’ signals about interventions always magnify the difference in exchange rate misalignment with and without transparency. Interestingly, this property continues to hold in the fuller dynamic models of Section 4 as well.

In order to examine the effects of transparency on the price of the peso, it is necessary to derive a result similar to Theorem 2.2 for the case in which the central bank credibly and publicly announces the value of \( \nu_1 \) in period one. The simplicity of the setup in this section makes it possible to isolate the truth-telling and signal-precision effects of transparency and determine exactly how each affects the equilibrium exchange rate. Let \( \tilde{e}_1 \) denote the exchange rate in period one if the central bank truthfully announces the value of \( \nu_1 \) to the investors.

**Theorem 2.3.** If the foreign central bank credibly and publicly announces the value of \( \nu_1 \) in period one, then the unique equilibrium exchange rate is given by

\[
\tilde{e}_1 = \mu_1 + \theta_\nu \mu_2 + (\theta_\nu + \gamma \tilde{\sigma}_1^2) \nu_1 + \tilde{\lambda}_1 \xi_1,
\]

where \( \tilde{\lambda} \) and \( \tilde{\sigma}_1^2 \) are given by the solution to

\[
\tilde{\lambda} = \frac{\tilde{\lambda} \theta^2 \tilde{\sigma}_1^2}{\theta^2 \tilde{\sigma}_1^2 + \tilde{\lambda}^2 \tilde{\sigma}_\xi^2} + \gamma \tilde{\sigma}_1^2,
\]

\[
\tilde{\sigma}_1^2 = \theta^2 \tilde{\sigma}_1^2 + \sigma_\kappa^2 - \frac{\theta_\mu^4 \tilde{\sigma}_\kappa^4}{\theta_\mu^2 \tilde{\sigma}_\xi^2 + \tilde{\lambda} \tilde{\sigma}_\xi^2}.
\]

In contrast to the system of equations from Theorem 2.2, this system of equations is simple enough to solve analytically. In this unique equilibrium with transparency, the effects of noise traders on the exchange rate again extend beyond the standard demand channel and bias investors’ average expectations of fundamentals. As in the equilibrium with no transparency, the difference between \( \tilde{\lambda} \) and \( \gamma \tilde{\sigma}_1^2 \) captures this extra effect and is equal to the bias of investors’ expectations. The final step is to compare the values of the parameters \( \lambda \) and \( \tilde{\lambda} \) and examine when and how transparency can in fact worsen exchange rate misalignment.
Theorem 2.4. There exists a unique threshold $\hat{\theta}_\nu > 0$ such that $\bar{\lambda} > \lambda$ if and only if $\theta_\nu < \hat{\theta}_\nu$. This threshold is given by $\hat{\theta}_\nu = \bar{\lambda} - \gamma \bar{\sigma}_1^2$, and satisfies

$$\lim_{\sigma_\xi \to 0} \hat{\theta}_\nu = \infty, \quad \lim_{\sigma_\xi \to \infty} \hat{\theta}_\nu = 0,$$

$$\lim_{\sigma_\nu \to 0} \hat{\theta}_\nu = \lim_{\theta_\nu \to 0} \hat{\theta}_\nu = 0, \quad \lim_{\sigma_\nu \to \infty} \hat{\theta}_\nu = \lim_{\theta_\nu \to \infty} \hat{\theta}_\nu = \frac{1}{\gamma \sigma_\xi^2},$$

$$\lim_{\sigma_\nu \to 0} \hat{\theta}_\nu = \frac{\gamma^2 \theta_\mu^2 \bar{\sigma}_1^2}{1 + \gamma^2 \theta_\mu^2 \sigma_\xi^2 \bar{\sigma}_1^2}, \quad \lim_{\sigma_\nu \to \infty} \hat{\theta}_\nu = 0,$$

$$\lim_{\gamma \to 0} \hat{\theta}_\nu = 0, \quad \lim_{\gamma \to \infty} \hat{\theta}_\nu = 0.$$

Corollary 2.5. If $\theta_\nu < \hat{\theta}_\nu$, then there exists a threshold $\hat{\xi} \in \mathbb{R}$ such that $\bar{\epsilon}_1 > e_1$ if and only if $\xi_1 \geq \hat{\xi}$.

Theorem 2.4 and Corollary 2.5 together present the main results of all of the models and extensions presented in this paper. They imply that if a currency is undervalued relative to fundamentals, then a policy of transparency and a central bank announcement about the size of its interventions will often further depreciate that currency. In these situations, a bank that wishes to push back against an undervalued and falling currency will benefit more from an ambiguous, secretive policy rather than a transparent one.

Theorem 2.4 is described by a threshold for the parameter that captures the extent of information revelation $\theta_\nu$. The theorem implies that it is only if the information revealed by the foreign central bank is sufficiently incomplete ($\theta_\nu < \hat{\theta}$) that exchange rate misalignment may be magnified by central bank transparency. Recall the two distinct effects of transparency: the truth-telling effect, which reduces currency mispricing, and the signal-precision effect, which magnifies currency mispricing. The truth-telling effect refers to the fact that any quantities the foreign central bank reveals to investors eliminate the role of the exchange rate as a signal of those quantities. The signal-precision effect refers to the fact that any quantities the bank reveals to investors also increase the precision of the exchange rate as a signal of other, still-unknown quantities. Theorem 2.4 states that it is precisely when information revelation is sufficiently incomplete that the truth-telling effect of transparency is small relative to the signal-precision effect of transparency. If information revelation is sufficiently complete ($\theta_\nu > \hat{\theta}$), on the other hand, Theorem 2.4 implies that the truth-telling effect will exceed the signal-precision effect and transparency will lessen exchange rate misalignment. This behavior is shown graphically in Figures 1, 2, and 3. The three different parameterizations are chosen both to match the parameterizations of the more realistic and complete dynamic models of Section 4 and to demonstrate how the relative values of $\lambda$ and
\[ \tilde{\lambda} \] change with the parameters of this model.

A more detailed discussion of the truth-telling and signal-precision effects of transparency is warranted. If the foreign central bank credibly and truthfully announces the value of \( \nu_1 \) in period one, then investors all learn this value perfectly and they no longer form expectations of it as part of their expectations of the fundamental value of the peso. This implies that if the demand of noise traders is negative so that the exchange rate is under-valued and investors’ expectations of fundamentals are biased downwards, then this public announcement eliminates the investors’ negative bias about \( \nu_1 \) and thus raises their demand for peso bonds and hence the value of the exchange rate. More precisely, the exchange rates with and without transparency are given by

\[
\tilde{e}_1 = \mu_1 + \theta_\mu \overline{E}_1[\mu_2] + \theta_\nu \nu_1 + \gamma \sigma_1^2 (\nu_1 + \xi_1)
\]

and

\[
e_1 = \mu_1 + \theta_\mu \overline{E}_1[\mu_2] + \theta_\nu \overline{E}_1[\nu_1] + \gamma \sigma_1^2 (\nu_1 + \xi_1),
\]

respectively, so it follows that expectations of \( \nu_1 \) are not part of the equilibrium exchange rate after a central bank announcement. This difference amounts to the smaller numerator of \( \tilde{\lambda} - \gamma \sigma_1^2 \) (the bias in the average expectations of investors with transparency) relative to \( \lambda - \gamma \sigma_1^2 \) (the bias in the average expectations of investors without transparency). According to equations (2.5) and (2.12), the difference between these numerators is equal to

\[
(\lambda - \tilde{\lambda}) \theta_\mu^2 \sigma_\epsilon^2 + \lambda \theta_\nu (\theta_\nu + \gamma \sigma_1^2) \sigma_\eta^2,
\]

indicating that the truth-telling effect of transparency is increasing in the extent of informa-
Figure 2: The value of $\lambda$ (dashed line) and $\tilde{\lambda}$ (solid line) as the level of information revelation $\theta_\nu$ increases. ($\sigma_\epsilon = 0.05$, $\sigma_\eta = 0.15$, $\sigma_\xi = 0.15$, $\sigma_\kappa = 0.1$, $\gamma = 5$, $\theta_\mu = 2$)

Figure 3: The value of $\lambda$ (dashed line) and $\tilde{\lambda}$ (solid line) as the level of information revelation $\theta_\nu$ increases. ($\sigma_\epsilon = 0.04$, $\sigma_\eta = 0.25$, $\sigma_\xi = 0.35$, $\sigma_\kappa = 0.1$, $\gamma = 5$, $\theta_\mu = 2$)
tion revelation $\theta$.

The second difference, which corresponds to the signal-precision effect of transparency, is that the precision of the exchange rate as a signal of the value of $\mu_2$ improves when the foreign central bank makes an announcement. This causes investors to weigh the exchange rate more heavily in their expectations of the fundamental value of the peso, which implies that negatively biased expectations about $\mu_2$ are magnified and an undervalued exchange rate declines even further. More precisely, the variance of the exchange rate in period one conditional on observing $\mu_1$, $x$, and $y$, is equal to $\theta^2 \sigma^2_\epsilon + (\theta_\nu + \gamma \sigma^2_1) \sigma^2_\eta + \lambda^2 \sigma^2_\zeta$ while, conditional on observing also $\nu_1$, this variance is equal to $\theta^2 \sigma^2_\epsilon + \tilde{\lambda}^2 \sigma^2_\zeta$. These variances determine how much weight Bayesian investors place on $e_1$ when forming their expectations about $e_2$, and the fact that the term $(\theta_\nu + \gamma \sigma^2_1) \sigma^2_\eta$ goes away after a central bank announcement represents the improved precision of this exchange rate signal. This difference amounts to the smaller denominator of $\tilde{\lambda} - \gamma \tilde{\sigma}^2_1$ relative to $\lambda - \gamma \sigma^2_1$. According to equations (2.5) and (2.12), the difference between these denominators is equal to

$$ (\lambda^2 - \tilde{\lambda}^2) \sigma^2_\zeta + (\theta_\nu + \gamma \sigma^2_1) \sigma^2_\eta. \quad (2.15) $$

It follows that the signal-precision effect of transparency grows relative to the truth-telling effect of transparency as this quantity grows relative to the quantity from equation (2.14) above.

Consider the special case in which $\theta_\nu = 0$. This parameterization of the model describes the case in which the foreign central bank’s intervention in period one neither conveys any information about future fundamentals nor has any direct effect on the exchange rate in period two. According to equations (2.14) and (2.15), if $\theta_\nu = 0$, then the truth-telling effect of transparency is smaller than the signal-precision effect of transparency. Theorem 2.4 confirms that this in indeed the case, since the threshold $\hat{\theta}_\nu$ is always positive and thus $\theta_\nu < \hat{\theta}_\nu$ is not possible whenever $\theta_\nu = 0$. In this special case, the bank’s intervention introduces only noise into the exchange rate in period one. There is no truth-telling effect since revealing the value of $\nu_1$ tells investors nothing about fundamentals, but there is a signal-precision effect since an announcement allows investors to filter out some of the noise in the exchange rate in period one.

As the parameter $\theta_\nu$ grows larger, the truth-telling effect grows relative to the signal precision effect. This can be seen from equations (2.14) and (2.15) since $\lambda$ increases in $\theta_\nu$ faster than $\gamma \sigma^2_1$ so that $\lambda \theta_\nu(\theta_\nu + \gamma \sigma^2_1)$ increases in $\theta_\nu$ faster than $(\theta_\nu + \gamma \sigma^2_1)^2$. The analysis so far ignores the effect that transparency has on the conditional variance of the exchange rate in period two (the value of $\sigma^2_1$ versus the value of $\tilde{\sigma}^2_1$), but Theorem 2.4 shows that
these effects generally do not negate the intuition about truth-telling and signal-precision described in the previous paragraphs.

Another instructive special case to consider is the case in which the foreign central bank’s intervention in period one fully reveals future exchange rate fundamentals. In this case, the setup is equivalent to a version of this benchmark model in which \( \theta_\mu = 0 \) since there is no part of fundamentals that is not correlated with interventions. Much of the early literature about sterilized foreign exchange interventions, such as Dominguez and Frankel (1993a) and Mussa (1981), posits an environment similar to this special case when arguing that transparency is desirable and effectively reduces exchange rate misalignment. Theorem 2.4 demonstrates that this benchmark model is consistent with these authors’ analysis, since \( \hat{\theta}_\nu \to 0 \) as \( \theta_\mu \to 0 \) and \( \theta_\nu \) is nonnegative by assumption. It is important to emphasize, however, that as the information about future fundamentals that is embedded in the central bank’s intervention declines, the benefits of transparency become more tenuous.

Corollary 2.5 follows from Theorem 2.4 since the exchange rate in period one is always given by an approximate expression of the form \( f - r_p + \lambda \xi_1 \) (with \( \hat{\lambda} \) replacing \( \lambda \) in the case of a central bank announcement), where \( f \) is equal to fundamentals and \( r_p \) is equal to the peso-bond risk premium. It is apparent, then, that a higher value of \( \hat{\lambda} \) relative to \( \lambda \) implies a lower value of \( \hat{e}_1 \) relative to \( e_1 \) for sufficiently low demand by noise traders \( \xi_1 \). In other words, if \( \hat{\lambda} > \lambda \), then transparency may exacerbate any existing misalignment between the exchange rate and fundamentals.

An important implication of Theorem 2.4 is that whether or not transparency magnifies exchange rate misalignment does not depend on the variance of investors’ private signals about central bank interventions \( \sigma_\eta \). This follows because the threshold \( \hat{\theta}_\nu \) is only a function of the exchange rate parameters \( \hat{\lambda} \) and \( \hat{\sigma}_1^2 \), which do not depend on \( \sigma_\eta \) since they correspond to a central bank policy of transparency (and hence \( \sigma_\eta = 0 \)). As mentioned earlier, an important implication of this fact is that changes in the precision of investors’ private signals of \( \nu_1 \) cannot swing the balance between the truth-telling and signal-precision effects of transparency. More precisely, if \( \lambda > \hat{\lambda} \) or \( \lambda < \hat{\lambda} \), then this relationship must hold for all \( \sigma_\eta^2 > 0 \).

In fact, I find that increases in \( \sigma_\eta^2 \) tend to magnify the difference between the parameters \( \lambda \) and \( \hat{\lambda} \). Because \( \hat{\lambda} \) does not change as \( \sigma_\eta^2 \) increases, this implies that \( \lambda \) is increasing in \( \sigma_\eta^2 \) whenever \( \lambda > \hat{\lambda} \) (and hence \( \theta_\nu > \hat{\theta}_\nu \)) and decreasing in \( \sigma_\eta^2 \) whenever \( \lambda < \hat{\lambda} \) (and hence \( \theta_\nu < \hat{\theta}_\nu \)). These properties are shown in Figure 4. This result is significant because it implies that all of these results about transparency and exchange rate misalignment apply even when the foreign central reduces rather than eliminates the variance of investors’ private signals about interventions. In reality, rather than choosing between full transparency and full ambiguity, central banks chose from a wide array of different policies that are distinguished
by their overall effect on the level of transparency.

Theorem 2.4 also describes how the threshold of information revelation $\hat{\theta}_\nu$ is affected by several of the model’s exogenous parameters. The most important of these comparative statics is that $\hat{\theta}_\nu \to \infty$ as the unpredictability of noise traders $\sigma_\xi$ disappears. This result remains intact even after the model is extended to an infinite horizon in Section 4, and it implies that as the precision of the exchange rate as a signal of fundamentals grows large, transparency magnifies currency mispricing even after extensive information revelation on the part of the foreign central bank. Intuitively, if the exchange rate is a very precise signal of fundamentals after the central bank makes an announcement (the announcement is essential, otherwise $\nu_1$ keeps this signal relatively imprecise), then the exchange rate will be very heavily weighted in investors’ expectations after this announcement so that there is a large signal-precision effect of transparency. This result is shown graphically in Figure 5.

The threshold of information revelation $\hat{\theta}_\nu$ is also increasing in investors’ private uncertainty about the value of future exchange rate fundamentals. If investors’ private signals are imprecise, then they rationally weigh the exchange rate heavily in their expectations about fundamentals $\mu_2$. This implies that any central bank policy that affects the precision of the exchange rate as a signal of fundamentals also has a large effect on the expectations of investors, which amounts to an increase in the relative importance of the signal-precision effect of transparency. I find that the extent of this effect in the limit as $\sigma_\epsilon \to \infty$ depends inversely on the predictability of noise traders. This result is shown graphically in Figure 6.
Once this model is extended to an infinite horizon, however, this comparative static result changes. In particular, I find that the difference between $\lambda$ and $\tilde{\lambda}$ is largest for intermediate values of $\sigma_\epsilon$ in the infinite-horizon model of Section 4.

This benchmark model formalizes the intuitive but vague justifications that central banks often provide for their ambiguous policies. Theorem 2.4 shows that banks are right to worry that unsuccessful transparent interventions might undermine the market’s confidence in their currencies, since transparency makes it easier for investors with different beliefs to learn each others’ information and hence for pessimism to intensify and spread. In other words, if investors observe a depreciated currency together with an extensive intervention, then they conclude that fundamentals are worse than they previously thought. This reasoning implies that both Mexico and Russia would have likely benefited from more ambiguous intervention policies during the financial crisis, as they eventually chose.

The benchmark model provides two key insights that grow and guide this intuition. First, it is only if the information that central banks reveal is sufficiently partial that transparency can magnify exchange rate misalignment. If central banks can credibly reveal enough information about fundamentals, then the effects of transparency are more standard and stabilizing. Second, if transparency does magnify currency mispricing, then ambiguity appreciates only an undervalued currency. This observation highlights the importance of the information advantage of central banks. In a world with rational expectations, it is only if a currency is undervalued that ambiguity can increase the effectiveness of an intervention designed to
appreciate that currency. If this model is interpreted literally, then it is natural to assume that the foreign central bank has more information about fundamentals than the investors since fundamentals are entirely determined by the bank’s policies. In a more realistic and complete model of exchange rate determination, however, there are many other components of exchange rate fundamentals that central banks are not more informed about.

Finally, I should emphasize that this model does not imply that a transparent intervention policy is always worse than a more ambiguous intervention policy. In fact, a transparent intervention policy is often better even if the conditions of Theorem 2.4 hold and $\tilde{\lambda} > \lambda$. This is because central bank policy is an important determinant of currency risk-premia and transparency can be an effective way to reduce these risk-premia. The purpose of my analysis is to examine and emphasize a mechanism by which transparency can in fact exacerbate exchange rate misalignment, rather than to capture all of the factors that affect exchange rates. While this mechanism is likely to be very important during times of great uncertainty about policy and fundamentals, it is unlikely to be as important during more normal times.
3 Policy as a Signal of Fundamentals

All of the results I have presented so far assume that either central bank interventions are independent of the underlying state of the economy or that investors are naive and unable to infer anything from the bank’s chosen policy of transparency. While this keeps the analysis tractable, it is an unrealistic assumption since there is plenty of evidence that central banks’ decisions whether or not to announce the size of their interventions are careful, highly strategic decisions. Rational investors are aware of this strategic element, and they use a bank’s chosen level of transparency to better infer the underlying state of the economy. In other words, central banks and investors play a Bayesian signalling game.

In this section, I relax this assumption and investigate how the benchmark model’s predictions are affected. I consider a Bayesian signalling game between the foreign central bank and the investors in which the central bank has a clear objective that investors are not naive about. With the example of a central bank defending a falling exchange rate in mind, I assume that the bank’s objective is to maximize the peso exchange rate. It is important to note, however, that all of this analysis is easily extended to a game in which the bank’s objective is to minimize the peso exchange rate. Furthermore, if the bank targets a publicly known value of the exchange rate, then the game that is played involves either the central bank maximizing the exchange rate—if the exchange rate is below the target—or the central bank minimizing the exchange rate—if the exchange rate is above the target. In either case, investors observe the value of the exchange rate relative to the target and are aware of the central bank’s desire to achieve either appreciation or depreciation.

This section’s main contribution is to construct a partially-separating Bayesian equilibrium in which the foreign central bank announces its intervention whenever the exchange rate is sufficiently overvalued. This equilibrium demonstrates that the previous results about central bank ambiguity reducing exchange rate misalignment are consistent with an environment in which policy choice is a signal to investors. Furthermore, the existence of a non-pooling equilibrium proves that self-fulfilling beliefs about the meaning of central bank transparency need not dwarf the effects I describe in the previous sections. In fact, self-fulfilling pooling equilibria often exist only together with highly unintuitive and implausible out-of-equilibrium beliefs.\footnote{The intuitive criterion of Cho and Kreps (1987) does not restrict the set of pooling equilibria in this game since the value of the central bank's policy is purely determined by the investors' interpretation of that policy. In other words, neither transparency nor ambiguity is ever strictly dominated.}

The Bayesian signalling game between the foreign central bank and investors takes place in the two-period setup of Section 2. As always, the foreign central bank chooses between two possible actions: either adopt a policy of transparency and announce the size of its
intervention in period one, or adopt a policy of ambiguity and do not announce anything. This decision is common public information, and investors rationally update their beliefs based on it.

A game of this kind together with a model that features asset-pricing under imperfect information presents many technical difficulties. Most significantly, investors’ beliefs about $\mu_2$ and $\nu_1$ are generally not normally distributed, a fact that makes it very difficult to characterize the investors’ aggregate demand for peso bonds and the equilibrium exchange rate. Indeed, investors’ utility functions are exponential, so if their beliefs about fundamentals are not normally distributed (which requires a normally distributed exchange rate in period one) then their demand is impossible to characterize analytically. If the demand of investors cannot be characterized, then the exchange rate in period one also cannot be characterized and it becomes very difficult to prove even simple equilibrium properties. Worse still, these technical difficulties do not go away even if exponential utility is replaced by mean-variance utility.\footnote{Although it may be possible to analytically characterize the investors’ demand for peso bonds with mean-variance utility, to characterize an equilibrium of this game one must also find a fixed point between investors’ beliefs about fundamentals and the exchange rate. Since investors’ beliefs are not normally distributed (beliefs are truncated in any partially-separating equilibrium), this is impossible to do analytically.}

I prove the existence of a partially-separating Bayesian equilibrium given a set of assumptions for the model’s primitives. One key to constructing this equilibrium is that absent any investor interpretation of transparency policy, the foreign central bank prefers one policy over another for some combination of fundamentals. This ensures that regardless of which policy is interpreted as a signal of currency overvaluation (a signal of currency overvaluation leads investors to reduce their demand and hence causes depreciation), the bank does not shun that policy in equilibrium. In this setup, a preference for one policy over another exists because the risk premium on peso bonds varies depending on both the conditional variance of the exchange rate in period two and the extent of central bank intervention (this alters the available supply of peso bonds). As long as different transparency policies imply different conditional variances, the central bank will never strictly prefer one policy over the other. More succinctly, if the exchange rate in period one is approximately given by

$$e_1 = \mu_1 + \theta_\mu \bar{E}_1[e_2] + \gamma \sigma_1^2 (\nu_1 + \xi_1), \quad (3.1)$$

then as long as the difference between $\bar{E}_1[e_2]$ with and without transparency is finite and $\sigma_1^2 \neq \tilde{\sigma}_1^2$, there will always be a nonempty set of fundamentals for which the central bank chooses each policy.

This also implies that self-fulfilling pooling equilibria often require highly unintuitive
out-of-equilibrium beliefs. Although large shifts in the exchange rate should be expected if central bank policy ever signals to investors that fundamentals are much different than what is implied by the value of the exchange rate, the preceding argument shows that for some range of fundamentals these shifts are less important than changes in the risk premium. Of course, this requires that the risk premium actually changes with the central bank’s transparency policy.

In the partially-separating equilibrium I construct, the bank makes an announcement only if the exchange rate is sufficiently overvalued in period one. The construction of this equilibrium is aided by a technical assumption that I make which ensures that less uncertainty about the exchange rate in period two reduces the risk premium on peso bonds and raises the peso exchange rate. Specifically, I assume that there is a fixed supply of peso bonds equal to \( S > 0 \) dollars and that the bank’s intervention \( \nu_1 \) is always less than this supply. This changes the risk premium term in equation (3.1) above to \((\theta \mu + \gamma \sigma_1^2)(\nu_1 - S)\) and ensures that this term is always positive.

**Assumption 3.1.** There is a positive net supply of peso bonds denoted by \( S > 0 \). The central bank’s intervention \( \nu_1 \) is bounded, so that \( |\nu_1| \leq \bar{\nu} < S \), and investors’ common prior for \( \nu_1 \) is uniform over the interval \([-\bar{\nu}, \bar{\nu}]\). The exchange rate in period two is exogenously given by

\[
e_2 = \theta_\mu \mu_2 + \theta_\nu (\nu_1 - S) + \kappa_2.
\]

Equation (3.2) generalizes equation (2.2) from Section 2 to include the fixed supply of peso bonds. Because an intervention in period one amounts simply to a change in the available supply of peso bonds, both \( \nu_1 \) and \( S \) multiply the same constant (with different signs) for the exchange rate in period two. Besides aiding with the technical details of Theorem 3.3 below, Assumption 3.1 better reflects the reality of a country for which transparency often reduces both the uncertainty and the risk premium of its assets. Indeed, a more realistic version of the benchmark model applied to risky assets certainly must assume that interventions are bounded and risk-premia are always positive (\( S > \nu_1 \)) and increasing in uncertainty.

**Definition 3.2.** A Bayesian equilibrium of this economy is a strategy for the foreign central bank and a function for the exchange rate in period one \( e_1 \), such that (i) the demand for peso bonds by each investor \( b_{i1} \) solves the maximization problem (2.1), where investor \( i \)'s information set consists of all common public information together with \( x_i, y_i, e_1 \), and, if the central bank announces its intervention policy, \( \nu_1 \) as well; (ii) the peso bond market clears: \( B_1 + \xi_1 + \nu_1 = S \); (iii) the exchange rate is a function of the central bank’s transparency policy, the demand for peso bonds by noise traders \( \xi_1 \), the supply of peso bonds \( S \), the
central bank’s intervention $\nu_1$, and the interest rate parameters $\mu_1$ and $\mu_2$; (iv) the central bank chooses its transparency policy so that the value of the exchange rate $e_1$ is maximized.

All expectations and variances in this game are functions of the bank’s policy choice. In order to emphasize this point, the conditional expectations with respect to the information set of investor $i$ in period one with and without transparency are denoted by $E_{i1}(T)[\cdot]$ and $E_{i1}(N)[\cdot]$, respectively. The conditional variances with respect to the information set of investor $i$ in period one with and without transparency are denoted by $\text{Var}_{i1}(T)[\cdot]$ and $\text{Var}_{i1}(N)[\cdot]$, respectively.

**Theorem 3.3.** There exist bounds $\hat{S}, \hat{\nu}, \hat{\sigma}_\xi > 0$ such that if $S \geq \hat{S}, \bar{\nu} \geq \hat{\nu},$ and $\sigma_\xi \leq \hat{\sigma}_\xi$, then there exists a partially-separating Bayesian equilibrium in which the foreign central bank announces the size of its intervention if and only if $\xi_1 \geq \hat{\xi}(\nu_1)$. In this equilibrium, the threshold function $\hat{\xi}(\nu_1)$ is positive and decreasing in $\nu_1$.

The proof of Theorem 3.3 is in Appendix A. The theorem states that there exists a partially-separating equilibrium in which the foreign central bank chooses a transparent policy if the exchange rate is sufficiently overvalued relative to fundamentals. Although rational investors infer that this policy choice is a sign of an overvalued currency and adjust their beliefs accordingly, the central bank still prefers to be transparent because it reduces the unpredictability of the exchange rate in period two and therefore lowers the risk premium on peso bonds (and thus raises the peso exchange rate). This is an important result because it demonstrates that the benchmark model’s predictions about central bank ambiguity reducing exchange rate misalignment are not overturned once signalling is introduced into the model.

Theorem 3.3 requires that the demand of noise traders be highly predictable (low variance). This ensures that investors’ beliefs about fundamentals are approximately normally distributed despite the fact that beliefs about $\xi_1$ are truncated above or below depending upon the central bank’s choice of policy. Without approximate normality, it is impossible to analytically characterize the equilibrium exchange rate, as mentioned earlier. If the exchange rate cannot be characterized in this way, even by approximation, then it is impossible to compare the value of the exchange rate under different transparency policies.

To better understand the role of this assumption about $\sigma_\xi$, consider a simple assumption. Let $f \in \mathbb{R}$ represent fundamentals and suppose that each investor $i$ observes both $f_i = f + \epsilon_i$ and the exchange rate in period one. In this simplified example, a central bank announcement reveals to investors that $\xi \geq \hat{\xi} > 0$. Suppose that $\tilde{e}_1 = f + \hat{\lambda}(\xi - \hat{\xi})$. This means that the distribution of $f$ conditional on the information of investor $i$ is truncated normal with mean $f_i + \frac{\sigma^2}{\sigma^2 + \hat{\lambda}^2 \sigma^2_\xi} \left( \tilde{e}_1 - f_i + \hat{\lambda} \hat{\xi} \right)$, variance $\frac{\sigma^2 \hat{\lambda}^2 \sigma^2_\xi}{\sigma^2 + \hat{\lambda}^2 \sigma^2_\xi}$, and truncation $f < \tilde{e}_1$. By l’Hôpital’s rule, this
implies that in the aggregate

\[ E_1(T) \left[ e^{-f} \right] \rightarrow e^{-\hat{e}_1} = e^{-f - \hat{\lambda} (\xi - \hat{\xi})} \]  

(3.3)
as \( \sigma_\xi \rightarrow 0 \). If \( e_2 = f + \kappa_2 \) and investors care only about \( e_2 \), it follows that \( \hat{e}_1 \rightarrow f + \hat{\lambda} (\xi - \hat{\xi}) \) as \( \sigma_\xi \rightarrow 0 \) and hence that the exchange rate in period one is indeed normally distributed in the limit.

On the other hand, if there is no central bank announcement then investors learn that \( \xi < \hat{\xi} \). Let \( e_1 = f + \lambda \xi \). This means that the distribution of \( f \) conditional on the information of investor \( i \) is truncated normal with mean \( f_i + \frac{\sigma_\xi^2}{\sigma_\xi^2 + \lambda \sigma_\xi^2} (e_1 - f_i) \), variance \( \frac{\sigma_\xi^2 \lambda^2 \sigma_\xi^2}{\sigma_\xi^2 + \lambda \sigma_\xi^2} \), and truncation \( f > e_1 - \lambda \hat{\xi} \). Average expectations are simpler this time, with

\[ E_1(N) \left[ e^{-f} \right] \rightarrow e^{-f - e_1 + f} = e^{-f - \lambda \xi} \]  

(3.4)
as \( \sigma_\xi \rightarrow 0 \). This follows because the truncation communicates nothing about \( f \) in the limit since the conditional mean of \( f \) is on average greater than the truncation. Once again, this implies that indeed \( e_1 \rightarrow f + \lambda \xi \) as \( \sigma_\xi \rightarrow 0 \), confirming the initial guess.

Although this example is simpler than full setup of this section, it does capture the role of the assumption \( \sigma_\xi \leq \hat{\sigma}_\xi \) in the proof of Theorem 3.3. One implication is that for \( \sigma_\xi \) small enough, the difference between \( e_1 \) and \( \hat{e}_1 \) is approximately given by

\[ e_1 - \hat{e}_1 = \xi (\lambda - \hat{\lambda}) + \hat{\lambda} \hat{\xi}. \]  

(3.5)
This relationship shows that if \( \hat{\lambda} > \lambda \), then it is not possible to construct an equilibrium in which the central bank only makes an announcement if \( \xi < \hat{\xi} \) in this setting. According to equation (3.5), regardless of the value of \( \hat{\xi} \) (or if \( \hat{\xi} \) multiplies \( \lambda \) instead of \( \hat{\lambda} \)), if \( \xi \) is sufficiently negative, then \( e_1 > \hat{e}_1 \) and the central bank prefers an ambiguous intervention policy. This is an important observation, because together with the existence of self-fulfilling pooling equilibria, another concern in this setup is that investors’ interpretation of central bank policy may dictate whether transparency signals an overvalued or undervalued currency in equilibrium. This example shows that this is often not possible.
4 Infinite-Horizon Model

Time is discrete and indexed by \( t \in \mathbb{N} \) and there are two countries. As in Section 2, I shall refer to the home country’s currency as the dollar and the foreign country’s currency as the peso. There is only one good for consumption and its price in each country is linked by the law of one price, so that \( e_t + p_t^* = p_t \) for all \( t \). As before, the exchange rate is defined as the dollar price of a peso, and its log in period \( t \) is given by \( e_t \).

In this infinite-horizon extension, three assets are traded in each period \( t \): a nominal one-period bond issued by the domestic central bank with return \( i_t \), a nominal one-period bond issued by the foreign central bank with return \( i_t^* \), and a risk-free technology with real return \( r \) in each period. As in the two-period model, I assume that the domestic central bank credibly commits to a constant domestic price level in all periods so that the interest rate on dollar bonds \( i_t \) is equal to \( r \) for all \( t \geq 1 \). This price level is normalized so that \( p_t = 0 \), which implies that the log-linearized real return on foreign bonds in period \( t \) is equal to

\[
-e_t + i_t^* = e_t + 1 - e_t + i_t^*.
\]

The foreign central bank’s interest rate policy is more complicated in this setup. In particular, I assume that the foreign central bank follows a Wicksellian interest rate rule in which the price target is equal to zero.\(^{15}\) This policy is subject to uncertainty, however, so that investors face risk when investing in peso bonds. Specifically, in each period \( t \), the interest rate on peso bonds is given by \( i_t^* = a p_t^* + \mu_t + r \), where \( \mu_t \) follows an AR1 process and \( a > 0 \) is a constant that measures the response of interest rate policy to deviations from the price target. The stochastic process for interest rate deviations is given by \( \mu_t = \rho \mu_{t-1} + \zeta_t \), where \( 0 < \rho < \infty \) is a constant and \( \zeta_t \) is i.i.d. normal, with mean zero and variance \( \sigma^2 \).

Note that the stochastic process for \( \mu_t \) is common knowledge among all investors, as is the value of \( \mu_t \) in period \( t \) since all current and past interest rates are publicly observable.

The economy is populated by overlapping generations of investors such that, in each period \( t \), a new generation of investors is born while the old generation of investors dies. Each newly born investor in period \( t \) chooses her portfolio and then, in period \( t + 1 \), liquidates her positions and consumes all of her realized wealth before dying. As in the previous section, investors are indexed by \( i \in [0,1] \) and each investor \( i \) born in period \( t \) solves the maximization problem

\[
\max_{b_{it} \in \mathbb{R}} -E_t \left[ e^{-\gamma c_{it+1}} \right], \quad \text{subject to} \quad c_{it+1} = (1 + i_t)w_{it} + (e_{t+1} - e_t + i_t^* - i_t)b_{it}, \tag{4.1}
\]

where \( w_{it} \in \mathbb{R} \) is investor \( i \)'s endowment of real wealth at birth, \( e_{t+1} - e_t + i_t^* - i_t \) is the

\(^{15}\)Woodford (2003) provides a detailed discussion of the implications of Wicksellian, price-targeting interest rate rules in cashless economies such as this one.
log-linearized excess return of peso bonds in period $t$, $b_{it}$ is the dollar amount of investor $i$’s purchases of peso bonds in period $t$, $c_{it+1}$ is the quantity of the economy’s only good consumed by investor $i$ in period $t + 1$, $\gamma > 0$ is the coefficient of absolute risk aversion, and $E_{it}[\cdot]$ denotes the conditional expectation with respect to the information set of investor $i$ in period $t$. The net supply of peso bonds is constant and equal to zero. In each period $t$, a mass of noise traders purchases $\xi_t$ dollars worth of peso bonds, where $\xi_t$ is i.i.d. normal, with mean zero and variance $\sigma^2_{\xi}$. Noise traders liquidate all their assets from the previous period before making any purchases.

As in the two-period model, the foreign central bank complements its interest rate policy by performing foreign exchange interventions in each period. I assume specifically that the central bank purchases $\nu_t \in \mathbb{R}$ dollars worth of peso bonds in each period $t$ and that these interventions follow an AR1 process, so that $\nu_t = \rho_{\nu}\nu_{t-1} + \delta_t$, where $0 < \rho_{\nu} < \infty$ is a constant and $\delta_t$ is i.i.d. normal, with mean zero and variance $\sigma^2_{\delta}$. The stochastic process for $\nu_t$ is common knowledge among all investors.

In this infinite-horizon model, I assume that in each period $t$ each investor $i$ receives the private signals $x_{it} = \mu_{t+1} + \epsilon_{it}$ and $y_{it} = \nu_t + \eta_{it}$, where $\epsilon_{it} \sim N(0, \sigma^2_{\epsilon})$, $\eta_{it} \sim N(0, \sigma^2_{\eta})$, $\epsilon_{it}$ and $\eta_{it}$ are both i.i.d. and independent of each other, and all noise terms are independent across investors. Following Bacchetta and van Wincoop (2006), I also assume that the generation of investors that is born in period $t$ inherits all of the private information from the generation that dies in period $t$. More precisely, I assume that in each period $t$, each newly born investor $i$ inherits all of the private information of investor $i$ from the generation born in period $t - 1$.

I shall consider two different specifications for the investors’ information. In the first, investors perfectly learn about past values of $\nu_t$ which causes higher-order expectations to collapse into more simple average beliefs. The exchange rate can be characterized analytically in this setup, and the equilibrium matches with the equilibrium from the two-period model in Section 2. It is not surprising, then, that most of the previous conclusions about transparency and exchange rate manipulation continue to be valid. In the second specification, investors never learn about past values of $\nu_t$ so that higher-order expectations remain part of the equilibrium exchange rate. This, however, makes an analytic solution impossible as has been widely documented by Bacchetta and van Wincoop (2006) and Lorenzoni (2009), among others. As a consequence, I solve numerically for an approximate steady-state solution using results from Nimark (2010a). Before specifying the details of investors’ information sets, it is useful to first solve for the equilibrium exchange rate without any assumptions about these information sets.

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16Note that investors already learn about current and past values of $\mu_t$ because interest rates are publicly observable.
In this infinite-horizon setup, I adopt notation similar to that from the benchmark model in the previous section. For all $t \in \mathbb{N}$, let $\mathcal{F}_t$ denote the information set consisting of all common public information in period $t$ together with $\mu_s$ and $\nu_s$ for all $s \in \mathbb{N}$ and $e_s$ for all $s \leq t$. The aggregate demand for peso bonds by the investors in period $t$ is equal to the average demand of the investors in period $t$ and is denoted by $B_t = E[b_{1t} | \mathcal{F}_t]$. It follows that the total demand for peso bonds in period $t$ is equal to $B_t + \nu_t + \xi_t$. Let $\mathbb{E}_t[\cdot] = E[E_{\mu}[\cdot] | \mathcal{F}_t]$ denote the average expectation of investors in period $t$, and let $\text{Var}_{\mu}[\cdot]$ denote the conditional variance with respect to the information set of investor $i$ in period $t$ and $\text{Var}_{\nu}[\cdot]$ the average conditional variance of investors in period $t$. I denote higher-order expectations in this environment by $\mathbb{E}^0_t[\cdot] = \cdot$, $\mathbb{E}^1_t[\cdot] = E_t[\cdot]$, and, in general, $\mathbb{E}^n_t[\cdot] = E_tE_{t+1} \cdots E_{t+n-1}[\cdot]$. The information set of investor $i$ in period $t$ is denoted by $\mathcal{G}_i$. Finally, let $\mathcal{G}_{i0} = \emptyset$, $\sigma^2_i = \text{Var}_t[e_{t+1}]$, and $\alpha = \frac{1}{1+n}$.

**Definition 4.1.** A steady-state equilibrium of this economy is a stochastic process for the exchange rate $\{e_t : t \in \mathbb{N}\}$, such that for all $t \in \mathbb{N}$ (i) the demand for peso bonds by each investor $i$ solves the maximization problem (4.1), where investor $i$’s information set $\mathcal{G}_i$ consists of all common public information in period $t$ together with $x_{it}$, $y_{it}$, $\mathcal{G}_{i,t-1}$, and, if the foreign central bank announces its intervention in period $t$, $\nu_t$ as well; (ii) the peso bond market clears: $B_t + \xi_t + \nu_t = 0$; (iii) the exchange rate is a linear function of current and past demand for peso bonds by noise traders $\{\xi_s : 1 \leq s \leq t\}$, the foreign central bank’s interventions $\{\nu_s : s \in \mathbb{N}\}$, the interest rate parameters $\{\mu_s : s \in \mathbb{N}\}$, and the disturbance terms $\{\delta_s : s \in \mathbb{N}\}$; (iv) the exchange rate is in a steady state: there exists $\sigma^2 > 0$ such that $\sigma^2_i = \sigma^2$ in all periods $t$.

**Lemma 4.2.** Suppose that the conditional variance $\text{Var}_u[e_{t+1}]$ is equal for all investors $i \in [0, 1]$ in all periods $t$ and that $e_{t+1}$ is normally distributed conditional on the information set of investor $i$ in period $t$. Then, the steady-state equilibrium exchange rate satisfies

$$e_t = \sum_{n=0}^{\infty} \alpha^{n+1} \mathbb{E}_t[\mu_{t+n}] + \gamma \sigma^2 \sum_{n=0}^{\infty} \alpha^{n+1} \mathbb{E}_t^\nu[\nu_{t+n}] + \alpha \gamma \sigma^2 \xi_t.$$  

(4.2)

**Proof.** If $e_{t+1}$ is normally distributed conditional on the information set of investor $i$ in period $t$, then problem (4.1) is a standard CARA-normal maximization and the demand for peso bonds by investor $i$ in period $t$ is given by

$$b_{it} = E_{\mu}[e_{t+1}] - e_t + i^*_t - i_t \quad \text{and} \quad \text{Var}_{\mu}[e_{t+1}] = \gamma \text{Var}_u[e_{t+1}] .$$

(4.3)

If the conditional variance $\text{Var}_u[e_{t+1}]$ is equal for all investors $i \in [0, 1]$, then $\text{Var}_u[e_{t+1}] = 31$.
\( \text{Var}_t[e_{t+1}] = \sigma_t^2 \) and hence
\[ B_t = \frac{\mathbb{E}_t[e_{t+1}] - e_t + i_t^* - i_t}{\gamma \sigma_t^2}. \tag{4.4} \]
Recall that in each period \( t \), the total demand for peso bonds is equal to \( B_t + \nu_t + \xi_t \) while the domestic and foreign interest rates are equal to \( r \) and \(-ae_t + \mu_t + r\), respectively. In the steady-state equilibrium, \( \sigma_t^2 = \sigma^2 \) for all \( t \), so that
\[ B_t = \frac{\mathbb{E}_t[e_{t+1}] - (1 + a)e_t + \mu_t}{\gamma \sigma^2}, \tag{4.5} \]
and then, by market clearing,
\[ e_t = \alpha \mathbb{E}_t[e_{t+1}] + \alpha \mu_t + \alpha \gamma \sigma^2 (\nu_t + \xi_t). \tag{4.6} \]

The noise traders’ demand is i.i.d. over time, so it follows that \( \mathbb{E}_t[\xi_{t+n}] = 0 \) for all \( n \geq 1 \). Forward iteration of equation (4.6), then, yields
\[ e_t = \alpha^2 \mathbb{E}_t[\mathbb{E}_{t+1}[e_{t+2}]] + \alpha^2 \mathbb{E}_t[\mu_{t+1}] + \alpha \mu_t + \alpha^2 \gamma \sigma^2 \mathbb{E}_t[\nu_{t+1}] + \alpha \gamma \sigma^2 \nu_t + \alpha \gamma \sigma^2 \xi_t \tag{4.7} \]
\[ = \alpha^3 \mathbb{E}_t^3[e_{t+3}] + \sum_{n=0}^{2} \alpha^{n+1} \mathbb{E}_t^n[\mu_{t+n}] + \gamma \sigma^2 \sum_{n=0}^{2} \alpha^{n+1} \mathbb{E}_t^n[\nu_{t+n}] + \alpha \gamma \sigma^2 \xi_t. \tag{4.8} \]
Finally, as demonstrated above, repeated forward iteration implies that the equilibrium exchange rate in period \( t \) must satisfy
\[ e_t = \sum_{n=0}^{\infty} \alpha^{n+1} \mathbb{E}_t^n[\mu_{t+n}] + \gamma \sigma^2 \sum_{n=0}^{\infty} \alpha^{n+1} \mathbb{E}_t^n[\nu_{t+n}] + \alpha \gamma \sigma^2 \xi_t, \tag{4.9} \]
which completes the proof.

Equation (4.2) shows that higher-order expectations are part of the steady-state equilibrium exchange rate unless special assumptions are made about the investors’ information. In the first specification of this model, I do make such special assumptions and this makes it possible to solve for the steady-state exchange rate explicitly. The equilibrium matches well with the equilibrium from the benchmark two-period model and generates a similar set of predictions about central bank transparency.
4.1 Common Knowledge of the Past

Suppose that in each period \( t > 1 \), the value of the previous period’s intervention \( \nu_{t-1} \) becomes common knowledge among all investors. This assumption implies that the higher-order expectations from equation (4.2) collapse into more simple average expectations.

I relax the assumption about public revelation of \( \nu_{t-1} \) in the next section. This creates an environment where investors are perpetually disparately informed and higher-order expectations are an important part of the equilibrium steady-state. I demonstrate that the same basic predictions about central bank announcements observed in this and the previous section’s models obtain in that setup as well. While this is certainly an important exercise since it verifies the robustness of this paper’s results, it is likely that some assumption about public revelation of past interventions is actually more realistic. Indeed, most interventions that are large enough to have a meaningful and lasting effect on the exchange rate will also have a sizeable effect on central bank foreign exchange reserves and are therefore very unlikely to go undetected forever. This is especially true as publicly available and standardized collections of data about bank reserves and other variables become more common.\(^{17}\)

This section’s assumptions about the investors’ information yield an equilibrium exchange rate that is very similar to the two-period model analyzed in Section 2. This is important because it provides the motivation for that section’s exogenous period-two exchange rate as given by equation (2.2). The major difference is that investors in this model have common knowledge of the stochastic processes for \( \mu_t \) and \( \nu_t \). Since investors know the value of \( \mu_t \) and \( \nu_{t-1} \) in period \( t \), the knowledge of these stochastic processes is equivalent to public signals of both \( \mu_{t+1} \) (because \( \mu_{t+1} = \rho_t \mu_t + \zeta_{t+1} \)) and \( \nu_t \) (because \( \nu_t = \rho_t \nu_{t-1} + \delta_t \)), much like a common prior. Unfortunately, this common prior complicates the solution to the model so that, unlike in the benchmark two-period model, an analytic solution for the case of central bank transparency is not possible. I am, however, able to characterize the equilibrium conditions analytically and prove results about transparency that are similar to those from the previous section.

As in the benchmark model, I present the equilibrium exchange rate with no central bank announcement before presenting the equilibrium exchange rate with a central bank announcement. These two cases are then compared, and the implications of transparency are stated and discussed. All proofs from this section are in Appendix B.

**Theorem 4.3.** If the value of \( \nu_{t-1} \) becomes common knowledge among all investors in period

\(^{17}\)A prominent example of this movement towards greater balance sheet transparency is the International Monetary Fund’s Special Data Dissemination Standard (SDDS). To date, there are 64 countries that subscribe to the SDDS.
the equilibrium exchange rate in period \( t \), then the steady-state equilibrium exchange rate is given by

\[
e_t = \alpha \mu_t + \psi \mu_{t+1} + \psi \nu_t + \lambda_\xi \xi_t + \lambda_\zeta \zeta_{t+1} + \lambda_\delta \delta_t,
\]

where \( \psi = \frac{\alpha_2}{1-\alpha_\rho} \), \( \psi_t = \frac{\alpha_2 \sigma^2}{1-\alpha_\rho} \), and \( \lambda_\xi, \lambda_\zeta, \lambda_\delta, \) and \( \sigma^2 \) are given by the solution to

\[
\lambda_\xi = \frac{\lambda_\xi \psi_t (\psi_t + \lambda_\zeta) (\sigma^2 + \sigma_\xi^2) \sigma^2 + \lambda_\zeta \rho \alpha \psi_t (\psi_t + \lambda_\delta) (\sigma^2 + \sigma_\xi^2) \sigma^2 \sigma_\delta^2 + \lambda_\zeta \rho \alpha \psi_t (\psi_t + \lambda_\zeta) (\sigma^2 + \sigma_\xi^2) \sigma^2 \sigma_\delta^2 + \alpha \gamma \sigma^2}{\Psi},
\]

\[
\lambda_\zeta = \frac{\alpha \rho \psi_t (\psi_t + \lambda_\zeta) (\sigma^2 + \sigma_\xi^2) \sigma^2 \sigma_\delta^2 - \psi_\mu ((\sigma^2 + \sigma_\xi^2) \lambda_\xi \sigma^2 + (\psi_t + \lambda_\delta) (\sigma^2 + \sigma_\xi^2) \sigma^2 \sigma_\delta^2)}{\Psi},
\]

\[
\lambda_\delta = \frac{\psi (\psi_t + \lambda_\zeta) (\psi_t + \lambda_\delta) (\sigma^2 + \sigma_\xi^2) \sigma^2 \sigma_\delta^2 - \lambda_\zeta \rho \psi_t ((\sigma^2 + \sigma_\xi^2) \lambda_\xi \sigma^2 + (\psi_t + \lambda_\delta) (\sigma^2 + \sigma_\xi^2) \sigma^2 \sigma_\delta^2)}{\Psi},
\]

\[
\sigma^2 = \frac{\psi^2}{\alpha^2} \sigma^2 + \rho^2 \psi^2 \sigma^2 + \lambda_\zeta \xi \sigma^2 + (\psi_t + \lambda_\zeta) \sigma^2 + (\psi_t + \lambda_\delta) \sigma^2
\]

\[
- \frac{\psi^2 \sigma^4}{\alpha^2 \Psi} \left[ (\sigma^2 + \sigma_\delta^2) (\lambda_\xi \sigma^2 + (\psi_t + \lambda_\zeta) \sigma^2) + (\psi_t + \lambda_\delta) \sigma^2 \sigma_\delta^2 \right]
\]

\[
- \frac{\rho^2 \psi^2 \sigma^4}{\Psi} \left[ (\sigma^2 + \sigma_\xi^2) (\lambda_\xi \sigma^2 + (\psi_t + \lambda_\delta) \sigma^2) + (\psi_t + \lambda_\zeta) \sigma^2 \sigma_\delta^2 \right]
\]

\[
- \frac{2 \rho \psi \psi_t (\psi_t + \lambda_\zeta) (\psi_t + \lambda_\delta) \sigma^2 \sigma^2 \sigma_\delta^2 \sigma_\xi^2 \sigma_\delta \sigma_\xi}{\alpha \Psi},
\]

with \( \Psi = (\psi_t + \lambda_\zeta) (\psi_t + \lambda_\delta) \sigma^2 \sigma^2 \sigma_\delta^2 + (\psi_t + \lambda_\delta) (\sigma^2 + \sigma_\xi^2) \sigma^2 \sigma_\delta^2 + (\sigma^2 + \sigma_\xi^2) \sigma_\delta^2 \sigma_\xi^2 \lambda_\delta \sigma^2 \).

Theorem 4.3 is very similar to Theorem 2.2 from the benchmark two-period model of Section 2. The most substantial difference between these two models is that investors in this dynamic model have common knowledge of the stochastic processes for interest rate spreads \( \mu_t \) and central bank interventions \( \nu_t \). In Theorem 4.3, this information is apparent since the equilibrium exchange rate in period \( t \) is a function of \( \zeta_{t+1} \) and \( \delta_t \) in addition to \( \xi_t \). In the benchmark two-period model, the equilibrium exchange rate in period one was only a function of \( \xi_1 \).

In Section 2, I assumed that the exchange rate in period two was exogenously given by \( e_2 = \theta_\mu \mu_2 + \theta_\nu \nu_1 + \kappa_2 \). I argued that a very similar expression for the future value of the exchange rate appears endogenously in an infinite-horizon extension of the two-period model. Theorem 4.3 demonstrates that this is indeed the case. To see this, consider the exchange
rate in period $t + 1$ implied by equation (4.10), which is given by

$$e_{t+1} = \alpha \mu_{t+1} + \psi_\mu \mu_{t+2} + \psi_\nu \nu_{t+1} + \lambda_\xi \xi_{t+1} + \lambda_\zeta \zeta_{t+2} + \lambda_\delta \delta_{t+1}$$

$$= \alpha \mu_{t+1} + \psi_\mu (\rho_\mu \mu_{t+1} + \zeta_{t+2}) + \psi_\nu (\rho_\nu \nu_{t+1} + \delta_{t+1}) + \lambda_\xi \xi_{t+1} + \lambda_\zeta \zeta_{t+2} + \lambda_\delta \delta_{t+1}$$

$$= \frac{\psi_\mu}{\alpha} \mu_{t+1} + \psi_\nu \rho_\nu \nu_{t} + \lambda_\xi \xi_{t+1} + (\psi_\mu + \lambda_\zeta) \zeta_{t+2} + (\psi_\nu + \lambda_\delta) \delta_{t+1}. \quad (4.15)$$

Equation (4.15) is identical to the exogenous expression for $e_2$ given by equation (2.2) in Section 2, with $\theta_\mu$ replaced by $\frac{\psi_\mu}{\alpha}$, $\theta_\nu$ replaced by $\psi_\nu \rho_\nu$, and $\kappa_2$ replaced by $\lambda_\xi \xi_{t+1} + (\psi_\mu + \lambda_\zeta) \zeta_{t+2} + (\psi_\nu + \lambda_\delta) \delta_{t+1}$. The only substantive difference is that the parameter $\psi_\nu$ depends on the endogenous steady-state variance $\sigma^2$. It not surprising, then, that the results about central bank transparency I proved in the two-period model also obtain in this infinite-horizon model.

Before presenting the results, however, it is first necessary to characterize the steady-state equilibrium exchange rate when the foreign central bank is transparent. I consider a policy of transparency to be a credible and truthful public announcement in period $t$ of the central bank’s intervention in period $t$. As in the benchmark model, let $\tilde{e}_t$ denote the exchange rate in period $t$ if the central bank announces the value of $\nu_t$ to the investors in period $t$.

**Theorem 4.4.** If the foreign central bank credibly and publicly announces the value of $\nu_t$ in period $t$, then the steady-state equilibrium exchange rate is given by

$$\tilde{e}_t = \alpha \mu_t + \psi_\mu \mu_{t+1} + \psi_\nu \nu_t + \tilde{\lambda}_\xi \xi_t + \tilde{\lambda}_\zeta \zeta_{t+1}, \quad (4.16)$$

where $\psi_\mu = \frac{\alpha^2}{1-\alpha_\mu^2}$, $\psi_\nu = \frac{\alpha \sigma^2}{1-\alpha_\rho_\nu}$, and $\tilde{\lambda}_\xi$, $\tilde{\lambda}_\zeta$, and $\tilde{\sigma}^2$ are given by the solution to

$$\tilde{\lambda}_\xi = \frac{\tilde{\lambda}_\xi \psi_\mu (\psi_\mu + \tilde{\lambda}_\zeta) \sigma^2 \sigma_{\tilde{\xi}}^2}{(\psi_\mu + \tilde{\lambda}_\zeta)^2 \alpha_\sigma^2 \sigma_{\tilde{\xi}}^2 + (\sigma^2 + \sigma_{\tilde{\xi}}^2) \tilde{\lambda}_\xi \sigma_{\tilde{\xi}}^2} + \alpha \gamma \tilde{\sigma}^2, \quad (4.17)$$

$$\tilde{\lambda}_\zeta = -\frac{\psi_\nu \sigma_{\tilde{\zeta}}^2 \lambda_\xi \sigma_{\tilde{\xi}}^2}{(\psi_\mu + \tilde{\lambda}_\zeta)^2 \sigma^2 \sigma_{\tilde{\xi}}^2 + (\sigma^2 + \sigma_{\tilde{\xi}}^2) \lambda_\xi \sigma_{\tilde{\xi}}^2}, \quad (4.18)$$

$$\tilde{\sigma}^2 = \frac{\psi_\mu^2 \sigma^2 + \lambda_\xi \sigma_{\tilde{\xi}}^2 + (\psi_\mu + \tilde{\lambda}_\zeta)^2 \sigma_{\tilde{\xi}}^2 + \psi_\nu^2 \sigma_{\tilde{\xi}}^2 - \frac{\psi_\mu \sigma_{\tilde{\xi}}^4 (\tilde{\lambda}_\xi \sigma_{\tilde{\xi}}^2 + (\psi_\mu + \tilde{\lambda}_\zeta)^2 \sigma_{\tilde{\xi}}^2)}{\alpha^2 (\psi_\mu + \tilde{\lambda}_\zeta)^2 \sigma_{\tilde{\xi}}^2 \sigma_{\tilde{\xi}}^2 + \alpha^2 (\sigma^2 + \sigma_{\tilde{\xi}}^2) \tilde{\lambda}_\xi \sigma_{\tilde{\xi}}^2}}. \quad (4.19)$$

Theorem 4.4 shows that the equilibrium exchange rate under a policy of central bank transparency does not depend on the innovation in the bank’s current intervention relative to the previous period’s intervention $\delta_t$. This follows because the size of this intervention is revealed to the public and hence investors no longer form expectations about it. The
equilibrium exchange rate formulations given by Theorems 4.3 and 4.4 feature parameters that multiply the innovations in the stochastic processes for \( \mu_t (\lambda_\xi, \tilde{\lambda}_\xi) \) and \( \nu_t (\lambda_\delta) \). An alternative formulation uses the fact that \( \mu_{t+1} = \rho_\mu \mu_t + \zeta_{t+1} \) and \( \nu_t = \rho_\nu \nu_{t-1} + \delta_t \) to write

\[
\begin{align*}
e_t &= (\alpha - \rho_\mu \lambda_\xi) \mu_t + (\psi_\mu + \lambda_\xi) \mu_{t+1} - \rho_\nu \lambda_\delta \nu_{t-1} + (\psi_\nu + \lambda_\delta) \nu_t + \lambda_\xi \xi_t, \\
\tilde{e}_t &= (\alpha - \rho_\mu \tilde{\lambda}_\xi) \mu_t + (\psi_\mu + \tilde{\lambda}_\xi) \mu_{t+1} + \psi_\nu \nu_t + \tilde{\lambda}_\xi \xi_t.
\end{align*}
\tag{4.20}
\]

I choose the formulation of Theorems 4.3 and 4.4 to emphasize the connection between this infinite-horizon model and the benchmark two-period model of Section 2. Furthermore, this formulation better contrasts the effects of transparency on the magnification of the various noise terms, which is the main focus of the paper.

**Theorem 4.5.** The parameters \( \lambda_\xi \) and \( \tilde{\lambda}_\xi \) satisfy

\[
\begin{align*}
\lim_{\sigma_\xi \to \infty} \lambda_\xi &> \lim_{\sigma_\xi \to \infty} \tilde{\lambda}_\xi = 0, & \lim_{\sigma_\xi \to 0} \lambda_\xi &< \lim_{\sigma_\xi \to 0} \tilde{\lambda}_\xi = \infty, \\
\lim_{\sigma_\zeta \to 0} \lambda_\xi = \lim_{\sigma_\zeta \to 0} \tilde{\lambda}_\xi = 0, & \lim_{\sigma_\delta \to 0} \lambda_\xi = \lim_{\sigma_\delta \to 0} \tilde{\lambda}_\xi > 0.
\end{align*}
\]

The limits of both \( \lambda_\xi \) and \( \tilde{\lambda}_\xi \) as either \( \sigma_\xi, \sigma_\zeta, \) or \( \sigma_\delta \) increases to infinity are undefined since the systems of equations that define the steady-state equilibria, given by Theorems 4.3 and 4.4, cease to have real solutions in those limits. Theorem 4.5 establishes several comparative statics about the value of the parameters \( \lambda_\xi \) and \( \tilde{\lambda}_\xi \). In particular, this theorem reproduces the result that \( \tilde{\lambda}_\xi > \lambda_\xi \) if the unpredictability of noise traders \( \sigma_\xi \) is sufficiently small.

The product \( \rho_\nu \psi_\nu \) from this infinite-horizon model replaces the parameter \( \theta_\nu \) from the two-period model of Section 2, so it is both important and not surprising that \( \lambda_\xi \) tends to be less than \( \tilde{\lambda}_\xi \) for smaller values of \( \rho_\nu \) and greater than \( \tilde{\lambda}_\xi \) for larger values of \( \rho_\nu \). In Figures 7, 8, and 9, I plot the value of \( \lambda_\xi \) and \( \tilde{\lambda}_\xi \) assuming various different values for the model’s parameters. This relative behavior of \( \lambda_\xi \) and \( \tilde{\lambda}_\xi \) is apparent.

The three parameterizations of the model I display are chosen to capture different characteristics of the model. Although these parameterizations all generate moments that are roughly consistent with those that are observed in most quarterly data, the spirit of these empirical exercises is to illustrate the mechanism by which exchange rate misalignment can be magnified rather than to create a quantitatively precise simulation. Indeed, all of the models that I discuss are highly stylized and intended to explore and characterize the interaction between the truth-telling and signal-precision effects of transparency rather than to produce a precise model of exchange rate determination.

The first parameterization, depicted in Figure 7, features a choice of parameters that implies that \( \lambda_\xi \) is less than \( \tilde{\lambda}_\xi \) for nearly all values of \( \rho_\nu \). The implied unconditional standard
Figure 7: The value of $\lambda_\xi$ (dashed line) and $\tilde{\lambda}_\xi$ (solid line) as the persistence of foreign central bank interventions $\rho_\nu$ increases. ($\sigma_\epsilon = 0.095$, $\sigma_\eta = 0.37$, $\sigma_\xi = 0.12$, $\sigma_\zeta = 0.025$, $\sigma_\delta = 0.09$, $\alpha = 0.92$, $\gamma = 5$, $\rho_\mu = 0.7$)

Figure 8: The value of $\lambda_\xi$ (dashed line) and $\tilde{\lambda}_\xi$ (solid line) as the persistence of foreign central bank interventions $\rho_\nu$ increases. ($\sigma_\epsilon = 0.075$, $\sigma_\eta = 0.25$, $\sigma_\xi = 0.08$, $\sigma_\zeta = 0.025$, $\sigma_\delta = 0.05$, $\alpha = 0.92$, $\gamma = 5$, $\rho_\mu = 0.7$)
deviation of the exchange rate, both with and without transparency, is equal to 10 percent given this first parameterization. The second parameterization, depicted by Figure 8, features a choice of parameters that implies $\lambda_\xi$ is much less than $\tilde{\lambda}_\xi$ for all values of $\rho_\nu$. In this case, a central bank announcement magnifies exchange rate misalignment regardless of how permanent the effects of intervention are. Furthermore, this parameterization implies that the level of exchange rate misalignment is much higher than in the previous parameterization (larger values of $\lambda_\xi$ and $\tilde{\lambda}_\xi$). The implied unconditional standard deviation of the exchange rate is equal to 10.3 percent given this second parameterization. The third parameterization, depicted by Figure 9, features a choice of parameters that implies that $\lambda_\xi$ and $\tilde{\lambda}_\xi$ are both very small and very close together. In this case, a central bank announcement has almost no effect on exchange rate misalignment for most values of $\rho_\nu$. The implied unconditional standard deviation of the exchange rate is equal to 8.6 percent given this third parameterization.

4.2 Imperfect Common Knowledge of the Past

Suppose that the value of $\nu_{t-1}$ does not become common knowledge among all investors in period $t$. Unlike in the previous section, this implies that higher-order expectations are part of the equilibrium exchange rate if the foreign central bank does not publicly announce the size of $\nu_t$ in period $t$. More specifically, in this section’s setup, it is no longer the case that
\[ E_t[n \nu_t + n] = \rho^n E_t[\nu_t] \] for all \( n > 1 \).

There have been a number of dynamic macroeconomic models that feature higher-order expectations, including the early models of Townsend (1983) and Singleton (1987), and more recently, the models of Bacchetta and van Wincoop (2006) and Lorenzoni (2009). With the exception of Townsend (1983), all of these setups are too difficult to solve directly and must instead be approximated. This is usually accomplished by assuming that the past exogenously becomes common knowledge with some lag, a technique that keeps the state space in these models finite and makes it possible to solve for the steady-state equilibrium using standard methods. There is, however, another technique for solving these models as described by Nimark (2010a). Rather than assuming that the past becomes common knowledge, Nimark (2010a) shows that the steady-state equilibrium of a model in which agents are perpetually disparately informed can be approximated arbitrarily well by exogenously bounding the order of agents’ expectations. The size of this bound becomes less and less important as the bound grows to infinity, and the approximate equilibrium converges to the true equilibrium.

In this section, I use this technique to consider the equilibrium of this infinite-horizon model when investors do not have common knowledge of the past. The solution is relatively complex and can only be approximated numerically. Before presenting the results, it is necessary to introduce more notation. In particular, the equilibrium exchange rate in this setup must be expressed as a function of higher-order expectations at time \( t \) only, so let \( \overline{E}(0)[\cdot] = \cdot, \overline{E}(1)[\cdot] = \overline{E}[\cdot], \) and in general, \( \overline{E}(j)[\cdot] = \overline{E}_t\overline{E}_{t+1} \cdots \overline{E}_t[\cdot] \) with the expectation repeated \( j \) times. For all \( 0 \leq j \leq k \), let

\[
q_{jt} = \left( \overline{E}(j)_t[\mu_{t+1}] \ \overline{E}(j)_t[\nu_t] \right), \tag{4.21}
\]

and for all \( t \in \mathbb{N} \), let

\[
Q_t(k) = \begin{pmatrix}
q_{0t} & q_{1t} & \cdots & q_{kt}
\end{pmatrix}^\prime, \tag{4.22}
\]

\[
w_t = \begin{pmatrix}
\sigma_{\zeta}^{-1} \zeta_{t+1} & \sigma_{\delta}^{-1} \delta_t & \sigma_{\xi}^{-1} \xi_t
\end{pmatrix}^\prime. \tag{4.23}
\]

Let \( h_1 = (1 \ 0 \ \cdots)^\prime \) and \( h_2 = (0 \ 1 \ 0 \ \cdots)^\prime \), and let the matrix \( H \) be given by

\[
H = \begin{pmatrix}
0_{2k \times 2} & I_{2k} \\
0_{2 \times 2k} & 0_{2 \times 2k}
\end{pmatrix}, \tag{4.24}
\]

where \( I_{2k} \) is equal to the identity matrix of dimension \( 2k \). This matrix evaluates the average
expectation of a vector and then annihilates the highest-order expectation, so that

$$ HQ_t(k) = \left( q'_{1t} \ q'_{2t} \ \cdots \ q'_{kt} \ 0 \ 0 \right)' = \left( \mathbb{E}_t[q'_{0t}] \ \mathbb{E}_t[q'_{1t}] \ \cdots \ \mathbb{E}_t[q'_{k-1t}] \ 0 \ 0 \right)' \quad (4.25) $$

If the foreign central bank announces the value of $\nu_t$ in period $t$, then the steady-state equilibrium exchange rate is simply given by Theorem 4.4 since knowledge of $\nu_t$ in period $t$ makes past values of $\nu_t$ irrelevant. Consequently, it is only necessary to present the equilibrium exchange rate in the absence of a central bank announcement. All proofs from this section are in Appendix B.

**Theorem 4.6.** If the value of $\nu_{t-1}$ does not become common knowledge among all investors in period $t$, then the steady-state equilibrium exchange rate is approximately given by the system of equations

$$ e_t = AQ_t(k) + \alpha \mu_t + \alpha \gamma \sigma^2 \xi_t, \quad (4.26) $$

$$ Q_t(k) = MQ_{t-1} + N \xi_t, \quad (4.27) $$

where the vector $A$ satisfies

$$ A = \frac{\alpha^2}{\rho_\nu(1 - \alpha \rho_\nu)} h'_1 M_1 H + \gamma \sigma^2 \sum_{n=0}^{\infty} \alpha^{n+1} h'_2 (M_1 H)^n. \quad (4.28) $$

As the order of truncation $k$ grows to infinity, the solution to this system of equations converges to the true steady-state equilibrium exchange rate.

The matrices $M$ and $N$ and the steady-state variance $\sigma^2$ must be approximated numerically. They are determined by the solution to a system of matrix equations as detailed in Appendix B. In Table 1, I report the value of the approximate equilibrium exchange rate with and without transparency, assuming the same three sets of values for the model’s parameters as in Figures 7, 8, and 9 from the previous section. In all parameterizations, I truncate higher-order expectations at $k = 50$ since I find that the results do not change if this parameter is increased even further.

The table shows that the predictions of this infinite-horizon model are similar with and without common knowledge of the past. Indeed, parameterizations one and two generate approximately a ten percent difference between $\hat{\lambda}_\xi$ and $\lambda_\xi$, much like in Section 4.1. The only substantive change is that this difference stays large for all values of $\rho_\nu$ (and even increases as $\rho_\nu$ increases) when investors are perpetually disparately informed. Parameterization three also reproduces the prediction of Section 4.1; the values of $\hat{\lambda}_\xi$ and $\lambda_\xi$ stay very close together for all values of $\rho_\nu$. These results indicate that including higher-order expectations slightly
the foreign interest rate process, it is first necessary to introduce more notation. For all

$$\sigma$$

Because investors only observe

$$\sigma$$

to have imperfect common knowledge about the value of

$$\sigma$$

does not significantly change the model’s predictions. magnifies the gap between the signal-precision and truth-telling effects of transparency, but does not significantly change the model’s predictions.

Before solving for the steady-state equilibrium exchange rate under this assumption about the foreign interest rate process, it is first necessary to introduce more notation. For all

$$0 \leq j \leq k,$$ let

$$\pi_{jt} = \left( \tilde{E}(j)_{t}[\mu_t] \quad E(j)_{t}[\nu_t] \right)' = 0.2362$$

$$\lambda_{\xi} = 0.2366$$

$$\lambda_{\xi} = 0.2566$$

Table 1: The value of $$\lambda_{\xi}$$ for various parameterizations of the model. Parameterization 1: $$\sigma_{\epsilon} = 0.095, \sigma_{\eta} = 0.37, \sigma_{\zeta} = 0.12, \sigma_{\xi} = 0.025, \sigma_{\delta} = 0.09$$. Parameterization 2: $$\sigma_{\epsilon} = 0.075, \sigma_{\eta} = 0.25, \sigma_{\zeta} = 0.08, \sigma_{\xi} = 0.025, \sigma_{\delta} = 0.05$$. Parameterization 3: $$\sigma_{\epsilon} = 0.01, \sigma_{\eta} = 0.1, \sigma_{\zeta} = 0.55, \sigma_{\xi} = 0.02, \sigma_{\delta} = 0.07$$. ($$\alpha = 0.92, \gamma = 5, \rho_{\mu} = 0.7, \text{and } k = 50$$ throughout)
and for all $t \in \mathbb{N}$, let
\[
\Pi_t(k) = \left(\pi'_{0t} \quad \pi'_{1t} \quad \cdots \quad \pi'_{kt}\right)', \quad (4.30)
\]
\[
w_t = \left(\sigma^{-1}_\zeta \zeta_t \quad \sigma^{-1}_\delta \delta_t \quad \sigma^{-1}_\chi \chi_t \quad \sigma^{-1}_\xi \xi_t\right)', \quad (4.31)
\]

**Theorem 4.7.** If the interest rate on peso bonds is given by $i^*_t = ap^*_t + \mu_t + \chi_t + r$ in each period $t$ and the value of $\nu_{t-1}$ does not become common knowledge among all investors in period $t$, then the steady-state equilibrium exchange rate is approximately given by the system of equations
\[
e_t = A\Pi_t(k) + \alpha \gamma \sigma^2 \xi_t, \quad (4.32)
\]
\[
\Pi_t(k) = M\Pi_{t-1}(k) + Nw_t, \quad (4.33)
\]
where the vector $A$ satisfies
\[
A = \sum_{n=0}^{\infty} \alpha^{n+1}(h_1' + \gamma \sigma^2 h_2')(MH)^n. \quad (4.34)
\]

As the order of truncation $k$ grows to infinity, the solution to this system of equations converges to the true steady-state equilibrium exchange rate.

If the interest rate on peso bonds is given by $i^*_t = ap^*_t + \mu_t + \chi_t + r$ in each period $t$, then a public announcement by the foreign central bank no longer implies that the steady-state equilibrium exchange rate is given by Theorem 4.4. Instead, investors continue to have imperfect common knowledge about $\mu_t$ while commonly learning the value of $\nu_t$. In order to compare a policy of transparency with a policy of secrecy, then, it is necessary to solve for the equilibrium exchange rate after a central bank announcement. First, I introduce more notation. For all $0 \leq j \leq k$, let $\tilde{\pi}_{jt} = \mathbb{E}(j)[\mu_t]$, and for all $t \in \mathbb{N}$, let
\[
\tilde{\Pi}_t(k) = \left(\tilde{\pi}_{0t} \quad \tilde{\pi}_{1t} \quad \cdots \quad \tilde{\pi}_{kt}\right)', \quad (4.35)
\]
\[
\tilde{H} = \begin{pmatrix} 0_{k+1 \times 1} & I_k \\ 0_{1 \times k} & 0_{1 \times k} \end{pmatrix}, \quad (4.36)
\]
\[
\tilde{w}_t = \left(\sigma^{-1}_\zeta \zeta_t \quad \sigma^{-1}_\chi \chi_t \quad \sigma^{-1}_\xi \xi_t\right)', \quad (4.37)
\]

**Theorem 4.8.** If the interest rate on peso bonds is given by $i^*_t = ap^*_t + \mu_t + \chi_t + r$ in each period $t$ and the foreign central bank credibly and publicly announces the value of $\nu_t$ in each period $t$, then the steady-state equilibrium exchange rate is approximately given by the system
of equations

\[ \tilde{e}_t = \tilde{A}\tilde{\Pi}_t(k) + \frac{\alpha \gamma \tilde{\sigma}^2}{1 - \alpha \rho_{\nu}} \nu_t + \alpha \gamma \tilde{\sigma}^2 \xi_t, \]  
\[ \tilde{\Pi}_t(k) = \tilde{M}\tilde{\Pi}_{t-1}(k) + \tilde{N}\tilde{w}_t, \]  
(4.38)  
(4.39)

where the vector \( \tilde{A} \) satisfies

\[ \tilde{A} = \sum_{n=0}^{\infty} \alpha^{n+1} h_1' (\tilde{M}\tilde{H})^n. \]  
(4.40)

As the order of truncation \( k \) grows to infinity, the solution to this system of equations converges to the true steady-state equilibrium exchange rate.

As in all parts of this section, the matrices \( \tilde{M} \) and \( \tilde{N} \) and the steady-state variance \( \tilde{\sigma}^2 \) must be approximated numerically. They are determined by the solution to a system of matrix equations as detailed in Appendix B. In Table 2, I report the value of the approximate equilibrium exchange rate with and without transparency, assuming that the interest rate on peso bonds in period \( t \) is given by \( i_t^* = \alpha p_t^* + \mu_t + \chi_t + r \). I assume the same three sets of values of the model’s parameters as in Table 1 (again with \( k = 50 \)) and Figures 7, 8, and 9.

<table>
<thead>
<tr>
<th>Parameterization</th>
<th>No Central Bank Announcement</th>
<th>Central Bank Announcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (( \rho_{\nu} = 0.1 ))</td>
<td>( \lambda_\xi = 0.0445 )</td>
<td>( \lambda_\xi = 0.0469 )</td>
</tr>
<tr>
<td>1 (( \rho_{\nu} = 0.5 ))</td>
<td>( \lambda_\xi = 0.0456 )</td>
<td>( \lambda_\xi = 0.0470 )</td>
</tr>
<tr>
<td>1 (( \rho_{\nu} = 0.95 ))</td>
<td>( \lambda_\xi = 0.0748 )</td>
<td>( \lambda_\xi = 0.0487 )</td>
</tr>
<tr>
<td>2 (( \rho_{\nu} = 0.1 ))</td>
<td>( \lambda_\xi = 0.0492 )</td>
<td>( \lambda_\xi = 0.0503 )</td>
</tr>
<tr>
<td>2 (( \rho_{\nu} = 0.5 ))</td>
<td>( \lambda_\xi = 0.0496 )</td>
<td>( \lambda_\xi = 0.0503 )</td>
</tr>
<tr>
<td>2 (( \rho_{\nu} = 0.95 ))</td>
<td>( \lambda_\xi = 0.0653 )</td>
<td>( \lambda_\xi = 0.0509 )</td>
</tr>
<tr>
<td>3 (( \rho_{\nu} = 0.1 ))</td>
<td>( \lambda_\xi = 0.0204 )</td>
<td>( \lambda_\xi = 0.0205 )</td>
</tr>
<tr>
<td>3 (( \rho_{\nu} = 0.5 ))</td>
<td>( \lambda_\xi = 0.0205 )</td>
<td>( \lambda_\xi = 0.0205 )</td>
</tr>
<tr>
<td>3 (( \rho_{\nu} = 0.95 ))</td>
<td>( \lambda_\xi = 0.0226 )</td>
<td>( \lambda_\xi = 0.0207 )</td>
</tr>
</tbody>
</table>

Table 2: The value of \( \lambda_\xi \) and \( \lambda_\xi \) for various parameterizations of the model, if the interest rate on peso bonds is given by \( i_t^* = \alpha p_t^* + \mu_t + \chi_t + r \) in each period \( t \). Parameterization 1: \( \sigma_\epsilon = 0.095, \sigma_\eta = 0.37, \sigma_\xi = 0.12, \sigma_\zeta = 0.025, \sigma_\delta = 0.09 \). Parameterization 2: \( \sigma_\epsilon = 0.075, \sigma_\eta = 0.25, \sigma_\xi = 0.08, \sigma_\zeta = 0.025, \sigma_\delta = 0.05 \). Parameterization 3: \( \sigma_\epsilon = 0.01, \sigma_\eta = 0.1, \sigma_\xi = 0.55, \sigma_\zeta = 0.02, \sigma_\delta = 0.07 \). (\( \alpha = 0.92, \gamma = 5, \rho_\mu = 0.7, \sigma_\chi = 0.005 \), and \( k = 50 \) throughout)

The table again shows that the predictions of this infinite-horizon model are similar if
investors continue to have imperfect common knowledge of the past after the foreign central bank announces the value of $\nu_t$. The assumption that the interest rate on peso bonds is given by $i^*_t = a p^*_t + \mu_t + \chi_t + r$ introduces significant noise into the model, so the parameters $\tilde{\lambda}_\xi$ and $\lambda_\xi$ are significantly smaller than in Table 1. However, what matters is the ratio of these parameters since this captures the relative magnification of exchange rate misalignment under different transparency policies. Table 2 shows that for medium-to-small values of $\rho_\nu$, this ratio is similar across all the different assumptions about investors’ information considered in this infinite-horizon model.

Although I do not show this in Table 2, another implication of this setup is that transitory changes in noise traders’ demand for peso bonds permanently change investors’ expectations of the interest rate parameter $\mu_t$ (although this change does gradually disappear over time). This behavior is typical of models with higher-order expectations as shown by Allen, Morris, and Shin (2006), Bacchetta and van Wincoop (2006), Lorenzoni (2009), and Nimark (2010b), and it implies that the central bank’s choice of transparency policy will also have effects on the persistence of exchange rate misalignment. More succinctly, if the exchange rate is more undervalued with a central bank policy, then all else equal, it stays more undervalued with that policy forever.

The goal of this section is to investigate the effects of perpetually disparately informed investors on the model’s predictions about central bank transparency and exchange rate misalignment. In reality, however, investors almost surely have common knowledge of past values for at least some parameters. Indeed, if the foreign central bank ever announces the value of $\nu_t$ in this model, then investors can never again be perpetually disparately informed about the intervention process. Furthermore, the trend in the past decades has been towards reporting changes in central banks’ foreign exchange reserves more often and more transparently. I emphasize that the purpose of this section is to examine how the model’s predictions change once the assumption about common knowledge of the past is relaxed rather than to present a more realistic representation of the world.
5 Conclusion

In this paper, I have theoretically examined the implications of central bank transparency during foreign exchange interventions. The central feature of all my models is that investors are heterogeneously informed about both interventions and fundamentals. Information about future fundamentals is embedded in the current exchange rate so that investors learn about these fundamentals when they observe the price of foreign currency. I have analyzed the effects of a transparent intervention policy in both two-period and infinite-horizon settings and in environments in which investors both do and do not have common knowledge of the past. I have also considered the effects of signalling in these environments. In all cases, this paper has identified and emphasized two distinct effects of transparency. The first is the truth-telling effect, which corresponds to the fact that any parameters the central bank reveals to investors eliminate the role of the exchange rate as a signal of those parameters. The second is the signal-precision effect, which corresponds to the fact that any parameters the central bank reveals to investors increase the precision of the exchange rate as a signal of other, still-unknown parameters. The truth-telling effect directly raises expectations of parameters for which average beliefs are too low, while the signal-precision effect indirectly lowers expectations of parameters for which average beliefs are too low. I find that the truth-telling effect grows relative to the signal-precision effect as the extent of information revelation increases.

The main prediction of the paper is that central bank transparency can in fact magnify any existing misalignment between the exchange rate and fundamentals. This occurs if the signal-precision effect of transparency is larger than the truth-telling effect of transparency, which is the case if a central bank can credibly reveal only partial information to market participants. The most important implication of this result is that a policy of ambiguity will often increase the effectiveness of central bank intervention during periods of crisis and large capital outflows. In these episodes, asymmetric information, pro-cyclical liquidity provision, and psychology often lead to excessive sales of risky assets and cause risky countries’ currencies to be undervalued. This prediction and the intuition behind it match well with the justification that central banks often provide for their ambiguous intervention policies.
A Appendix: Benchmark Two-Period Model

This appendix presents the proofs of Theorems 2.2, 2.3, 2.4, and 3.3, and Corollary 2.5.

Proof of Theorem 2.2 Suppose that the exchange rate in period two is normally distributed conditional on investor \( i \)'s information set. Then, the investors’ problem (2.1) is a standard CARA-normal maximization problem, and the demand for peso bonds by investor \( i \) is given by

\[
 b_{i1} = \frac{E_{i1}[e_2] - e_1 + \mu_1}{\gamma \text{Var}_{i1}[e_2]}.
\]

(A.1)

Suppose also that \( \text{Var}_{i1}[e_2] \) is equal for all \( i \in [0, 1] \) and hence that \( \text{Var}_1[e_2] = \text{Var}_{i1}[e_2] \). It follows that \( \sigma_1^2 = \text{Var}_{i1}[e_2] \) and that the aggregate investor demand for peso bonds in period one is given by

\[
 B_1 = \frac{E_1[e_2] - e_1 + \mu_1}{\gamma \sigma_1^2},
\]

(A.2)

which, together with the market clearing condition in the peso bond market, implies that

\[
e_1 = \frac{E_1[e_2] + \gamma \sigma_1^2(\nu_1 + \xi_1)}{\nu_1 + \gamma \sigma_1^2}.
\]

(A.3)

The exchange rate in period two is given by \( e_2 = \theta_\mu \mu_2 + \theta_\nu \nu_1 + \kappa_2 \), so that \( E_{i1}[e_2] = \theta_\mu E_{i1}[\mu_2] + \theta_\nu E_{i1}[\nu_1]S_1 \). I am interested in the rational expectations equilibrium of this economy, so investors must take into account the fact that the value of the exchange rate in period one is a signal of both \( \mu_2 \) and \( \nu_1 \). In other words, the exchange rate \( e_1 \) is part of investors’ information sets in period one.

Let \( E_i[\cdot] \), \( \text{Var}_i[\cdot] \), and \( \text{Cov}_i[\cdot, \cdot] \) denote, respectively, the expected value, variance, and covariance with respect to the information set consisting only of \( \mu_1 \) and the private signals \( x_i \) and \( y_i \). In equilibrium, the exchange rate in period one is of the form

\[
e_1 = \mu_1 + \theta_\mu \mu_2 + (\theta_\nu + \gamma \sigma_1^2)\nu_1 + \lambda \xi_1,
\]

(A.4)

so that \( \text{Cov}_i[\mu_2, e_1] = \theta_\mu \sigma_1^2 \) and \( \text{Cov}_i[\nu_1, e_1] = (\theta_\nu + \gamma \sigma_1^2)\sigma_1^2 \). The goal is to solve for the undetermined coefficients \( \lambda \) and \( \sigma_1^2 \) in equation (A.4). Standard Bayesian inference implies that the exchange rate in period two is normally distributed conditional on investor \( i \)'s information set (this justifies the assumption of conditional normality) and that

\[
 E_{i1}[\mu_2] = E_i[\mu_2] + \frac{\text{Cov}_i[\mu_2, e_1]}{\text{Var}_i[e_1]}(e_1 - E_i[e_1])
\]

\[
 = x_i + \frac{\theta_\mu \sigma_1^2}{\theta_\mu^2 \sigma_1^2 + (\theta_\nu + \gamma \sigma_1^2)^2 \sigma_1^2 + \lambda^2 \sigma_1^2} (e_1 - \mu_1 - \theta_\mu x_i - (\theta_\nu + \gamma \sigma_1^2)y_i),
\]

and

\[
 E_{i1}[\nu_1] = E_i[\nu_1] + \frac{\text{Cov}_i[\nu_1, e_1]}{\text{Var}_i[e_1]}(e_1 - E_i[e_1])
\]

\[
 = y_i + \frac{(\theta_\nu + \gamma \sigma_1^2)\sigma_1^2}{\theta_\mu^2 \sigma_1^2 + (\theta_\nu + \gamma \sigma_1^2)^2 \sigma_1^2 + \lambda^2 \sigma_1^2} (e_1 - \mu_1 - \theta_\mu x_i - (\theta_\nu + \gamma \sigma_1^2)y_i).
\]
It follows, then, that
\[ E_1[\mu_2] = \mu_2 + \frac{\lambda \theta_\mu \sigma_\epsilon^2}{\theta_\mu^2 \sigma_\epsilon^2 + (\theta_\nu + \gamma \sigma_1^2) \sigma_\eta^2 + \lambda^2 \sigma_\xi^2} \]  
(A.5)
and
\[ E_1[\nu_1] = \nu_1 + \frac{\lambda (\theta_\nu + \gamma \sigma_1^2) \sigma_\eta^2}{\theta_\nu^2 \sigma_\epsilon^2 + (\theta_\nu + \gamma \sigma_1^2) \sigma_\eta^2 + \lambda^2 \sigma_\xi^2} \]  
(A.6)
Substituting equations (A.5) and (A.6) into equation (A.3) above yields
\[ e_1 = \mu_1 + \theta_\mu \mu_2 + (\theta_\nu + \gamma \sigma_1^2) \nu_1 + \left( \frac{\lambda \theta_\mu \sigma_\epsilon^2 + \lambda \theta_\nu (\theta_\nu + \gamma \sigma_1^2) \sigma_\eta^2}{\theta_\mu^2 \sigma_\epsilon^2 + (\theta_\nu + \gamma \sigma_1^2) \sigma_\eta^2 + \lambda^2 \sigma_\xi^2} \right) \xi_1. \]  
(A.7)

The next step is to solve for \( \sigma_1^2 \), the conditional variance of the exchange rate in period two. Because \( e_2 = \theta_\mu \mu_2 + \theta_\nu \nu_1 + \kappa_2 \), this conditional variance is given by
\[ \sigma_1^2 = \theta_\mu^2 \text{Var}_1[\mu_2] + \theta_\nu^2 \text{Var}_1[\nu_1] + \sigma_\kappa^2 + 2 \theta_\nu \theta_\nu \text{Cov}_1[\mu_2, \nu_1]. \]  
As before, standard Bayesian inference implies that
\[ \text{Var}_1[\mu_2] = \text{Var}_1[\nu_1] = \frac{\text{Cov}_1[\mu_2, e_1]^2}{\text{Var}_1[e_1]} = \sigma^2 - \frac{\theta_\mu^2 \sigma_\epsilon^2}{\theta_\mu^2 \sigma_\epsilon^2 + (\theta_\nu + \gamma \sigma_1^2) \sigma_\eta^2 + \lambda^2 \sigma_\xi^2}, \]
and that
\[ \frac{\text{Cov}_1[\mu_2, \nu_1]}{\text{Var}_1[\nu_1]} = \frac{\text{Cov}_1[\mu_2, e_1]}{\text{Var}_1[e_1]} = \frac{-\theta_\nu (\theta_\nu + \gamma \sigma_1^2) \sigma_\eta^2}{\theta_\mu^2 \sigma_\epsilon^2 + (\theta_\nu + \gamma \sigma_1^2) \sigma_\eta^2 + \lambda^2 \sigma_\xi^2}. \]
\[ \text{Var}_1[\mu_2, \nu_1] = \text{Cov}_1[\mu_2, \nu_1] = \text{Cov}_1[\mu_2, e_1] \text{Cov}_1[\nu_1, e_1] \text{Var}_1[e_1] = -\frac{\theta_\nu (\theta_\nu + \gamma \sigma_1^2) \sigma_\eta^2}{\theta_\mu^2 \sigma_\epsilon^2 + (\theta_\nu + \gamma \sigma_1^2) \sigma_\eta^2 + \lambda^2 \sigma_\xi^2}. \]
It follows, then, that
\[ \sigma_1^2 = \theta_\mu^2 \sigma_\epsilon^2 + \theta_\nu^2 \sigma_\eta^2 + \sigma_\kappa^2 - \left( \frac{\theta_\mu^2 \sigma_\epsilon^2 + \theta_\nu (\theta_\nu + \gamma \sigma_1^2) \sigma_\eta^2}{\theta_\mu^2 \sigma_\epsilon^2 + (\theta_\nu + \gamma \sigma_1^2) \sigma_\eta^2 + \lambda^2 \sigma_\xi^2} \right)^2. \]  
(A.8)
Note that this justifies the assumption that the conditional variance is equal for all investors \( i \). The proof of existence is complete once we equate the undetermined coefficients from equation (A.4) above with the implied expressions from equations (A.7) and (A.8).

It can be verified that there exists only one rational solution to equation (2.5) and one rational solution to equation (2.6). These equations can separately be solved analytically, but it is not possible to obtain an analytic solution for both equations simultaneously. However, it can be verified numerically that there exists one unique rational solution to this system of equations. This unique rational solution corresponds to the unique equilibrium exchange rate \( e_1 \). \( \Box \)

**Proof of Theorem 2.3** This proof follows the proof of Theorem 2.2 very closely. If I again assume that the exchange rate in period two is normally distributed conditional on investor \( i \)'s information set, then it can be shown in a similar manner to before that market clearing in the peso bond market implies that \( e_1 = \overline{E}_1[e_2] + \mu_1 + \gamma \sigma_1^2 (\nu_1 + \xi_1) \). In equilibrium, this exchange rate is of the form \( e_1 = \mu_1 + \theta_\mu \mu_2 + (\theta_\nu + \gamma \sigma_1^2) \nu_1 + \lambda \xi_1 \), so that standard Bayesian inference both justifies the assumption of conditional normality and yields aggregate expectations about \( \mu_2 \) that are similar.
to those when $\nu_1$ remained unknown:

$$E_1[\mu_2] = \mu_2 + \frac{\tilde{\lambda}\theta_\mu \sigma_\mu^2}{\theta_\mu^2 \sigma_\nu^2 + \tilde{\lambda}^2 \sigma_\xi^2} \xi_1. \quad (A.9)$$

Substituting this equation into the expression for the exchange rate in period one yields

$$\mu_1 + \theta_\mu \mu_2 + (\theta_\nu + \gamma \tilde{\sigma}_1^2) \nu_1 + \left(\frac{\tilde{\lambda}\theta_\mu \sigma_\mu^2}{\theta_\mu^2 \sigma_\nu^2 + \tilde{\lambda}^2 \sigma_\xi^2} + \gamma \tilde{\sigma}_1^2\right) \xi_1. \quad (A.10)$$

The conditional variance of the exchange rate in period two, $\tilde{\sigma}_1^2$, is also determined in a manner similar to the previous proof. In particular, standard Bayesian inference implies that

$$\sigma_\nu^2 = \frac{\theta_\mu^2 \sigma_\nu^2}{\theta_\mu^2 \sigma_\nu^2 + \tilde{\lambda}^2 \sigma_\xi^2},$$

where $\sigma_\nu^2$ is the conditional variance of the exchange rate in period two.

This shows that the conditional variance is equal for all investors $i$, and together with equation (A.10) completes the proof of existence. In this simpler case, the system of equations (2.12) and (2.13) can be solved analytically. There exists only one rational solution to this system and, as in the previous proof, this unique rational solution corresponds to the unique equilibrium exchange rate $\hat{e}_1$. \hfill \Box

**Proof of Theorem 2.4** I first show that $\lambda > \tilde{\lambda}$ whenever $\lambda < \theta_\nu + \gamma \sigma_1^2$ and $\lambda < \tilde{\lambda}$ whenever $\lambda > \theta_\nu + \gamma \sigma_1^2$, and then show that $\lambda - \gamma \tilde{\sigma}_1^2 > \theta_\nu$ whenever $\lambda - \gamma \sigma_1^2 > \theta_\nu$ and $\lambda - \gamma \tilde{\sigma}_1^2 < \theta_\nu$ whenever $\lambda - \gamma \sigma_1^2 < \theta_\nu$. Together, these two facts imply that $\lambda > \tilde{\lambda}$ whenever $\theta_\nu > \lambda - \gamma \tilde{\sigma}_1^2$ and $\lambda < \tilde{\lambda}$ whenever $\theta_\nu < \lambda - \gamma \tilde{\sigma}_1^2$.

According to equation (2.6) from Theorem 2.2,

$$\sigma_1^2 = \theta_\mu^2 \sigma_\nu^2 + \theta_\nu^2 \sigma_\xi^2 + \sigma_\nu^2 - \frac{\theta_\mu^4 \sigma_\nu^4 + 2\theta_\mu^2 \theta_\nu \sigma_\nu \sigma_\xi^2}{\theta_\mu^2 \sigma_\nu^2 + (\theta_\nu + \gamma \sigma_1^2)^2 \sigma_\xi^2 + \lambda^2 \sigma_\xi^2} - \frac{\theta_\nu^2 \sigma_\nu^4}{\theta_\mu^2 \sigma_\nu^2 + (\theta_\nu + \gamma \sigma_1^2)^2 \sigma_\xi^2 + \lambda^2 \sigma_\xi^2} - \frac{\theta_\nu^2 \sigma_\nu^4}{\theta_\mu^2 \sigma_\nu^2 + (\theta_\nu + \gamma \sigma_1^2)^2 \sigma_\xi^2 + \lambda^2 \sigma_\xi^2} - \frac{\theta_\nu^2 \sigma_\nu^4}{\theta_\mu^2 \sigma_\nu^2 + (\theta_\nu + \gamma \sigma_1^2)^2 \sigma_\xi^2 + \lambda^2 \sigma_\xi^2}, \quad (A.12)$$

so that by equation (2.5) also

$$\lambda = \frac{\lambda \theta_\mu^2 \sigma_\nu^2 + \lambda \theta_\nu (\theta_\nu + \gamma \sigma_1^2) \sigma_\nu^2 + \gamma \sigma_\nu^2}{\theta_\mu^2 \sigma_\nu^2 + (\theta_\nu + \gamma \sigma_1^2)^2 \sigma_\xi^2 + \lambda^2 \sigma_\xi^2 + \gamma \sigma_\nu^2 + \frac{\gamma (\gamma \sigma_1^2)^2 \theta_\mu^2 \sigma_\nu^2 \sigma_\xi^2 + (\gamma \sigma_1^2)^2 \theta_\nu \sigma_\nu^2 \sigma_\xi^2 + \gamma \lambda^2 \sigma_1^2 \theta_\mu^2 \sigma_\nu^2 \sigma_\xi^2 + \gamma \lambda^2 \sigma_1^2 \theta_\nu \sigma_\nu^2 \sigma_\xi^2}{\theta_\mu^2 \sigma_\nu^2 + (\theta_\nu + \gamma \sigma_1^2)^2 \sigma_\xi^2 + \lambda^2 \sigma_\xi^2}. \quad (A.13)$$
Similarly, equation (2.13) from Theorem 2.3 implies that 
\[ \hat{\sigma}^2_1 = \sigma^2_\xi + \frac{\lambda^2 \theta^2_\mu \sigma^2_\varepsilon}{\theta^2_\mu \sigma^2_\varepsilon + \lambda^2 \sigma^2_\xi} \]
so that by equation (2.12) also
\[ \hat{\lambda} = \frac{\lambda^2 \theta^2_\mu \sigma^2_\varepsilon + \lambda^2 \gamma \theta^2_\mu \sigma^2_\varepsilon}{\theta^2_\mu \sigma^2_\varepsilon + \lambda^2 \sigma^2_\xi} + \gamma \sigma^2_\xi. \]  
(A.14)

Equations (A.13) and (A.14) imply that
\[ \lambda^2 \sigma^2_\xi (\lambda - \gamma \theta^2_\mu \sigma^2_\varepsilon - \gamma \sigma^2_\nu) = \gamma \sigma^2_\nu \theta^2_\mu \sigma^2_\varepsilon + \gamma \sigma^2_\xi (\theta^2_\mu \gamma^2 \sigma^2_\varepsilon + \lambda^2 \theta^2_\mu \sigma^2_\xi + \sigma^2_\nu (\theta_\nu + \gamma \sigma^2_1) - \lambda \sigma^2_\nu (\theta_\nu + \gamma \sigma^2_1)) , \]
and
\[ \lambda^2 \sigma^2_\xi (\hat{\lambda} - \gamma \theta^2_\mu \sigma^2_\varepsilon - \gamma \sigma^2_\nu) = \gamma \sigma^2_\nu \theta^2_\mu \sigma^2_\varepsilon. \]  
(A.15)

Let
\[ \Delta = \theta^2_\mu \gamma^2 \sigma^2_\varepsilon + \lambda^2 \theta^2_\mu \sigma^2_\xi + \sigma^2_\nu (\theta_\nu + \gamma \sigma^2_1)^2 - \lambda \sigma^2_\nu (\theta_\nu + \gamma \sigma^2_1), \]  
(A.16)
so that
\[ \lambda^2 \sigma^2_\xi (\lambda - \gamma \theta^2_\mu \sigma^2_\varepsilon - \gamma \sigma^2_\nu) = \gamma \sigma^2_\nu \theta^2_\mu \sigma^2_\varepsilon + \gamma \sigma^2_\xi \Delta \]  
(A.17)
and also
\[ \lambda = \theta^2_\mu \sigma^2_\varepsilon + \sigma^2_\nu + \frac{\gamma \sigma^2_\xi \theta^2_\mu \sigma^2_\varepsilon}{\lambda^2 \sigma^2_\xi} + \lambda \sigma^2_\xi \Delta \]  
(A.18)

It follows that \( \hat{\lambda} \) is increasing in \( \Delta \) with \( \lambda = \hat{\lambda} \) if and only if \( \Delta = 0 \) or \( \sigma^2_\eta = 0 \). Equation (A.18) also implies that \( \lambda > \hat{\lambda} \) whenever \( \Delta > 0 \) and \( \sigma^2_\eta > 0 \), and \( \lambda < \hat{\lambda} \) whenever \( \Delta < 0 \) and \( \sigma^2_\eta > 0 \). The bulk of this proof amounts to showing that \( \Delta > 0 \) whenever \( \theta_\nu > \lambda - \gamma \sigma^2_1 \) and that \( \Delta < 0 \) whenever \( \theta_\nu < \lambda - \gamma \sigma^2_1 \).

Before proving these inequalities, note that equation (2.5) implies that
\[ \lambda - \gamma \sigma^2_1 = \frac{\lambda \theta^2_\mu \sigma^2_\varepsilon + \lambda \theta_\nu (\theta_\nu + \gamma \sigma^2_1) \sigma^2_\eta}{\theta^2_\mu \sigma^2_\varepsilon + (\theta_\nu + \gamma \sigma^2_1)^2 \sigma^2_\eta + \lambda^2 \sigma^2_\xi}, \]
so that
\[ (\lambda - \gamma \sigma^2_1)^2 (\theta^2_\mu \sigma^2_\varepsilon + (\theta_\nu + \gamma \sigma^2_1)^2 \sigma^2_\eta + \lambda^2 \sigma^2_\xi) = \lambda \theta^2_\mu \sigma^2_\varepsilon + \lambda \theta_\nu (\theta_\nu + \gamma \sigma^2_1) \sigma^2_\eta. \]  
(A.19)

Some algebra then yields
\[ \lambda^2 \sigma^2_\xi (\lambda - \gamma \sigma^2_1) = \gamma \sigma^2_\nu \theta^2_\mu \sigma^2_\varepsilon + (\theta_\nu + \gamma \sigma^2_1) (\lambda \theta_\nu - (\lambda - \gamma \sigma^2_1) (\theta_\nu + \gamma \sigma^2_1)) \sigma^2_\eta \]
\[ = \gamma \sigma^2_\nu \theta^2_\mu \sigma^2_\varepsilon + \gamma \sigma^2_1 (\theta_\nu + \gamma \sigma^2_1)(\theta_\nu + \gamma \sigma^2_1) - \lambda \sigma^2_\nu, \]
so that
\[ \lambda^2 \sigma^2_\xi \theta_\nu = \frac{\gamma \sigma^2_\nu \theta^2_\mu \sigma^2_\varepsilon + \gamma \sigma^2_1 (\theta_\nu + \gamma \sigma^2_1)(\theta_\nu + \gamma \sigma^2_1) - \lambda \sigma^2_\nu}{\lambda - \gamma \sigma^2_1}. \]  
(A.20)

Equation (A.20) is crucial to the proof of Theorem 2.4. It implies that \( \lambda^2 \sigma^2_\xi \theta_\nu > \gamma \sigma^2_\nu \theta^2_\mu \sigma^2_\varepsilon \) and
\[ \frac{\lambda - \gamma \sigma^2_1}{\gamma \sigma^2_1} > \frac{\theta^2_\mu \sigma^2_\varepsilon}{\lambda \sigma^2_\xi}, \]  
whenever \( \lambda < \theta_\nu + \gamma \sigma^2_1 \), and also that \( \lambda^2 \sigma^2_\xi \theta_\nu < \gamma \sigma^2_\nu \theta^2_\mu \sigma^2_\varepsilon \) and \( \frac{\lambda - \gamma \sigma^2_1}{\gamma \sigma^2_1} < \frac{\theta^2_\mu \sigma^2_\varepsilon}{\lambda \sigma^2_\xi} \) whenever \( \lambda > \theta_\nu + \gamma \sigma^2_1 \).

Suppose that \( \theta_\nu > \lambda - \gamma \sigma^2_1 \), so that \( \lambda < \theta_\nu + \gamma \sigma^2_1 \). As I just showed in equation (A.20), this
implies that both $\lambda^2 \sigma_\varepsilon^2 \theta_\nu > \gamma \sigma_\varepsilon^2 \theta_\mu \sigma_\varepsilon^2$ and $\frac{\lambda - \gamma \sigma_1^2}{\gamma \sigma_1^2} < \frac{\theta_\mu^2 \sigma_\varepsilon^2}{\lambda \sigma_\varepsilon^2}$. It follows that

$$
\gamma^2 \sigma_\varepsilon^4 \theta_\mu \sigma_\varepsilon^2 + \lambda^2 \theta_\nu^2 \sigma_\varepsilon^2 + \sigma_\varepsilon^2 (\theta_\nu + \gamma \sigma_1^2) > (\gamma \sigma_1^2)^2 \theta_\mu \sigma_\varepsilon^2 + \gamma \sigma_1^2 \theta_\nu \sigma_\varepsilon^2 + \sigma_\varepsilon^2 (\theta_\nu + \gamma \sigma_1^2)^2
\hspace{1cm} = \gamma \sigma_1^2 (\theta_\nu + \gamma \sigma_1^2) \theta_\mu \sigma_\varepsilon^2 + \sigma_\varepsilon^2 (\theta_\nu + \gamma \sigma_1^2)^2
\hspace{1cm} = (\theta_\nu + \gamma \sigma_1^2)^2 \left( \gamma \sigma_\varepsilon^2 + \gamma \theta_\mu \sigma_\varepsilon^2 \frac{\gamma \sigma_1^2}{\theta_\nu + \gamma \sigma_1^2} \right).
\quad \text{(A.21)}
$$

Suppose now that $\Delta \leq 0$. It follows by equation (A.18), then, that $\lambda \leq \gamma \theta_\mu^2 \sigma_\varepsilon^2 + \gamma \sigma_1^2 + \frac{\gamma \sigma_\varepsilon^2 \theta_\mu^2 \sigma_\varepsilon^2}{\lambda \sigma_\varepsilon^2}$ and hence

$$
\gamma \sigma_1^2 = \lambda - \frac{(\lambda - \gamma \sigma_1^2) \gamma \sigma_1^2}{\gamma \sigma_1^2} = \gamma \theta_\mu^2 \sigma_\varepsilon^2 + \gamma \sigma_1^2 + \frac{\gamma \sigma_\varepsilon^2 \theta_\mu^2 \sigma_\varepsilon^2}{\lambda \sigma_\varepsilon^2} - (\lambda - \gamma \sigma_1^2).
\quad \text{(A.22)}
$$

Because $\frac{\lambda - \gamma \sigma_1^2}{\gamma \sigma_1^2} > \frac{\theta_\mu^2 \sigma_\varepsilon^2}{\lambda \sigma_\varepsilon^2}$ in this case, inequality (A.22) implies that

$$
\gamma \sigma_1^2 < \gamma \theta_\mu^2 \sigma_\varepsilon^2 + \gamma \sigma_1^2 + \frac{(\lambda - \gamma \sigma_1^2) \gamma \sigma_1^2}{\gamma \sigma_1^2} - (\lambda - \gamma \sigma_1^2) = \gamma \theta_\nu \sigma_\varepsilon^2 + \gamma \sigma_1^2 + \frac{\lambda - \gamma \sigma_1^2}{\gamma \sigma_1^2} (\gamma \sigma_1^2 - \gamma \sigma_1^2),
$$

which then implies that

$$
\gamma \sigma_1^2 \left( 1 + \frac{\lambda - \gamma \sigma_1^2}{\gamma \sigma_1^2} \right) < \gamma \sigma_1^2 \left( 1 + \frac{\lambda - \gamma \sigma_1^2}{\gamma \sigma_1^2} \right) + \gamma \theta_\mu^2 \sigma_\varepsilon^2.
\quad \text{(A.23)}
$$

Inequality (A.23) yields

$$
\gamma \sigma_1^2 < \gamma \sigma_1^2 + \frac{\lambda - \gamma \sigma_1^2}{\gamma \sigma_1^2} \gamma \sigma_1^2,
$$

from which it follows that

$$
\lambda(\theta_\nu + \gamma \sigma_1^2) \gamma \sigma_1^2 < \lambda(\theta_\nu + \gamma \sigma_1^2) \gamma \sigma_1^2 + (\theta_\nu + \gamma \sigma_1^2) \gamma \sigma_1^2 \gamma \sigma_1^2
\hspace{1cm} < (\theta_\nu + \gamma \sigma_1^2)^2 \left( \gamma \sigma_1^2 + \gamma \theta_\mu \sigma_\varepsilon^2 \frac{\gamma \sigma_1^2}{\theta_\nu + \gamma \sigma_1^2} \right).
\quad \text{(A.24)}
$$

Of course, inequality (A.24) together with inequality (A.21) from above implies that

$$
\lambda(\theta_\nu + \gamma \sigma_1^2) \gamma \sigma_1^2 < \gamma^2 \sigma_\varepsilon^4 \theta_\mu \sigma_\varepsilon^2 + \lambda^2 \theta_\nu^2 \sigma_\varepsilon^2 + \sigma_\varepsilon^2 (\theta_\nu + \gamma \sigma_1^2)^2,
$$

which, because $\Delta = \theta_\mu^2, \gamma \sigma_1^2 \sigma_\varepsilon^2 + \lambda^2 \theta_\nu^2 \sigma_\varepsilon^2 + \sigma_\varepsilon^2 (\theta_\nu + \gamma \sigma_1^2)^2 - \lambda \sigma_\varepsilon^2 (\theta_\nu + \gamma \sigma_1^2)$ by equation (A.16), contradicts the assumption that $\Delta \leq 0$ and proves that $\Delta > 0$ whenever $\lambda < \theta_\nu + \gamma \sigma_1^2$.

Suppose that $\theta_\nu < \lambda - \gamma \sigma_1^2$, so that $\lambda > \theta_\nu + \gamma \sigma_1^2$. As shown above in equation (A.20), this implies that both $\lambda^2 \sigma_\varepsilon^2 \theta_\nu < \gamma \sigma_\varepsilon^2 \theta_\mu \sigma_\varepsilon^2$ and $\frac{\lambda - \gamma \sigma_1^2}{\gamma \sigma_1^2} < \frac{\theta_\mu^2 \sigma_\varepsilon^2}{\lambda \sigma_\varepsilon^2}$. It follows that

$$
\gamma^2 \sigma_\varepsilon^4 \theta_\mu \sigma_\varepsilon^2 + \lambda^2 \theta_\nu^2 \sigma_\varepsilon^2 + \sigma_\varepsilon^2 (\theta_\nu + \gamma \sigma_1^2)^2 < (\gamma \sigma_1^2)^2 \theta_\mu \sigma_\varepsilon^2 + \gamma \sigma_1^2 \theta_\nu \sigma_\varepsilon^2 + \sigma_\varepsilon^2 (\theta_\nu + \gamma \sigma_1^2)^2
\hspace{1cm} = \gamma \sigma_1^2 (\theta_\nu + \gamma \sigma_1^2) \theta_\mu \sigma_\varepsilon^2 + \sigma_\varepsilon^2 (\theta_\nu + \gamma \sigma_1^2)^2
\hspace{1cm} = (\theta_\nu + \gamma \sigma_1^2)^2 \left( \gamma \sigma_\varepsilon^2 + \gamma \theta_\mu \sigma_\varepsilon^2 \frac{\gamma \sigma_1^2}{\theta_\nu + \gamma \sigma_1^2} \right).
\quad \text{(A.25)}
$$
Suppose now that $\Delta \geq 0$. It follows by equation (A.18), then, that $\lambda \geq \gamma \theta_{\mu}^2 \sigma_\varepsilon^2 + \gamma \sigma_k^2 + \frac{\sigma_\tau^2 \theta_{\mu}^2 \sigma_\varepsilon^2}{\lambda \sigma_\xi^2}$ and hence

$$\gamma \sigma_1^2 = \lambda - (\lambda - \gamma \sigma_1^2) \geq \gamma \theta_{\mu}^2 \sigma_\varepsilon^2 + \gamma \sigma_k^2 + \frac{\sigma_\tau^2 \theta_{\mu}^2 \sigma_\varepsilon^2}{\lambda \sigma_\xi^2} - (\lambda - \gamma \sigma_1^2). \tag{A.26}$$

Because $\frac{\lambda - \gamma \sigma_1^2}{\gamma \sigma_1^2} < \frac{\theta_{\mu}^2 \sigma_\varepsilon^2}{\lambda \sigma_\xi^2}$ in this case, inequality (A.26) implies that

$$\gamma \sigma_1^2 > \gamma \theta_{\mu}^2 \sigma_\varepsilon^2 + \gamma \sigma_k^2 + \frac{(\lambda - \gamma \sigma_1^2) \gamma \sigma_k^2}{\gamma \sigma_1^2} - (\lambda - \gamma \sigma_1^2) = \gamma \theta_{\mu}^2 \sigma_\varepsilon^2 + \gamma \sigma_k^2 + \frac{\lambda - \gamma \sigma_1^2}{\gamma \sigma_1^2} (\gamma \sigma_k^2 - \gamma \sigma_1^2),$$

which then implies that

$$\gamma \sigma_1^2 \left(1 + \frac{\lambda - \gamma \sigma_1^2}{\gamma \sigma_1^2}\right) > \gamma \sigma_k^2 \left(1 + \frac{\lambda - \gamma \sigma_1^2}{\gamma \sigma_1^2}\right) + \gamma \theta_{\mu}^2 \sigma_\varepsilon^2. \tag{A.27}$$

Inequality (A.27) yields

$$\gamma \sigma_1^2 > \gamma \sigma_k^2 + \gamma \theta_{\mu}^2 \sigma_\varepsilon^2 \frac{\gamma \sigma_1^2}{\lambda},$$

from which it follows that

$$\lambda(\theta_{\nu} + \gamma \sigma_1^2) \gamma \sigma_1^2 > \lambda(\theta_{\nu} + \gamma \sigma_1^2) \gamma \sigma_k^2 + (\theta_{\nu} + \gamma \sigma_1^2) \gamma \theta_{\mu}^2 \sigma_\varepsilon^2 \gamma \sigma_1^2 \gamma \sigma_1^2 > (\theta_{\nu} + \gamma \sigma_1^2)^2 \left(\gamma \sigma_k^2 + \gamma \theta_{\mu}^2 \sigma_\varepsilon^2 \frac{\gamma \sigma_1^2}{\theta_{\nu} + \gamma \sigma_1^2}\right). \tag{A.28}$$

Of course, inequality (A.28) together with inequality (A.25) from above implies that

$$\lambda(\theta_{\nu} + \gamma \sigma_1^2) \gamma \sigma_1^2 > \gamma \sigma_k^2 + \gamma \theta_{\mu}^2 \sigma_\varepsilon^2 + \lambda \theta_{\mu}^2 \sigma_\varepsilon^2 + \sigma_k^2 (\theta_{\nu} + \gamma \sigma_1^2)^2,$$

which, because $\Delta = \theta_{\mu}^2 \gamma \sigma_1^2 \sigma_k^2 + \lambda \theta_{\mu}^2 \sigma_\varepsilon^2 + \sigma_k^2 (\theta_{\nu} + \gamma \sigma_1^2)^2 - \lambda \sigma_1^2 (\theta_{\nu} + \gamma \sigma_1^2)$, contradicts the assumption that $\Delta \geq 0$ and proves that $\Delta < 0$ whenever $\lambda > \theta_{\nu} + \gamma \sigma_1^2$. These two inequalities also imply that $\Delta = 0$ if and only if $\lambda = \theta_{\nu} + \gamma \sigma_1^2$, which by equation (A.18) and continuity implies that if $\lambda > \hat{\lambda}$ (for $\lambda < \hat{\lambda}$) for all $\sigma_\eta^2 > 0$, then $\lambda > \hat{\lambda}$ (for $\lambda < \hat{\lambda}$) for all $\sigma_\eta^2 > 0$.\(^{18}\)

The final step of the proof is to show that $\hat{\lambda} - \gamma \sigma_1^2 > \theta_{\nu}$ whenever $\hat{\lambda} - \gamma \sigma_1^2 > \theta_{\nu}$ and $\hat{\lambda} - \gamma \sigma_1^2 < \theta_{\nu}$ whenever $\hat{\lambda} - \gamma \sigma_1^2 < \theta_{\nu}$. Suppose that $\hat{\lambda} - \gamma \sigma_1^2 > \theta_{\nu}$, $\hat{\lambda} - \gamma \sigma_1^2 > \theta_{\nu}$. As was just proved, this implies that $\hat{\lambda} - \gamma \sigma_1^2 > \theta_{\nu}$, for all $\sigma_\eta^2 > 0$. Since $\hat{\lambda} = \hat{\lambda}$ and $\sigma_\eta^2 = \sigma_\eta^2$ for $\sigma_\eta^2 = 0$, it follows by continuity that $\hat{\lambda} - \gamma \sigma_1^2 = \theta_{\nu} = \hat{\lambda} - \gamma \sigma_1^2$ if $\sigma_\eta^2 = 0$. But, I just proved that this implies that $\hat{\lambda} - \gamma \sigma_1^2 = \theta_{\nu}$ for all $\sigma_\eta^2 > 0$ as well, so there is a contradiction and it must be that $\hat{\lambda} - \gamma \sigma_1^2 > \theta_{\nu}$. A similar argument proves that $\hat{\lambda} - \gamma \sigma_1^2 < \theta_{\nu}$ whenever $\hat{\lambda} - \gamma \sigma_1^2 < \theta_{\nu}$ as well.

\(^{18}\)This requires that also $\frac{\partial \hat{\lambda}}{\partial \sigma_\eta} = \frac{\partial \gamma \sigma_1^2}{\partial \sigma_\eta} = 0$ whenever $\lambda = \theta_{\nu} + \gamma \sigma_1^2$ (and hence $\Delta = 0$), which is not difficult to show.

\[ \boxempty \]

Proof of Corollary 2.5 Recall that $e_1 = \mu_1 + \theta_{\mu} \mu_2 + (\theta_{\nu} + \gamma \sigma_1^2) \nu_1 + \lambda \xi_1$ and that a similar expression describes $\hat{e}_1$, with $\lambda$ and $\sigma_1^2$ replacing $\lambda$ and $\sigma_1^2$, respectively. It is immediate, then, that $\hat{e}_1 - e_1$ is strictly increasing in $\xi_1$ whenever $\hat{\lambda} > \lambda$ and that for $\xi_1$ large enough, this quantity is greater than zero regardless of the value of $\nu_1$. This implies the existence of a unique threshold $\xi \in \mathbb{R}$ such that $\hat{e}_1 > e_1$ if and only if $\xi_1 > \xi$. \[ \boxempty \]
Proof of Theorem 3.3 Suppose that the foreign central bank announces its intervention if and only if \( \xi_1 \geq \hat{\xi}(\nu_1) \), where \( \hat{\xi}(\nu_1) \) is positive, bounded, and decreasing in \( \nu_1 \). It is important to emphasize that investors only know the exact value of \( \hat{\xi} \) if they learn \( \nu_1 \) via a central bank announcement, otherwise they are only aware of the equilibrium relationship between these variables.

Suppose that \( \hat{\epsilon}_1 = \mu_1 + \theta_{\nu}\mu_2 + (\theta_{\nu} + \gamma \hat{\sigma}_1^2)(\nu_1 - S) + \lambda(\xi_1 - \hat{\xi}(\nu_1)) \), where \( \lambda \) and \( \hat{\sigma}_1^2 \) are given by the solution to equations (2.12) and (2.13) from Theorem 2.3. Because investors observe that \( \hat{\epsilon}_1 - \mu_1 - (\theta_{\nu} + \gamma \hat{\sigma}_1^2)(\nu_1 - S) + \hat{\lambda}(\xi_1 - \hat{\xi}(\nu_1)) \geq \hat{\lambda}(\xi_1 - \hat{\xi}(\nu_1)) \). Bayesian inference implies that for each investor \( i \), the distribution of \( \theta_{\nu}\mu_2 \) conditional on investor \( i \)'s information set is truncated normal, with mean \( \theta_{\nu}x_i + \frac{\theta_{\nu}^2 \hat{\sigma}_1^2}{\theta_{\nu}^2 \hat{\sigma}_1^2 + \lambda^2 \hat{\sigma}_1^2} (\hat{\epsilon}_1 - \mu_1 - (\theta_{\nu} + \gamma \hat{\sigma}_1^2)(\nu_1 - S) + \hat{\lambda}(\xi_1 - \theta_{\nu}x_i)) \), variance \( \frac{\lambda^2 \hat{\sigma}_1^2}{\theta_{\nu}^2 \hat{\sigma}_1^2 + \lambda^2 \hat{\sigma}_1^2} \), and truncation \( \theta_{\nu}\mu_2 \leq \hat{\epsilon}_1 - \mu_1 - (\theta_{\nu} + \gamma \hat{\sigma}_1^2)(\nu_1 - S) \).

The difference between the truncation and the mean of \( \theta_{\nu}\mu_2 \) is equal to

\[
\frac{\lambda^2 \hat{\sigma}_1^2}{\theta_{\nu}^2 \hat{\sigma}_1^2 + \lambda^2 \hat{\sigma}_1^2} (\hat{\epsilon}_1 - \mu_1 - (\theta_{\nu} + \gamma \hat{\sigma}_1^2)(\nu_1 - S) + \hat{\lambda}(\xi_1 - \mu_2 x_i)) - \hat{\lambda}(\xi_1).
\]

Because \( \hat{\xi}(\nu_1) \) is positive for all \( \nu_1 \in [-\bar{\nu}, \bar{\nu}] \) and \( \bar{\lambda}\sigma_\xi \to 0 \) as \( \sigma_\xi \to 0 \) (this is not hard to prove), it follows that this difference converges to a strictly negative value as \( \sigma_\xi \to 0 \). In this case, Lemma A.1 implies that

\[
\lim_{\sigma_\xi \to 0} E_{\nu_i}(T) \left[ e^{-\theta_{\nu}\mu_2} \right] = \lim_{\sigma_\xi \to 0} e^{-\left(\hat{\epsilon}_1 - \mu_1 - (\theta_{\nu} + \gamma \hat{\sigma}_1^2)(\nu_1 - S) + \frac{1}{2} \left( \frac{\lambda^2 \hat{\sigma}_1^2}{\theta_{\nu}^2 \hat{\sigma}_1^2 + \lambda^2 \hat{\sigma}_1^2} \right) \right)}.
\]

Utility is exponential, so equation (A.29) implies that each investor \( i \)'s demand for peso bonds in period one is satisfies

\[
\lim_{\sigma_\xi \to 0} b_{i1} = \lim_{\sigma_\xi \to 0} \frac{\theta_{\nu}\mu_2 + \hat{\lambda}(\xi_1 - \hat{\xi}(\nu_1)) + \theta_{\nu}\nu_1 - \hat{\epsilon}_1 + \mu_1}{\gamma \text{Var}_{\nu_i}(T)[e_2]},
\]

so that

\[
\lim_{\sigma_\xi \to 0} B_1 = \lim_{\sigma_\xi \to 0} \frac{\theta_{\nu}\mu_2 + \hat{\lambda}(\xi_1 - \hat{\xi}(\nu_1)) + \theta_{\nu}\nu_1 - \hat{\epsilon}_1 + \mu_1}{\gamma \hat{\sigma}_1^2}.
\]

This last equality together with the market clearing condition for the peso bond market implies that

\[
\lim_{\sigma_\xi \to 0} \hat{\epsilon}_1 = \lim_{\sigma_\xi \to 0} \mu_1 + \theta_{\nu}\mu_2 + (\theta_{\nu} + \gamma \hat{\sigma}_1^2)(\nu_1 - S) + \hat{\lambda}(\xi_1 - \hat{\xi}(\nu_1)), \tag{A.30}
\]

where \( \hat{\lambda} \) and \( \hat{\sigma}_1^2 \) are given by the solution to equations (2.12) and (2.13) from Theorem 2.3. Of course, if \( \hat{\epsilon}_1 \to \mu_1 + \theta_{\nu}\mu_2 + (\theta_{\nu} + \gamma \hat{\sigma}_1^2)(\nu_1 - S) + \hat{\lambda}(\xi_1 - \hat{\xi}(\nu_1)) \) as \( \sigma_\xi \to 0 \), then all of the above statements are true in the limit and it follows that the limit (A.30) indeed holds.

Suppose that \( \epsilon_1 = \mu_1 + \theta_{\nu}\mu_2 + (\theta_{\nu} + \gamma \hat{\sigma}_1^2)(\nu_1 - S) + \lambda\xi_1 \), where \( \lambda \) and \( \sigma_\xi^2 \) are given by the solution to equations (2.5) and (2.6) from Theorem 2.2. Because investors observe that the foreign
Bayesian inference implies that for each investor $i$, the distribution of $\theta_i \mu + \theta_i \nu_i$ is truncated normal, with mean
\[ \theta_i x_i + \theta_i y_i + \frac{\theta_i^2 \sigma_i^2}{\theta_i^2 \sigma_i^2 + (\theta_i + \gamma \sigma_i^2)^2 \sigma_i^2 + \lambda^2 \sigma_i^2} (e_1 - \mu_i - \theta_i x_i - (\theta_i + \gamma \sigma_i^2)(y_i - S)), \]
variance
\[ \frac{\theta_i^2 \sigma_i^2 + \theta_i^2 \sigma_i^2 - (\theta_i^2 \sigma_i^2 + (\theta_i + \gamma \sigma_i^2)^2 \sigma_i^2 + \lambda^2 \sigma_i^2)^2}{\theta_i^2 \sigma_i^2 + (\theta_i + \gamma \sigma_i^2)^2 \sigma_i^2 + \lambda^2 \sigma_i^2}, \]
and truncations $\theta_i \mu + \theta_i \nu_i > e_1 - \mu_i - \gamma \sigma_i^2 \nu_i + (\theta_i + \gamma \sigma_i^2) S - \hat{\xi} (\nu_i)$ and $\theta_i \mu + \theta_i \nu_i \leq \theta_i \mu + \theta_i \nu_i \leq \theta_i \mu + \theta_i \nu_i \leq \theta_i \mu + \theta_i \nu_i \leq \theta_i \mu + \theta_i \nu_i$.

The difference between the mean and the first truncation of $\theta_i \mu + \theta_i \nu_i$ is equal to
\[ \gamma \sigma_i^2 \nu_i + \lambda \hat{\xi} (\nu_i) - \frac{\lambda^2 \sigma_i^2}{\theta_i^2 \sigma_i^2 + (\theta_i + \gamma \sigma_i^2)^2 \sigma_i^2 + \lambda^2 \sigma_i^2} (e_1 - \mu_i - \theta_i x_i - (\theta_i + \gamma \sigma_i^2)(y_i - S)). \]

Because $\hat{\xi} (\nu_i)$ is positive for all $\nu_i \in [-\bar{\nu}, \bar{\nu}]$, it follows that this difference converges to a strictly positive value as $\sigma_i \to 0$. In this case, Lemma A.1 implies that
\[ \lim \lim_{\sigma_i \to 0 \bar{\nu} \to \infty} E_i [e^{e_1 - \theta_i \mu - \theta_i \nu_i}] = \lim \lim_{\sigma_i \to 0 \bar{\nu} \to \infty} -\frac{\theta_i \mu x_i - \theta_i y_i - \theta_i \nu_i \hat{\xi} (\nu_i) - \theta_i \nu_i \nu_i}{\theta_i^2 \sigma_i^2 + (\theta_i + \gamma \sigma_i^2)^2 \sigma_i^2 + \lambda^2 \sigma_i^2} \left( e_1 - \mu_i - \theta_i x_i - (\theta_i + \gamma \sigma_i^2)(y_i - S) \right). \]

(A.31)

Utility is exponential, so equation (A.31) implies that each investor $i$’s demand for peso bonds in period one satisfies
\[ \lim \lim_{\sigma_i \to 0 \bar{\nu} \to \infty} b_i = \lim \lim_{\sigma_i \to 0 \bar{\nu} \to \infty} \frac{\theta_i x_i + \theta_i y_i + \theta_i x_i + (\theta_i + \gamma \sigma_i^2) \sigma_i^2}{\theta_i^2 \sigma_i^2 + (\theta_i + \gamma \sigma_i^2)^2 \sigma_i^2 + \lambda^2 \sigma_i^2} \left( \lambda \xi_i - \theta_i \epsilon_i - (\theta_i + \gamma \sigma_i^2) \eta_i \right) - e_1 + \mu_1, \]
so that by dominated convergence
\[ \lim \lim_{\sigma_i \to 0 \bar{\nu} \to \infty} B_1 = \lim \lim_{\sigma_i \to 0 \bar{\nu} \to \infty} \frac{\theta_i \nu_i + \theta_i \nu_i + \lambda \theta_i x_i + (\theta_i + \gamma \sigma_i^2) \sigma_i^2}{\theta_i^2 \sigma_i^2 + (\theta_i + \gamma \sigma_i^2)^2 \sigma_i^2 + \lambda^2 \sigma_i^2} \xi_i - e_1 + \mu_1, \]

This last equality together with the market clearing condition for the peso bond market implies that
\[ \lim \lim_{\sigma_i \to 0 \bar{\nu} \to \infty} e_1 = \lim \lim_{\sigma_i \to 0 \bar{\nu} \to \infty} \mu_i + \theta_i \mu_i + (\theta_i + \gamma \sigma_i^2)(\nu_i - S) + \lambda \xi_i, \]
where $\lambda$ and $\sigma_i^2$ are given by the solution to equations (2.5) and (2.6) from Theorem 2.2. Of course, if $e_1 \to \mu_1 + \theta_i \mu_i + (\theta_i + \gamma \sigma_i^2)(\nu_i - S) + \lambda \xi_i$ as $\sigma_i \to 0$ and $\bar{\nu} \to \infty$, then all of the above statements are true in the limit and it follows that the limit (A.32) indeed holds.

I have shown that if the foreign central bank announces its intervention if and only if $\xi_1 \geq \hat{\xi} (\nu_i)$,
where $\xi(\nu_1)$ is positive and decreasing in $\nu_1$, then as $\sigma_1 \to 0$ and $\nu \to \infty$, if there is a central bank announcement, the exchange rate in period one is arbitrarily close to $\mu_1 + \theta_1 \mu_2 + \theta_0 + \gamma \hat{\sigma}_1^2(\nu_1 - S) + \lambda(\xi - \xi(\nu_1))$, where $\lambda$ and $\hat{\sigma}_1^2$ are given by the solution to equations (2.12) and (2.13), and if there is no central bank announcement, the exchange rate in period one is arbitrarily close to $\mu_1 + \theta_1 \mu_2 + (\theta_0 + \gamma \hat{\sigma}_1^2)(\nu_1 - S) + \lambda(\xi - \xi(\nu_1))$, where $\lambda$ and $\hat{\sigma}_1^2$ are given by the solution to equations (2.5) and (2.6). Equations (2.12) and (2.13) imply that $\lambda \to \infty$ and $\hat{\sigma}_1^2 \to \sigma_1^2$ as $\sigma_1 \to 0$ (see Theorem 2.4), while equations (2.5) and (2.6) imply that $\lim_{\sigma_1 \to 0} \lambda < \infty$ and $\lim_{\sigma_1 \to 0} \sigma_1^2 > \sigma_2^2$. It follows that as $\nu \to \infty$ and $\sigma_1 \to 0$, the difference $e_1 - \hat{e}_1$ is arbitrarily close to

$$\gamma(\nu_1 - S)(\sigma_1^2 - \sigma_2^2) + \lambda \xi_1 + \hat{\lambda}(\xi(\nu_1) - \xi_1).$$

As long as $S > \nu$, then for each $\nu_1 \in [-\bar{\nu}, \bar{\nu}]$, there exists $\hat{\xi}(\nu_1)$ such that $e_1 - \hat{e}_1 = 0$ whenever $\xi_1 = \hat{\xi}(\nu_1)$, $e_1 - \hat{e}_1 < 0$ whenever $\xi_1 > \hat{\xi}(\nu_1)$, and $e_1 - \hat{e}_1 > 0$ whenever $\xi_1 < \hat{\xi}(\nu_1)$ and such that $\hat{\xi}(\nu_1)$ is always strictly positive and decreasing in $\nu_1$.

**Lemma A.1.** Let $x \sim N(\mu_x(z), \sigma_x^2(z))$ with $z > 0$, and suppose that $\lim_{z \to 0} \sigma_x^2(z) = 0$ and that $\lim_{z \to 0} \mu_x(z)$ and $\lim_{z \to 0} \hat{x}(z)$ exist or are equal to plus or minus infinity. Also, suppose that all functions are continuously differentiable and

$$\frac{d}{dz} \left( \frac{\sigma_x^2(z)}{\sigma_x(z)} \right) \to 0$$

as $z \to 0$. Then

$$\lim_{z \to 0} E \left[ e^{x} \mid x < \hat{x}(z) \right] = \lim_{z \to 0} e^{\mu_x(z) + \frac{1}{2} \sigma_x^2(z)}$$

whenever $\lim_{z \to 0} \hat{x}(z) - \mu_x(z) \geq 0$, and

$$\lim_{z \to 0} E \left[ e^{x} \mid x < \hat{x}(z) \right] = \lim_{z \to 0} e^{\hat{x}(z) + \frac{1}{2} \sigma_x^2(z)}$$

whenever $\lim_{z \to 0} \hat{x}(z) - \mu_x(z) < 0$.

**Proof.** Let $x \sim N(\mu_x(z), \sigma_x^2(z))$ with $z > 0$, and suppose that $\lim_{z \to 0} \sigma_x^2(z) = 0$ and that $\lim_{z \to 0} \mu_x(z)$ and $\lim_{z \to 0} \hat{x}(z)$ exist or are equal to plus or minus infinity. For all $z > 0$,

$$E \left[ e^{x} \mid x < \hat{x}(z) \right] \Phi \left( \frac{\hat{x}(z) - \mu_x(z)}{\sigma_x(z)} \right) = e^{\mu_x(z) + \frac{1}{2} \sigma_x^2(z)} \Phi \left( \frac{\hat{x}(z) - \mu_x(z) - \sigma_x^2(z)}{\sigma_x(z)} \right),$$

and hence

$$\lim_{z \to 0} E \left[ e^{x} \mid x < \hat{x}(z) \right] \Phi \left( \frac{\hat{x}(z) - \mu_x(z)}{\sigma_x(z)} \right) = \lim_{z \to 0} e^{\mu_x(z) + \frac{1}{2} \sigma_x^2(z)} \Phi \left( \frac{\hat{x}(z) - \mu_x(z) - \sigma_x^2(z)}{\sigma_x(z)} \right).$$

If $\lim_{z \to 0} \hat{x}(z) - \mu_x(z) \geq 0$, then it is immediate by equation (A.36) that

$$\lim_{z \to 0} E \left[ e^{x} \mid x < \hat{x}(z) \right] = \lim_{z \to 0} e^{\mu_x(z) + \frac{1}{2} \sigma_x^2(z)}.$$
and by assumption,

$$\lim_{z \to 0} \frac{\Phi \left( \frac{\hat{x}(z) - \mu_x(z) - \sigma_x^2(z)}{\sigma_x(z)} \right)}{\Phi \left( \frac{\hat{x}(z) - \mu_x(z)}{\sigma_x(z)} \right)} = \lim_{z \to 0} \frac{\Phi \left( \frac{\hat{x}(z) - \mu_x(z) - \sigma_x^2(z)}{\sigma_x(z)} \right)}{\Phi \left( \frac{\hat{x}(z) - \mu_x(z)}{\sigma_x(z)} \right)} = \lim_{z \to 0} \exp \left\{ \frac{(\hat{x}(z) - \mu_x(z))^2}{2\sigma_x^2(z)} + \frac{(\hat{x}(z) - \mu_x(z))^2}{2\sigma_x^2(z)} \right\} = \lim_{z \to 0} e^{\hat{x}(z) - \mu_x(z)}.$$

It follows that $\lim_{z \to 0} E[e^x | x < \hat{x}(z)] = \lim_{z \to 0} e^{\hat{x}(z) + \frac{1}{2} \sigma_x^2(z)}$ in this case.

B Appendix: Infinite-Horizon Model

This appendix presents the proofs of Theorems 4.3, 4.4 4.5, 4.6, 4.7, and 4.8.

**Proof of Theorem 4.3** Suppose that the steady-state equilibrium exchange rate in period $t+1$ is normally distributed conditional on investor $i$’s information set in period $t$ and that the conditional variance $\text{Var}_{it}[\epsilon_{t+1}]$ is equal for all investors $i$ (it must be equal in all periods $t$ by definition). Lemma 4.2 then implies that the equilibrium exchange rate in period $t$ must satisfy

$$e_t = \alpha \mu_t + \sum_{n=1}^{\infty} \alpha^{n+1} \bar{E}_t[\mu_{t+n}] + \gamma \sigma^2 \sum_{n=0}^{\infty} \alpha^{n+1} \bar{E}_t[\nu_{t+n}] + \alpha \gamma \sigma^2 \zeta_t.$$

(B.1)

The exchange rate in period $t$ is of the form

$$e_t = \alpha \mu_t + \psi_\mu \mu_{t+1} + \psi_\nu \nu_t + \lambda_\xi \xi_t + \lambda_\zeta \zeta_{t+1} + \lambda_\delta \delta_t,$$

(B.2)

so the goal is to solve for the coefficients $\psi_\mu, \psi_\nu, \lambda_\xi, \lambda_\zeta$, and $\lambda_\delta$, which requires solving for the steady-state variance $\sigma^2$ as well.

The next step, then, is to solve for the average expectations $\bar{E}_t[\mu_{t+n}]$ and $\bar{E}_t[\nu_{t+n}]$. This requires first solving for the individual expectations $E_{it}[\mu_{t+1}]$ and $E_{it}[\nu_{t+1}]$, with the latter equal to $\rho_t E_{it}[\nu_t]$ since investors in period $t$ have private signals of $\nu_t$ only. These expectations are more difficult to compute now that investors have prior distributions.

Let $E^0_{it}[:,], \text{Var}^0_{it}[:,], \text{Cov}^0_{it}[:,]$ denote, respectively, the expected value, variance, and covariance with respect to the information set consisting only of $\mu_t$ and the private signals $x_{it}$ and $y_{it}$. If the form of the exchange rate in equation (B.2) is taken as given, then Bayesian inference implies both that the exchange rate in period $t+1$ is conditionally normally distributed (this justifies the assumption of conditional normality) and that

$$\begin{pmatrix}
E_{it}[\mu_{t+1}] \\
E_{it}[\nu_{t}] 
\end{pmatrix} = \begin{pmatrix}
x_{it} \\
y_{it} 
\end{pmatrix} + \begin{pmatrix}
\sigma^2 & 0 \\
0 & \sigma^2_\eta 
\end{pmatrix} \begin{pmatrix}
\psi_\mu \sigma^2_{\mu} & 0 \\
0 & \psi_\nu \sigma^2_{\nu} 
\end{pmatrix} \begin{pmatrix}
\sigma^2 + \sigma^2_\xi & 0 \\
0 & \sigma^2_\eta + \sigma^2_\zeta 
\end{pmatrix}^{-1} \begin{pmatrix}
\pi_\mu & \pi_\nu & \text{Var}^0_{it}[\epsilon_t] \\
\rho_t \mu_t - x_{it} & \rho_t \nu_t - y_{it} & e_{it} - E^0_{it}[\epsilon_t] 
\end{pmatrix},$$

where $\pi_\mu = \psi_\mu \sigma^2_\mu - \lambda_\xi \sigma^2_\xi$ and $\pi_\nu = \psi_\nu \sigma^2_\nu - \lambda_\delta \sigma^2_\delta$. The inverse of the variance matrix in the above
expression is equal to

\[
\frac{1}{\Psi} \begin{pmatrix}
(\sigma^2_{\eta} + \sigma^2_{\phi}) \Var_{\eta}[e_t] - \pi^2_{\nu} & \pi_{\mu} \pi_{\nu} & -(-\sigma^2_{\eta} + \sigma^2_{\phi}) \pi_{\mu} \\
\pi_{\mu} \pi_{\nu} & (\sigma^2_{\nu} + \sigma^2_{\xi}) \Var_{\nu}[e_t] - \pi^2_{\nu} & -(-\sigma^2_{\nu} + \sigma^2_{\xi}) \pi_{\mu} \\
-(-\sigma^2_{\eta} + \sigma^2_{\phi}) \pi_{\mu} & -(-\sigma^2_{\nu} + \sigma^2_{\xi}) \pi_{\mu} & (\sigma^2_{\nu} + \sigma^2_{\xi})(\sigma^2_{\eta} + \sigma^2_{\phi})
\end{pmatrix},
\]  
(B.3)

where

\[
\Psi = (\sigma^2_{\nu} + \sigma^2_{\xi}) \Var_{\nu}[e_t] - (\sigma^2_{\eta} + \sigma^2_{\phi}) \pi_{\mu} - (\sigma^2_{\nu} + \sigma^2_{\xi}) \pi_{\mu} \\
= (\sigma^2_{\nu} + \sigma^2_{\xi})(\sigma^2_{\eta} + \sigma^2_{\phi}) \lambda^2_{2} \sigma^2_{2} + (\psi^2_{\nu} + 2\lambda_{\eta} \lambda_{\xi} + \lambda^2_{2})(\sigma^2_{\eta} + \sigma^2_{\phi}) \sigma^2_{2} \sigma^2_{2} + (\psi^2_{\nu} + 2\lambda_{\eta} \lambda_{\delta} + \lambda^2_{3})(\sigma^2_{\nu} + \sigma^2_{\xi}) \sigma^2_{2} \sigma^2_{2} \\
= (\psi_{\mu} + \lambda_{\nu})^2(\sigma^2_{\nu} + \sigma^2_{\xi}) \sigma^2_{2} \sigma^2_{2} + (\psi_{\nu} + \lambda_{\delta})^2(\sigma^2_{\nu} + \sigma^2_{\xi}) \sigma^2_{2} \sigma^2_{2} + (\sigma^2_{\nu} + \sigma^2_{\xi})(\sigma^2_{\eta} + \sigma^2_{\phi}) \lambda^2_{2} \sigma^2_{2}.  
\]  
(B.4)

Note that \( \mathcal{E}_{t}[x_{\mu}] = \mu_{t+1}, \mathcal{E}_{t}[y_{\mu}] = \nu_{t}, \) and \( \mathcal{E}_{t}[e_{t} - E^0_{\mu}[e_{t}]] = \lambda_{\xi} \xi_t + \lambda_{\zeta} \zeta_{t+1} + \lambda_{\delta} \delta_t, \) since \( E[x_{\mu} | \mathcal{F}_t] = \mu_{t+1} \) and \( E[y_{\mu} | \mathcal{F}_t] = \nu_t \) for all \( i \in [0,1] \) and all \( t \in \mathbb{N}. \) Let

\[
\Delta_{\mu} = \psi_{\mu}(\sigma^2_{\nu} + \sigma^2_{\xi})(\sigma^2_{\eta} + \sigma^2_{\phi}) - (\sigma^2_{\eta} + \sigma^2_{\phi}) \pi_{\mu} = (\psi_{\mu} + \lambda_{\nu}) (\sigma^2_{\nu} + \sigma^2_{\xi}) \sigma^2_{2},
\]

and

\[
\Delta_{\nu} = \psi_{\nu}(\sigma^2_{\nu} + \sigma^2_{\xi})(\sigma^2_{\eta} + \sigma^2_{\phi}) - (\sigma^2_{\nu} + \sigma^2_{\xi}) \pi_{\nu} = (\psi_{\nu} + \lambda_{\delta}) (\sigma^2_{\nu} + \sigma^2_{\xi}) \sigma^2_{2}.
\]

Because \( \Var_{\nu}[e_{t}] = \psi_{\mu}^2 \sigma^2_{\nu} + \psi_{\nu} \sigma^2_{\nu} + \lambda_{\nu}^2 \sigma^2_{\xi} + \lambda_{\xi}^2 \sigma^2_{\nu} + \lambda_{\delta}^2 \sigma^2_{\phi}, \) it follows that

\[
\mathcal{E}_{t}[\mu_{t+1}] = \mu_{t+1} + \lambda_{\xi} \Delta_{\mu} \sigma^2_{\xi} \zeta_{t} + \frac{\sigma^2_{\nu}}{\Psi}((\sigma^2_{\nu} + \sigma^2_{\xi}) \pi_{\nu} \psi_{\mu} - \pi_{\mu} \pi_{\nu} + \lambda_{\delta} \Delta_{\mu}) \delta_t \\
- \frac{\sigma^2_{\eta}}{\Psi}((\sigma^2_{\eta} + \sigma^2_{\phi}) \psi_{\mu} - (\sigma^2_{\eta} + \sigma^2_{\phi}) \pi_{\mu}) \mu_{t+1} + \lambda_{\xi} \Delta_{\mu} \sigma^2_{\xi} \zeta_{t+1} \\
\mathcal{E}_{t}[\nu_{t}] = \nu_{t} + \lambda_{\xi} \Delta_{\nu} \sigma^2_{\xi} \zeta_{t} + \frac{\sigma^2_{\nu}}{\Psi}((\sigma^2_{\nu} + \sigma^2_{\xi}) \pi_{\nu} \psi_{\mu} - \pi_{\mu} \pi_{\nu} + \lambda_{\delta} \Delta_{\nu}) \delta_t \\
- \frac{\sigma^2_{\eta}}{\Psi}((\sigma^2_{\eta} + \sigma^2_{\phi}) \psi_{\mu} - (\sigma^2_{\eta} + \sigma^2_{\phi}) \pi_{\mu}) \mu_{t+1} + \lambda_{\xi} \Delta_{\nu} \sigma^2_{\xi} \zeta_{t+1}.
\]  
(B.5)

Similarly, it follows that

\[
\mathcal{E}_{t}[\nu_{t}] = \nu_{t} + \lambda_{\xi} \Delta_{\nu} \sigma^2_{\xi} \zeta_{t} + \frac{\sigma^2_{\nu}}{\Psi}((\sigma^2_{\nu} + \sigma^2_{\xi}) \pi_{\nu} \psi_{\mu} - \pi_{\mu} \pi_{\nu} + \lambda_{\delta} \Delta_{\nu}) \zeta_{t+1} \\
- \frac{\sigma^2_{\eta}}{\Psi}((\sigma^2_{\eta} + \sigma^2_{\phi}) \psi_{\mu} - (\sigma^2_{\eta} + \sigma^2_{\phi}) \pi_{\mu}) \mu_{t+1} + \lambda_{\xi} \Delta_{\nu} \sigma^2_{\xi} \zeta_{t+1},
\]  
(B.5)
so that
\[ \mathbb{E}_t[\nu_t] = \nu_t + \lambda_\xi(\psi_\nu + \lambda_\delta)(\sigma_\xi^2 + \sigma_\zeta^2)\sigma_{\xi\zeta}^2 \xi_t \]
\[ + \frac{(\psi_\mu + \lambda_\zeta)(\psi_\nu + \lambda_\delta)\sigma_\mu^2\sigma_\nu^2\sigma_{\xi\zeta}^2\xi_{t-1} - \sigma_\nu^2 \left( (\sigma_\xi^2 + \sigma_\zeta^2)\lambda_\xi^2\sigma_\zeta^2 + (\psi_\mu + \lambda_\zeta)^2\sigma_\mu^2\sigma_\nu^2 \right) \delta_t}{\psi} \]  
(B.6)

Equations (B.5) and (B.6) state that both \( \mathbb{E}_t[\mu_{t+1}] \) and \( \mathbb{E}_t[\nu_t] \) are not functions of past noise trades or disturbances, so that higher-order beliefs collapse. More precisely, higher-order expectations are such that \( \mathbb{E}_t^n[\mu_{t+n}] = \rho_\mu^n \mathbb{E}_t[\mu_{t+1}] \) and \( \mathbb{E}_t^n[\nu_{t+n}] = \rho_\nu^n \mathbb{E}_t[\nu_t] \) for all \( n > 1 \). This important observation implies that the expression from equation (B.1) simplifies to
\[ e_t = \alpha \mu_t + \alpha \gamma \sigma^2 \nu_t + \gamma \sigma^2 \sum_{n=1}^{\infty} \alpha^{n+1} \rho_\mu^n \mathbb{E}_t[\nu_t] + \alpha \gamma \sigma^2 \xi_t \]
\[ = \alpha \mu_t + \frac{\alpha^2}{1 - \alpha \rho_\mu} \mathbb{E}_t[\mu_{t+1}] + \alpha \gamma \sigma^2 \nu_t + \gamma \sigma^2 \frac{\alpha^2 \rho_\nu}{1 - \alpha \rho_\nu} \mathbb{E}_t[\nu_t] + \alpha \gamma \sigma^2 \xi_t. \]  
(B.7)

Substituting equations (B.5) and (B.6) into equation (B.7) yields
\[ e_t = \alpha \mu_t + \psi_\mu \mu_{t+1} + \psi_\nu \nu_t + \lambda_\xi \xi_t + \lambda_\zeta \zeta_{t+1} + \lambda_\delta \delta_t, \]  
(B.8)

where \( \psi_\mu = \frac{\alpha^2}{1 - \alpha \rho_\mu} \) and \( \psi_\nu = \frac{\alpha \gamma \sigma^2}{1 - \alpha \rho_\nu} \), and \( \lambda_\xi, \lambda_\zeta, \) and \( \lambda_\delta \) are given by the solution to equations (4.11), (4.12), and (4.13).

The final step is to solve for \( \sigma^2 \), the steady-state variance of the exchange rate, which is accomplished by first solving for \( \text{Var}_t[\mu_{t+1}], \text{Var}_t[\nu_t], \) and \( \text{Cov}_t[\mu_{t+1}, \nu_t] \). Bayesian inference implies that
\[
\left( \begin{array}{c}
\text{Var}_t[\mu_{t+1}] \\
\text{Cov}_t[\mu_{t+1}, \nu_t]
\end{array} \right) = \left( \begin{array}{cc}
\sigma_\mu^2 & 0 \\
0 & \sigma_\nu^2
\end{array} \right)
- \left( \begin{array}{cc}
\psi \sigma_\mu^2 & \psi \sigma_\nu^2 \\
\psi \sigma_\mu^2 & \psi \sigma_\nu^2
\end{array} \right) \left( \begin{array}{cc}
\sigma_\mu^2 + \sigma_\zeta^2 & 0 \\
0 & \sigma_\nu^2 + \sigma_\delta^2
\end{array} \right)
\left( \begin{array}{c}
\pi_{\mu} \\
\pi_{\nu}
\end{array} \right)
\text{Var}_t[\varepsilon_t]^{-1}
\left( \begin{array}{cc}
\sigma_\mu^2 & 0 \\
\psi \sigma_\mu^2 & \psi \sigma_\nu^2
\end{array} \right),
\]
where \( \pi_{\mu} = \psi \mu \sigma_\xi^2 - \lambda_\xi \sigma_\zeta^2 \) and \( \pi_{\nu} = \psi \nu \sigma_\eta^2 - \lambda_\delta \sigma_\delta^2 \) as before. It follows by equation (B.3) that
\[ \text{Var}_t[\mu_{t+1}] = \sigma_\mu^2 - \frac{\sigma_\mu^4}{\psi} \left( (\sigma_\eta^2 + \sigma_\zeta^2) \text{Var}_t[\varepsilon_t] - \pi_{\mu} - 2 \psi_\mu (\sigma_\eta^2 + \sigma_\zeta^2) \pi_{\mu} + \psi_\mu (\sigma_\xi^2 + \sigma_\zeta^2) (\sigma_\eta^2 + \sigma_\zeta^2) (\sigma_\eta^2 + \sigma_\zeta^2) \right) \]
\[ = \sigma_\mu^2 - \frac{\sigma_\mu^4}{\psi} \left( (\sigma_\eta^2 + \sigma_\zeta^2) \left( \psi_\mu \sigma_\eta^2 + \psi_\mu \sigma_\zeta^2 + \lambda_\xi^2 \sigma_\zeta^2 + \lambda_\xi^2 \sigma_\zeta^2 + \lambda_\delta^2 \sigma_\zeta^2 - 2 \psi_\mu \pi_{\mu} + \psi_\mu (\sigma_\xi^2 + \sigma_\zeta^2) \right) - \pi_{\mu} \right) \]
\[ = \sigma_\mu^2 - \frac{\sigma_\mu^4}{\psi} \left( (\sigma_\eta^2 + \sigma_\zeta^2) \left( \psi_\mu \sigma_\eta^2 + \psi_\mu \sigma_\zeta^2 + \lambda_\xi^2 \sigma_\zeta^2 + \lambda_\xi^2 \sigma_\zeta^2 + \lambda_\delta^2 \sigma_\zeta^2 - 2 \psi_\mu \pi_{\mu} + \psi_\mu (\sigma_\xi^2 + \sigma_\zeta^2) \right) - \pi_{\mu} \right) \]
\[ = \sigma_\mu^2 - \frac{\sigma_\mu^4}{\psi} \left( (\sigma_\eta^2 + \sigma_\zeta^2) \left( \lambda_\xi^2 \sigma_\zeta^2 + (\psi_\mu + \lambda_\zeta)^2 \sigma_\xi^2 + (\psi_\mu + \lambda_\zeta)^2 \sigma_\eta^2 \right) \right) \]
that
\[
\text{Var}_t[\nu_t] = \sigma^2_n - \frac{\sigma^4_n}{\Psi} \left( (\sigma^2 + \sigma^2) \text{Var}_n[\varepsilon_t] - \pi^2_n - 2 \nu_\sigma (\sigma^2 + \sigma^2) \pi_\nu + \nu^2_\sigma (\sigma^2 + \sigma^2) (\sigma^2_n + \sigma^2) \right)
\]
\[
= \sigma^2_n - \frac{\sigma^4_n}{\Psi} \left[ (\sigma^2 + \sigma^2) \left( \psi^2_\sigma \sigma^2 + \psi^2_\nu \sigma^2_n + \lambda^2_\sigma \sigma^2 + \lambda^2_\sigma \sigma^2 + \lambda^2_\sigma \sigma^2_n - 2 \nu_\sigma \pi_\nu + \nu^2_\sigma (\sigma^2_n + \sigma^2) \right) \right] - \pi^2_n
\]
\[
= \sigma^2_n - \frac{\sigma^4_n}{\Psi} \left[ (\sigma^2 + \sigma^2) \left( \psi^2_\sigma \sigma^2 + \lambda^2_\sigma \sigma^2 + \lambda^2_\sigma \sigma^2 + (\nu_\sigma + \lambda_\sigma) \sigma^2 \sigma^2_n \right) - (\psi_\sigma \sigma^2_n - \lambda^2_\sigma \sigma^2) \right]
\]
\[
= \sigma^2_n - \frac{\sigma^4_n}{\Psi} \left[ (\sigma^2 + \sigma^2) \left( \psi^2_\sigma \sigma^2 + (\nu_\sigma + \lambda_\sigma) \sigma^2 \right) + (\psi_\sigma + \lambda_\sigma) \sigma^2 \sigma^2 \right],
\]
and that
\[
\text{Cov}_t[\mu_{t+1}, \nu_t] = -\frac{\sigma^2_n}{\Psi} (\pi_\sigma \pi_\nu - \psi_\sigma (\sigma^2 + \sigma^2) \pi_\nu - \psi_\sigma (\sigma^2_n + \sigma^2) \pi_\mu + \psi_\sigma \nu_\sigma (\sigma^2 + \sigma^2) (\sigma^2_n + \sigma^2))
\]
\[
= -\frac{\sigma^2_n}{\Psi} (\nu_\sigma (\sigma^2_n + \sigma^2) (\psi_\sigma + \lambda_\sigma) \sigma^2 - \pi_\nu (\psi_\sigma + \lambda_\sigma) \sigma) = -\frac{(\psi_\sigma + \lambda_\sigma) \sigma^2}{\Psi}.
\]
As before, \( \Psi \) is given by equation (B.4). Equation (B.8) implies that the steady-state variance is equal to
\[
\sigma^2 = \frac{\psi^2_\sigma}{\alpha^2} \text{Var}_{[\mu_{t+1}]} + \rho^2_\psi \psi^2_\sigma \text{Var}_{[\nu_t]} + \frac{2\rho_\psi \psi_\sigma \psi^2_\sigma}{\alpha} \text{Cov}_{[\mu_{t+1}, \nu_t]} + \lambda^2_\sigma \sigma^2 + (\theta_\sigma + \lambda_\sigma) \sigma^2 + (\theta_\nu + \lambda_\sigma) \sigma^2
\]
which justifies the assumption that the conditional variance is equal for all investors \( i \). Equation (4.14) follows.

**Proof of Theorem 4.4** This proof follows the proof of Theorem 4.3 very closely. Suppose that the steady-state equilibrium exchange rate in period \( t + 1 \) is normally distributed conditional on investor \( i \)'s information set in period \( t \) and that the conditional variance \( \text{Var}_t[\varepsilon_{t+1}] \) is equal for all investors \( i \). Lemma 4.2 then implies that the equilibrium exchange rate in period \( t \) satisfies equation (B.1). The exchange rate in period \( t \) is again of the form \( \tilde{e}_t = \alpha \mu_t + \psi_\sigma \mu_{t+1} + \psi_\nu \nu_t + \lambda_\sigma \xi_t + \lambda_\sigma \zeta_{t+1} \) and the goal remains to solve for the coefficients \( \psi_\sigma, \psi_\nu, \lambda_\sigma \), and \( \lambda_\sigma \) as well as the conditional variance \( \bar{\sigma}^2 \).

Bayesian inference again implies that the exchange rate in period \( t + 1 \) is conditionally normally distributed, so the initial assumption is justified. As in the previous proof, \( \bar{E}_t[\varepsilon_{t+1}] = \mu_{t+1} \) and \( \bar{E}_t[\varepsilon_t - E^0_{t+1}[\varepsilon_t]] = \lambda_\sigma \xi_t + \lambda_\sigma \zeta_{t+1} \). Furthermore, the average expectation of \( \nu_t \) is equal to \( \nu_t \) itself since the intervention is common knowledge, and so it follows that the average expectation of \( \mu_{t+1} \) is given by
\[
\bar{E}_t[\mu_{t+1}] = \mu_{t+1} + \left( \sigma^2 + \psi_\sigma \sigma^2 \right) \left( \frac{\psi_\sigma \sigma^2 - \lambda_\sigma \sigma^2}{\psi_\sigma \sigma^2 + \lambda_\sigma \sigma^2} \right)^{-1} \left( \frac{-\lambda_\sigma \sigma^2}{\lambda_\sigma \sigma^2 + \lambda_\sigma \sigma^2} \right)
\]
\[
= \mu_{t+1} + \frac{1}{D} \left( \sigma^2 + \psi_\sigma \sigma^2 \right) \left( \frac{\psi_\sigma \sigma^2 + \lambda_\sigma \sigma^2 + \lambda_\sigma \sigma^2}{\psi_\sigma \sigma^2 + \lambda_\sigma \sigma^2} \right) \left( \frac{-\lambda_\sigma \sigma^2}{\lambda_\sigma \sigma^2 + \lambda_\sigma \sigma^2} \right),
\]

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where \( D = (\psi_{\mu} + \tilde{\lambda}_{\xi})^2 \sigma_{\epsilon}^2 \sigma_{\xi}^2 + (\sigma_{\epsilon}^2 + \sigma_{\xi}^2) \tilde{\lambda}_{\xi}^2 \sigma_{\epsilon}^2 \). It follows that

\[
\begin{align*}
\mathbb{E}_t[\mu_{t+1}] &= \mu_{t+1} + \frac{1}{D} \left( \left( \tilde{\lambda}_{\xi}^2 \sigma_{\epsilon}^2 + \tilde{\lambda}_{\xi} (\psi_{\mu} + \tilde{\lambda}_{\xi}) \sigma_{\xi}^2 \right) \sigma_{\epsilon}^2 \left( \tilde{\lambda}_{\xi} + \psi_{\mu} \right) \sigma_{\xi}^2 \right) \\
&= \mu_{t+1} + \frac{\tilde{\lambda}_{\xi} (\psi_{\mu} + \tilde{\lambda}_{\xi}) \sigma_{\xi}^2 \sigma_{\epsilon}^2 \xi_t - \tilde{\lambda}_{\xi}^2 \sigma_{\epsilon}^2 \sigma_{\xi}^2 \zeta_{t+1}}{(\psi_{\mu} + \tilde{\lambda}_{\xi})^2 \sigma_{\epsilon}^2 \sigma_{\xi}^2 + (\sigma_{\epsilon}^2 + \sigma_{\xi}^2) \tilde{\lambda}_{\xi}^2 \sigma_{\epsilon}^2}.
\end{align*}
\]

Equation (B.9) states that \( \mathbb{E}_t[\mu_{t+1}] \) is not a function of past noise trades or disturbances, so it follows that higher-order beliefs again collapse in this case. Furthermore, investors have no information about future values of \( \nu_t \) besides knowledge of the current value of \( \nu_t \) and the stochastic process that governs its motion. This implies that \( \mathbb{E}_t[\mu_{t+n}] = \rho_n^{-1} \mathbb{E}_t[\mu_{t+1}] \) and \( \mathbb{E}_t[\nu_{t+n}] = \rho_n^{\nu} \nu_t \) for all \( n > 1 \), so that equation (B.1) simplifies to

\[
\tilde{\epsilon}_t = \alpha \mu_t + \frac{\alpha^2}{1 - \alpha \rho_{\mu}} \mathbb{E}_t[\mu_{t+1}] + \frac{\alpha \gamma \sigma^2}{1 - \alpha \rho_{\nu}} \nu_t + \alpha \gamma \sigma^2 \xi_t
\]

(B.10)

Substituting equation (B.9) into equation (B.10) yields

\[
\tilde{\epsilon}_t = \alpha \mu_t + \psi_{\mu} \mu_{t+1} + \psi_{\nu} \nu_t + \tilde{\lambda}_{\xi} \xi_t + \tilde{\lambda}_{\xi} \zeta_{t+1},
\]

(B.11)

where \( \psi_{\mu} = \frac{\alpha^2}{1 - \alpha \rho_{\mu}} \) and \( \psi_{\nu} = \frac{\alpha \gamma \sigma^2}{1 - \alpha \rho_{\nu}} \), and \( \tilde{\lambda}_{\xi} \) and \( \tilde{\lambda}_{\xi} \) are given by the solution to equations (4.17) and (4.18).

The final step of the proof is to solve for the steady-state variance of the exchange rate, \( \tilde{\sigma}^2 \). If investors know the value of \( \nu_t \) in period \( t \), then standard Bayesian inference implies that

\[
\begin{align*}
\text{Var}_t[\mu_{t+1}] &= \sigma_{\epsilon}^2 - (\sigma_{\epsilon}^2 \psi_{\mu} \sigma_{\epsilon}) \left( \psi_{\mu} \sigma_{\epsilon}^2 - \tilde{\lambda}_{\xi} \sigma_{\epsilon}^2 \psi_{\mu}^2 \sigma_{\epsilon}^2 + \tilde{\lambda}_{\xi} \sigma_{\epsilon}^2 \psi_{\mu} \sigma_{\epsilon}^2 + \tilde{\lambda}_{\xi} \sigma_{\epsilon}^2 \right)^{-1} \left( \sigma_{\epsilon}^2 \psi_{\mu} \sigma_{\epsilon}^2 \right) \\
&= \frac{\sigma_{\epsilon}^2}{D} \left( \psi_{\mu} \sigma_{\epsilon}^2 + \tilde{\lambda}_{\xi} \sigma_{\epsilon}^2 + \tilde{\lambda}_{\xi} \psi_{\mu} \sigma_{\epsilon}^2 \right) \left( \psi_{\mu} \sigma_{\epsilon}^2 \right) \\
&= \frac{\sigma_{\epsilon}^4 \left( \tilde{\lambda}_{\xi} \sigma_{\epsilon}^2 + (\psi_{\mu} + \tilde{\lambda}_{\xi})^2 \sigma_{\epsilon}^2 \right) \left( \psi_{\mu} \sigma_{\epsilon}^2 \right)}{(\psi_{\mu} + \tilde{\lambda}_{\xi})^2 \sigma_{\epsilon}^2 \sigma_{\xi}^2 + (\sigma_{\epsilon}^2 + \sigma_{\xi}^2) \tilde{\lambda}_{\xi}^2 \sigma_{\epsilon}^2}.
\end{align*}
\]

Equation (B.11) implies that the steady-state variance is equal to

\[
\tilde{\sigma}^2 = \frac{\psi_{\mu}^2}{\alpha^2} \text{Var}_t[\mu_{t+1}] + \tilde{\lambda}_{\xi} \sigma_{\epsilon}^2 + (\psi_{\mu} + \tilde{\lambda}_{\xi}) \sigma_{\xi}^2 - \psi_{\mu} \sigma_{\epsilon}^2.
\]

which justifies the assumption that the conditional variance is equal for all investors \( i \). Equation (4.19) follows. \( \square \)
Proof of Theorem 4.5 Let $\tilde{\Psi} = (\psi + \lambda)\sigma(\sigma^2 + (\sigma^2 + \lambda^2)\lambda^2\sigma^2)$, and recall that
\[
\frac{\psi}{\sigma^2 + \sigma^2} = (\psi + \lambda)^2 \sigma^2 + (\psi + \lambda^2)(\sigma^2 + \lambda^2)\lambda^2\sigma^2 + (\sigma^2 + \lambda^2)\lambda^2\sigma^2.
\] (B.12)

According to equations (4.14) and (4.19),
\[
\sigma^2 = \frac{\psi^2\sigma^2\lambda^2\sigma^2 + \lambda^2\sigma^2 + (\psi + \lambda^2)(\sigma^2 + \lambda^2)\lambda^2\sigma^2}{\alpha^2\Psi} + \frac{\rho^2\psi^2\sigma^2\lambda^2\sigma^2 + (\psi + \lambda)^2\lambda^2\sigma^2 + (\psi + \lambda)^2\lambda^2\sigma^2}{\alpha^2\Psi} + \frac{2\rho\psi\lambda(\psi + \lambda)^2\lambda^2\sigma^2}{\alpha^2\Psi}.
\] (B.13)

and
\[
\sigma^2 = \frac{\psi^2\sigma^2\lambda^2\sigma^2 + \lambda^2\sigma^2 + (\psi + \lambda)^2\lambda^2\sigma^2 + \psi^2\lambda^2\sigma^2}{\alpha^2\Psi}.
\] (B.14)

Throughout this proof, I assume that the parameters of the model are such that there exist real solutions $\lambda^2$ and $\tilde{\lambda}^2$ to the systems of equations given by Theorems 4.3 and 4.4.

Consider the limit of $\lambda^2$, $\tilde{\lambda}^2$ as $\sigma^2 \to 0$ and suppose that $\tilde{\lambda}^2$ does not diverge to infinity. In this case, $\tilde{\lambda}^2\sigma^2 \to 0$ so that by equations (4.18) and (4.17) it follows that $\lambda^2 \to 0$ and $\lim_{\sigma^2 \to 0} \lambda^2 = \lambda^2$, $\lambda^2 + \alpha\gamma\sigma^2$. Of course, the limit of $\lambda^2$ and $\lambda^2 + \alpha\gamma\sigma^2$ can only be equal if either $\lambda^2 \to 0$ or $\tilde{\lambda}^2 \to \infty$. Equation (B.14) implies that $\sigma^2 \geq \psi^2\sigma^2 > 0$ in the limit, however, so it must be that $\tilde{\lambda}^2 \to \infty$ as $\sigma^2 \to 0$. On the other hand, if $\sigma^2 = 0$, then equations (B.12), (B.11), and (B.13) imply that
\[
\lambda^2 = \frac{\lambda^2\psi^2\lambda^2\psi^2 + \lambda^2\psi^2 + \lambda^2\alpha\rho\psi\lambda^2\psi^2 + \lambda^2\lambda^2\psi^2}{(\psi + \lambda^2)^2(\sigma^2 + \sigma^2\lambda^2\sigma^2 + (\psi + \lambda^2)^2(\sigma^2 + \lambda^2\sigma^2))} + \alpha\gamma\sigma^2,
\]
where
\[
\sigma^2 = \frac{\psi^2\sigma^2\lambda^2\sigma^2 + \lambda^2\sigma^2 + \alpha^2(\psi + \lambda^2)^2(\sigma^2 + \lambda^2\sigma^2)\sigma^2 + \sigma^2}{\alpha^2(\psi + \lambda^2)^2(\sigma^2 + \lambda^2\sigma^2)\sigma^2 + \sigma^2} + \frac{\sigma^2}{\alpha^2(\psi + \lambda^2)^2(\sigma^2 + \lambda^2\sigma^2)\sigma^2 + \sigma^2}.
\]
As long as $\sigma^2 > 0$, it follows that $\lambda^2 \not\to \infty$.

Consider the limit of $\lambda^2$, $\tilde{\lambda}^2$ as $\sigma^2 \to \infty$. According to equation (4.18), $\tilde{\lambda}^2 \to -\psi^2$ in this case so that $\lim_{\sigma^2 \to \infty} \lambda^2 = \lim_{\sigma^2 \to \infty} \alpha\gamma\sigma^2$. Equation (B.14) implies that
\[
\lim_{\sigma^2 \to \infty} \sigma^2 = \lim_{\sigma^2 \to \infty} \lambda^2\sigma^2 + \psi^2\lambda^2\sigma^2 = \lim_{\sigma^2 \to \infty} \alpha^2\gamma^2\lambda^2\sigma^2 + \frac{\alpha^2\gamma^2\lambda^2\sigma^2}{(1 + \alpha\rho\psi)^2}\lambda^2\sigma^2.
\]

The only real solution to the equation $\sigma^2 = \alpha^2\gamma^2\lambda^2\sigma^2 + \frac{\alpha^2\gamma^2\lambda^2\sigma^2}{(1 + \alpha\rho\psi)^2}\lambda^2\sigma^2$ is $\sigma^2 = 0$, so it follows that both $\sigma^2 \to 0$ and $\tilde{\lambda}^2 \to 0$ as $\sigma^2 \to \infty$. According to equation (B.12),
\[
\lim_{\sigma^2 \to \infty} \frac{\psi}{\sigma^2} = \lim_{\sigma^2 \to \infty} (\psi + \lambda)^2(\sigma^2 + \lambda^2\sigma^2)\lambda^2\sigma^2 + (\psi^2 + \lambda^2\sigma^2 + (\sigma^2 + \lambda^2\lambda^2\sigma^2)
\]
so that, much like in the case of $\tilde{\lambda}^2$, equation (4.12) implies that $\lambda^2 \to -\psi^2$ as $\sigma^2 \to \infty$. These
properties imply that
\[
\lim_{\sigma \to \infty} \lambda_\xi = \lim_{\sigma \to \infty} \frac{\lambda_\xi \alpha \rho_\nu \psi_\nu (\psi_\nu + \lambda_\delta) \sigma_\eta^2 \sigma_\delta^2}{(\psi_\nu + \lambda_\delta)^2 \sigma_\eta^2 + (\sigma_\eta^2 + \sigma_\delta^2) \lambda_\xi^2} + \alpha \gamma \sigma^2.
\] (B.15)

The key equation is equation (4.13), which implies that
\[
\lim_{\sigma \to \infty} \lambda_\delta = \lim_{\sigma \to \infty} \frac{-\alpha \rho_\nu \psi_\nu \sigma_\eta^2 \lambda_\xi^2 \sigma_\delta^2}{(\psi_\nu + \lambda_\delta)^2 \sigma_\eta^2 + (\sigma_\eta^2 + \sigma_\delta^2) \lambda_\xi^2} + \alpha \gamma \sigma^2,
\]
so that \(\psi_\nu + \lambda_\delta\) does not converge to zero as long as \(\alpha \rho_\nu < 1\). All that remains is to show that \(\sigma^2\) and hence \(\psi_\nu\) does not converge to zero as \(\sigma_e \to \infty\). This follows by equation (B.13), which implies that
\[
\lim_{\sigma \to \infty} \sigma^2 = \lim_{\sigma \to \infty} \frac{\psi_\nu^2}{\alpha^2 \sigma_\xi^2} + \rho_\nu^2 \psi_\nu \sigma_\eta^2 \sigma_\delta^2 + \lambda_\xi^2 \sigma_\xi^2 + (\psi_\nu + \lambda_\delta)^2 \sigma_\delta^2.
\] (B.16)

The solution to this equation in the limit must be greater than zero since it contains the constant term \(\frac{\psi_\nu^2}{\alpha^2 \sigma_\xi^2} > 0\). It follows by equation (B.15) that \(\lambda_\xi\) converges to a constant greater than zero as \(\sigma_e \to \infty\).

Consider the limit of \(\lambda_\xi, \lambda_\delta\) as \(\sigma_\xi \to 0\). As in the case of \(\sigma \to \infty\), equation (4.18) implies that \(\lambda_\xi \to -\psi_\nu\) in this case and hence by equation (B.14) it follows that \(\sigma^2 \to 0\) and \(\lambda_\xi \to 0\). It is not difficult to show that a limit equation identical to equation (B.15) obtains for this case where \(\sigma_\xi \to 0\), and that a similar equation to equation (B.16) also obtains. The key difference, however, is that if \(\sigma_\xi \to 0\), equation (B.16) changes so that
\[
\lim_{\sigma_\xi \to 0} \sigma^2 = \lim_{\sigma_\xi \to 0} \frac{\psi_\nu^2}{\alpha^2 \sigma_\xi^2} + \rho_\nu^2 \psi_\nu \sigma_\eta^2 \sigma_\delta^2 + \lambda_\xi^2 \sigma_\xi^2 + (\psi_\nu + \lambda_\delta)^2 \sigma_\delta^2,
\]
and hence both \(\sigma^2\) and \(\psi_\nu\) converge to zero in the limit. It follows by equation (B.15) that \(\lambda_\xi \to 0\) as \(\sigma_\xi \to 0\).

Consider the limit of \(\lambda_\xi, \lambda_\delta\) as \(\sigma_\xi \to 0\). Equation (B.12) implies that
\[
\lim_{\sigma_\xi \to 0} \Psi = \lim_{\sigma_\xi \to 0} (\psi_\mu + \lambda_\xi)^2 \sigma_\xi^2 \sigma_\eta^2 + (\sigma_\xi^2 + \sigma_\eta^2) \lambda_\xi^2 \sigma_\xi^2,
\]
and hence equations (4.11) and (B.13) imply that
\[
\lim_{\sigma_\xi \to 0} \lambda_\xi = \lim_{\sigma_\xi \to 0} \frac{\lambda_\xi \psi_\mu (\psi_\mu + \lambda_\xi) \sigma_\eta^2 \sigma_\xi^2}{(\psi_\mu + \lambda_\xi)^2 \sigma_\eta^2 \sigma_\xi^2 + (\sigma_\xi^2 + \sigma_\eta^2) \lambda_\xi^2 \sigma_\xi^2},
\]
and
\[
\lim_{\sigma_\xi \to 0} \sigma^2 = \lim_{\sigma_\xi \to 0} \frac{\psi_\mu^2 \sigma_\eta^2 \lambda_\xi^2 \sigma_\xi^2}{\alpha^2 (\psi_\mu + \lambda_\xi)^2 \sigma_\eta^2 \sigma_\xi^2 + \alpha^2 (\sigma_\xi^2 + \sigma_\eta^2) \lambda_\xi^2 \sigma_\xi^2} + \lambda_\xi^2 \sigma_\xi^2 + (\psi_\mu + \lambda_\xi)^2 \sigma_\xi^2.
\]
Equation (4.12) also implies that
\[
\lim_{\sigma_\xi \to 0} \lambda_\xi = \lim_{\sigma_\xi \to 0} \frac{\psi_\mu \sigma_\eta^2 \lambda_\xi^2 \sigma_\xi^2}{(\psi_\mu + \lambda_\xi)^2 \sigma_\eta^2 \sigma_\xi^2 + (\sigma_\xi^2 + \sigma_\eta^2) \lambda_\xi^2 \sigma_\xi^2}.
\]
Meanwhile, equations (4.17), (4.18), and (B.14) imply that an identical set of equations jointly
determine the value of $\tilde{\lambda}_t$ as $\sigma_\delta \to 0$, so it follows that $\lim_{\sigma_\delta \to 0} \lambda_t = \lim_{\sigma_\delta \to 0} \tilde{\lambda}_t$. \qed

**Proof of Theorem 4.6** Suppose that the steady-state equilibrium exchange rate in period $t + 1$ is normally distributed conditional on investor $i$’s information set in period $t$. Suppose also that the conditional variance $\text{Var}_{it}[e_{t+1}]$ is equal for all investors $i$. Lemma 4.2 then implies that the equilibrium exchange rate in period $t$ must satisfy

$$e_t = \sum_{n=0}^{\infty} \alpha^{n+1} E_t^{n}[\mu_{t+n}] + \gamma \sigma^2 \sum_{n=0}^{\infty} \alpha^{n+1} E_t^{n}[v_{t+n}] + \alpha \gamma \sigma^2 \xi_t.$$  \hfill (B.17)

The exchange rate in period $t$ is of the form

$$e_t = AQ_t(k) + \alpha \mu_t + \alpha \gamma \sigma^2 \xi_t,$$  \hfill (B.18)

$$Q_t(k) = MQ_{t-1}(k) + Nw_t,$$  \hfill (B.19)

where $k > 0$ is the level at which higher-order expectations are truncated in the model. The goal is to solve for the equilibrium conditions that characterize the matrices $M$ and $N$, the vector $A$, and the steady-state variance $\sigma^2$.

As in Section 4.1, the investors in this economy publicly observe the value of $\mu_t$ in each period $t$, and so there are no higher-order expectations of this interest rate parameter in equilibrium. This follows because common knowledge of $\mu_t$ in each period $t$ implies that $E_t^{n}[\mu_{t+n}] = \rho^{-1}_\mu E_t^{n}[\mu_{t+1}]$ for all $n \geq 1$. Expectations of the foreign central bank’s interventions $\nu_t$ are more complicated, however, since investors do not learn the value of $\nu_{t-1}$ publicly as they did in the previous section. The definition of the higher-order expectations vector $Q_t(k)$ and the matrices $M$ and $H$ implies that $E_t^{n}[v_{t+n}] = h_t^2(MH)^n Q_t(k)$ for all $n \geq 1$, so it follows by equation (B.17) that

$$e_t = \frac{\alpha^2}{1 - \alpha \rho_\mu} E_t^{n}[\mu_{t+1}] + \gamma \sigma^2 \sum_{n=0}^{\infty} \alpha^{n+1} h_t^2(MH)^n Q_t(k) + \alpha \mu_t + \alpha \gamma \sigma^2 \xi_t.$$  \hfill (B.20)

The fact that $\rho^{-1}_\mu E_t^{n}[\mu_{t+2}] = E_t^{n}[\mu_{t+1}]$ implies that $E_t^{n}[\mu_{t+1}] = \rho^{-1}_\mu h_t^1 MH Q_t(k)$, so it follows by equations (B.18) and (B.20) that the vector $A$ must satisfy

$$A = \frac{\alpha^2}{\rho_\mu(1 - \alpha \rho_\mu)} h_t^1 MH + \gamma \sigma^2 \sum_{n=0}^{\infty} \alpha^{n+1} h_t^2(MH)^n.$$

Note that this equation matches equation (4.28) exactly, so that all that remains of this proof is to characterize the state transition matrices $M$ and $N$ and the steady-state variance $\sigma^2$.

Let $\tilde{e}_t = e_t - \alpha \mu_t$. In each period $t$, each investor $i$ observes

$$z_{it} = \begin{pmatrix} x_{it} \\ y_{it} \\ \mu_t \\ \tilde{e}_t \end{pmatrix} = DQ_t(k) + R \begin{pmatrix} \sigma_{e^{-1}}^{-1} \epsilon_{it} \\ \sigma_{\eta^{-1}}^{-1} \eta_{it} \\ \sigma_{\xi^{-1}}^{-1} \xi_{it} \end{pmatrix}.$$  

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where
\[
D = \begin{pmatrix}
1 & 0 \\
0 & 1 \\
\rho^{-1} & 0 \\
\end{pmatrix}_{3 \times 2k},
\]
and \(R = (R_1 \quad R_2),\) with
\[
R_1 = \begin{pmatrix}
\sigma_e & 0 \\
0 & \sigma_\eta \\
0 & 0
\end{pmatrix}, \\
R_2 = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
-\rho^{-1} & 0 & 0 \\
0 & 0 & \alpha \gamma \sigma^2 \sigma_\xi
\end{pmatrix}.
\]

If the state vector of higher-order expectations evolves according to equation (B.19), then Bayesian updating implies both that the exchange rate in period \(t + 1\) is conditionally normally distributed (this justifies the assumption of conditional normality) and that
\[
E_{it}[Q_t(k)] = ME_{it-1}[Q_{t-1}(k)] + K (z_{it} - DME_{it-1}[Q_{t-1}(k)]),
\]
where \(K\) is the Kalman gain matrix. Averaging this equation over all investors yields
\[
\bar{E}_t[Q_t(k)] = M\bar{E}_{t-1}[Q_{t-1}(k)] + K (DMQ_{t-1}(k) + (DN + R_2)w_t - DMA_{t-1}[Q_{t-1}(k)]) \\
= (M - KDM)\bar{E}_{t-1}[Q_{t-1}(k)] + KDMQ_{t-1}(k) + K(DN + R_2)w_t.
\]
Equation (B.21) implies that
\[
Q_t(k) = \left[\begin{array}{c}
q_{0t} \\
\bar{E}_t[Q_t(k - 1)]
\end{array}\right] = M \left[\begin{array}{c}
q_{0t-1} \\
\bar{E}_{t-1}[Q_{t-1}(k - 1)]
\end{array}\right] + Nw_t = MQ_{t-1}(k) + Nw_t,
\]
where
\[
M = \begin{pmatrix}
\rho \mu & 0 \\
0 & \rho \nu \\
0 & 0
\end{pmatrix}_{2k \times 2k} + \begin{pmatrix}
0_{2k \times 2k+2} \\
\sigma_\eta_{2k \times 2k+2} \\
0_{2k \times 2k+2}
\end{pmatrix} = \begin{pmatrix}
0_{2k \times 2k+2} \\
\sigma_\eta_{2k \times 2k+2} \\
0_{2k \times 2k+2}
\end{pmatrix} + \begin{pmatrix}
0_{2k \times 2k+2} \\
\sigma_\eta_{2k \times 2k+2} \\
0_{2k \times 2k+2}
\end{pmatrix},
\]
\[
N = \begin{pmatrix}
\sigma_\zeta & 0 & 0 \\
0 & \sigma_\delta & 0 \\
0 & 0 & 0
\end{pmatrix}_{2k \times 2k} + \begin{pmatrix}
0_{2k \times 2k+2} \\
\sigma_\eta_{2k \times 2k+2} \\
0_{2k \times 2k+2}
\end{pmatrix} = \begin{pmatrix}
0_{2k \times 2k+2} \\
\sigma_\eta_{2k \times 2k+2} \\
0_{2k \times 2k+2}
\end{pmatrix} + \begin{pmatrix}
0_{2k \times 2k+2} \\
\sigma_\eta_{2k \times 2k+2} \\
0_{2k \times 2k+2}
\end{pmatrix},
\]
and \([M - KDM]_-\) is the matrix \(M - KDM\) with the last two rows and columns removed and \([KDM]_-\) and \([K(DN + R_2)]_-\) are, respectively, the matrices \(KDM\) and \(K(DN + R_2)\) with the last two rows removed. The Kalman gain matrix \(K\) is given by
\[
K = (PD' + NR_2')(DPD' + RR')^{-1},
\]
where \(P\) satisfies the matrix Riccati equation
\[
P = M \left( P - (PD' + NR_2')(DPD' + RR')^{-1}(PD' + NR_2') \right) M' + NN'.
\]
The next step is to solve for the steady-state variance of the exchange rate \(\sigma^2\). In order to do
this, it is necessary to compute the variance-covariance matrix

\[ \hat{P} = \text{Var}_{it} \begin{bmatrix} Q_{t+1}(k) \\ \mu_{t+1} \\ \xi_{t+1} \end{bmatrix} = \text{Var}_{t} \begin{bmatrix} Q_{t+1}(k) \\ \mu_{t+1} \\ \xi_{t+1} \end{bmatrix}, \]

which depends on the steady-state dynamics of a system slightly more general than the system from equation (B.19). Note that

\[ \left( \begin{array}{c} Q_t(k) \\ \mu_t \\ \xi_t \end{array} \right) = \left( \begin{array}{ccc} M & 0_{2k+2 \times 2} & 0 \\ 0_{2 \times 2k+2} & \rho_\mu & 0 \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{c} Q_{t-1}(k) \\ \mu_{t-1} \\ \xi_{t-1} \end{array} \right) + \left( \begin{array}{ccc} 0_{2k+2 \times 1} & N_1 & N_2 \\ \sigma_\zeta & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{c} \sigma_{-1} \xi_t \\ \xi_{t+1} \\ \delta_t \xi_t \end{array} \right), \]

where \( N_1 \) and \( N_2 \) consist, respectively, of the first two columns and the last column of the matrix \( N \) from equation (B.24) above, and that

\[ z_{it} = \left( \begin{array}{c} x_{it} \\ y_{it} \\ \mu_t \\ \xi_t \end{array} \right) = \left( \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ A & 0 & 0 & \alpha \gamma \sigma^2 \end{array} \right) \left( \begin{array}{c} Q_t(k) \\ \mu_t \\ \xi_t \end{array} \right) + \left( \begin{array}{c} 1 \\ 0 \\ 0 \\ 1 \end{array} \right) \left( \begin{array}{c} \epsilon_{it} \\ \eta_{it} \end{array} \right). \]

This system of equations both justifies the assumption that the conditional variance is equal for all investors \( i \) and implies that the matrix \( \hat{P} \) is given by the solution to the Riccati equation

\[ \hat{P} = \hat{M} \left( \hat{P} - \hat{P} \hat{D}' (\hat{D} \hat{P} \hat{D}' + \hat{R} \hat{R}')^{-1} \hat{D} \hat{P}' \right) \hat{M}' + \hat{N} \hat{N}', \quad (B.27) \]

where

\[ \hat{M} = \left( \begin{array}{ccc} M & 0_{2k+2 \times 2} & 0 \\ 0_{2 \times 2k+2} & \rho_\mu & 0 \\ 0 & 0 & 0 \end{array} \right), \quad \hat{N} = \left( \begin{array}{ccc} 0_{2k+2 \times 1} & N_1 & N_2 \\ \sigma_\zeta & 0 & 0 \\ 0 & 0 & 0 \end{array} \right), \]

\[ \hat{D} = \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ A & 0 & \alpha \gamma \sigma^2 \end{array} \right), \quad \hat{R} = \left( \begin{array}{ccc} \sigma_\epsilon & 0 \\ 0 & \sigma_\eta \end{array} \right). \]

Because \( \epsilon_{t+1} = A Q_{t+1}(k) + \alpha \mu_{t+1} + \alpha \gamma \sigma^2 \xi_{t+1} \), it follows that

\[ \sigma^2 = \left( A \alpha \alpha \gamma \sigma^2 \right) \hat{P} \left( A \alpha \alpha \gamma \sigma^2 \right)'. \quad (B.28) \]

I conclude that the matrices \( M \) and \( N \) and the steady-state variance \( \sigma^2 \) from the approximate equilibrium of Theorem 4.6 are given by the joint solution to equations (B.23), (B.24), (B.25), (B.26), (B.27), and (B.28). The fact that this approximation converges to the true steady-state equilibrium of this model is shown by Nimark (2010a).

\[ \square \]

**Proof of Theorem 4.7** Suppose that the steady-state equilibrium exchange rate in period \( t+1 \) is normally distributed conditional on investor \( i \)'s information set in period \( t \). Suppose also that
the conditional variance \( \text{Var}_t[e_{t+1}] \) is equal for all investors \( i \). Lemma 4.2 then implies that the equilibrium exchange rate in period \( t \) must satisfy

\[
e_t = \sum_{n=0}^{\infty} \alpha^{n+1} E_t^n[\mu_{t+n}] + \gamma \sigma^2 \sum_{n=0}^{\infty} \alpha^{n+1} E_t^n[\nu_{t+n}] + \alpha \gamma \sigma^2 \xi_t. \tag{B.29}
\]

The exchange rate in period \( t \) is of the form

\[
e_t = A \Pi_t(k) + \alpha \gamma \sigma^2 \xi_t, \tag{B.30}
\]

\[
\Pi_t(k) = M \Pi_{t-1}(k) + N \omega_t, \tag{B.31}
\]

where \( k > 0 \) is the level at which higher-order expectations are truncated in the model. The goal is to solve for the equilibrium conditions that characterize the matrices \( M \) and \( N \), the vector \( A \), and the steady-state variance \( \sigma^2 \).

Unlike Theorem 4.6, the investors now do not publicly observe the value of \( \mu_t \) in each period \( t \), and so higher-order expectations of this interest rate parameter are part of the equilibrium exchange rate. All expectations of \( \mu_t \) are computed in the same way as the expectations of \( \nu_t \), so it follows that \( E_t^n[\mu_{t+n}] = h_1'(MH)^n \Pi_t(k) \) and \( E_t^n[\nu_{t+n}] = h_2'(MH)^n \Pi_t(k) \) for all \( n \geq 1 \). Equation (B.29) then implies that

\[
e_t = \sum_{n=0}^{\infty} \alpha^{n+1}(h_1' + \gamma \sigma^2 h_2')(MH)^n \Pi_t(k) + \alpha \gamma \sigma^2 \xi_t,
\]

so it follows by equation (B.30) that the vector \( A \) must satisfy

\[
A = \sum_{n=0}^{\infty} \alpha^{n+1}(h_1' + \gamma \sigma^2 h_2')(MH)^n.
\]

Note that this equation matches equation (4.34) exactly, so that all that remains of this proof is to characterize the state transition matrices \( M \) and \( N \) and the steady-state variance \( \sigma^2 \).

Recall that \( \tilde{i}_t = i_t^* - ap_t^* - r = \mu_t + \chi_t \). In each period \( t \), each investor \( i \) observes

\[
z_{it} = \begin{pmatrix}
x_{it} \\
y_{it} \\
\tilde{i}_t \\
e_t
\end{pmatrix} = D \Pi_t(k) + R \begin{pmatrix}
\sigma_{\epsilon \epsilon}^{-1} \epsilon_{it} \\
\sigma_{\eta \eta}^{-1} \eta_{it} \\
\sigma_{\zeta \zeta}^{-1} \zeta_t \\
\sigma_{\delta \delta}^{-1} \delta_t \\
\sigma_{\chi \chi}^{-1} \chi_t \\
\sigma_{\xi \xi}^{-1} \xi_t
\end{pmatrix},
\]

where

\[
D = \begin{pmatrix}
1 & 0 \\
0 & 1 \\
0 & 0 \end{pmatrix} \times \mathbb{I}_{3 \times 2k}
\]

\[
R = \begin{pmatrix}
A
\end{pmatrix},
\]

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and \( R = (R_1 \ R_2) \), with
\[
R_1 = \begin{pmatrix}
\sigma_\varepsilon & 0 \\
0 & \sigma_\eta \\
0 & 0 \\
0 & 0
\end{pmatrix}, \quad R_2 = \begin{pmatrix}
0 & 0 \\
0 & 0 \\
\sigma_\chi & 0 \\
0 & \alpha \gamma \sigma_\varepsilon^2 \sigma_\xi
\end{pmatrix}.
\]

If the state vector of higher-order expectations evolves according to equation (B.31), then Bayesian updating implies both that the exchange rate in period \( t + 1 \) is conditionally normally distributed (this justifies the assumption of conditional normality) and that
\[
E_{it}[\Pi_t(k)] = ME_{it-1}[\Pi_{t-1}(k)] + K (z_{it} - DME_{it-1}[\Pi_{t-1}(k)]),
\]
where \( K \) is the Kalman gain matrix. Averaging this equation over all investors yields
\[
\overline{E}_t[\Pi_t(k)] = ME_{t-1}[\Pi_{t-1}(k)] + K (DM \Pi_{t-1}(k) + (DN + R_2)w_t - DME_{t-1}[\Pi_{t-1}(k)]) = (M - KDM)\overline{E}_{t-1}[\Pi_{t-1}(k)] + KDM\Pi_{t-1}(k) + (DN + R_2)w_t.
\]
Equation (B.32) implies that
\[
\Pi_t(k) = \begin{pmatrix}
\overline{E}_t[\Pi_t(k - 1)] \\
\overline{E}_t[\Pi_t(k - 1)]
\end{pmatrix} = M \begin{pmatrix}
\overline{E}_{t-1}[\Pi_{t-1}(k - 1)] \\
\overline{E}_{t-1}[\Pi_{t-1}(k - 1)]
\end{pmatrix} + N w_t = M\Pi_{t-1}(k) + N w_t,
\]
where
\[
M = \begin{pmatrix}
\rho_\mu & 0 & 0 \\
0 & \rho_\nu & 0 \\
0 & 0 & \rho_\omega
\end{pmatrix} + \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix} + \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix},
\]
\[
N = \begin{pmatrix}
\sigma_\zeta & 0 \\
0 & \sigma_\delta \\
0 & 0
\end{pmatrix} \begin{pmatrix}
0 & 0 \\
0 & 0 \\
[DN + R_2]_-
\end{pmatrix}.
\]
and \( [M - KDM]_- \) is the matrix \( M - KDM \) with the last two rows and columns removed and \( [KDM]_- \) and \( [DN + R_2]_- \) are, respectively, the matrices \( KDM \) and \( (DN + R_2) \) with the last two rows removed. The Kalman gain matrix \( K \) is given by
\[
K = (PD' + NR_2')(DPD' + RR')^{-1},
\]
where \( P \) satisfies the matrix Riccati equation
\[
P = M \left( P - (PD' + NR_2')(DPD' + RR')^{-1}(PD' + NR_2') \right) M' + NN'.
\]
As in the proof of Theorem 4.6, the next step is to solve for the steady-state variance of the exchange rate \( \sigma^2 \). In order to do this, it is necessary to compute the variance-covariance matrix
\[
\hat{\sigma}^2 = \text{Var}_t \begin{pmatrix}
\Pi_{t+1}(k) \\
\xi_{t+1}
\end{pmatrix} = \text{Var}_t \begin{pmatrix}
\Pi_{t+1}(k) \\
\xi_{t+1}
\end{pmatrix},
\]
which depends on the steady-state dynamics of a system slightly more general than the system.
from equation (B.31). Note that

$$
\begin{pmatrix}
\Pi_t(k) \\
\xi_t
\end{pmatrix} =
\begin{pmatrix}
M \\
0_{2k+2 \times 1}
\end{pmatrix}
\begin{pmatrix}
\Pi_{t-1}(k) \\
\xi_{t-1}
\end{pmatrix} +
\begin{pmatrix}
N_1 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix}
\begin{pmatrix}
\sigma_{\xi}^{-1} \xi_t \\
\sigma_{\delta}^{-1} \delta_t \\
\sigma_{\chi}^{-1} \chi_t \\
\sigma_{\xi}^{-1} \xi_t
\end{pmatrix},
$$

where $N_1$ and $N_2$ consist, respectively, of the first two columns and the last two columns of the matrix $N$ from equation (B.35) above, and that

$$
z_{it} =
\begin{pmatrix}
x_{it} \\
y_{it} \\
\epsilon_{it}
\end{pmatrix} =
\begin{pmatrix}
1 & 0 \\
0 & 1 \\
1 & 0
\end{pmatrix}
\begin{pmatrix}
\Pi_t(k) \\
\xi_t
\end{pmatrix} +
\begin{pmatrix}
1 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix}
\begin{pmatrix}
\epsilon_{it} \\
\eta_{it} \\
\chi_{it}
\end{pmatrix}.
$$

This system of equations both justifies the assumption that the conditional variance is equal for all investors $i$ and implies that the matrix $\hat{P}$ is given by the solution to the Riccati equation

$$
\hat{P} = \hat{M} \left( \hat{P} - (\hat{P} \hat{D}^t + \hat{N} \hat{R}_2^t)(\hat{D} \hat{P} \hat{D}^t + \hat{R} \hat{R}^t)^{-1}(\hat{D} \hat{P} \hat{D}^t + \hat{N} \hat{R}_2^t)^t \right) \hat{M}' + \hat{N} \hat{N}', \tag{B.38}
$$

where

$$
\hat{M} =
\begin{pmatrix}
M \\
0_{2k+2 \times 1}
\end{pmatrix}, \quad \hat{N} =
\begin{pmatrix}
N_1 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix}, \quad \hat{D} =
\begin{pmatrix}
1 & 0 \\
0 & 1 \\
0 \\
0
\end{pmatrix}
\begin{pmatrix}
0_{3 \times 2k+1} \\
A & \alpha \gamma \sigma^2
\end{pmatrix},
$$

$$
\hat{R} = \begin{pmatrix} \hat{R}_1 \hat{R}_2 \end{pmatrix}, \quad \hat{R}_1 = \begin{pmatrix} \sigma_{\epsilon} & 0 \\ 0 & \sigma_{\eta} \end{pmatrix}, \quad \hat{R}_2 = \begin{pmatrix} 0_{2 \times 4} \\ 0 \end{pmatrix}.
$$

Because $e_{t+1} = A \Pi_{t+1}(k) + \alpha \gamma \sigma^2 \xi_{t+1}$, it follows that

$$
\sigma^2 = (A \alpha \gamma \sigma^2 \hat{P} \left( A \alpha \gamma \sigma^2 \right)') \tag{B.39}
$$

I conclude that the matrices $M$ and $N$ and the steady-state variance $\sigma^2$ from the approximate equilibrium of Theorem 4.7 are given by the joint solution to equations (B.34), (B.35), (B.36), (B.37), (B.38), and (B.39). The fact that this approximation converges to the true steady-state equilibrium of this model is shown by Nimark (2010a).

**Proof of Theorem 4.8** Suppose that the steady-state equilibrium exchange rate in period $t + 1$ is normally distributed conditional on investor $i$’s information set in period $t$. Suppose also that the conditional variance $\text{Var}_{it}[\hat{e}_{t+1}]$ is equal for all investors $i$. Lemma 4.2 then implies that the equilibrium exchange rate in period $t$ must satisfy

$$
\hat{e}_t = \sum_{n=0}^{\infty} \alpha^{n+1} \mathbb{E}_t^m [\mu_{t+n}] + \gamma \sigma^2 \sum_{n=0}^{\infty} \alpha^{n+1} \mathbb{E}_t^m [\nu_{t+n}] + \alpha \gamma \sigma^2 \xi_t. \tag{B.40}
$$
The exchange rate in period $t$ is of the form

$$
\tilde{e}_t = \tilde{A}\tilde{\Pi}_t(k) + \frac{\alpha\gamma\tilde{\sigma}^2}{1 - \alpha\rho_v}\nu_t + \alpha\gamma\tilde{\sigma}^2\xi_t,
$$

(B.41)

$$
\Pi_t(k) = \tilde{M}\tilde{\Pi}_{t-1}(k) + \tilde{N}\tilde{\omega}_t,
$$

(B.42)

where $k > 0$ is the level at which higher-order expectations are truncated in the model. The goal is to solve for the equilibrium conditions that characterize the matrices $\tilde{M}$ and $\tilde{N}$, the vector $\tilde{A}$, and the steady-state variance $\tilde{\sigma}^2$.

As in Theorem 4.7, the investors do not publicly observe the value of $\mu_t$ in each period $t$, and so higher-order expectations of this interest rate parameter are part of the equilibrium exchange rate. However, unlike in Theorem 4.7, the investors do publicly observe $\nu_t$ and hence there are no higher-order expectations of current or future interventions. It follows that $E_n[\mu_t] = h_1'(\tilde{M}\tilde{H})^n\tilde{\Pi}_t(k)$ for all $n \geq 1$ as before, while now $E_n[\nu_t] = \rho^n\nu_t$ for all $n \geq 1$. Equation (B.40) then implies that

$$
\tilde{e}_t = \sum_{n=0}^{\infty} \alpha^{n+1}h'_1(\tilde{M}\tilde{H})^n\tilde{\Pi}_t(k) + \frac{\alpha\gamma\tilde{\sigma}^2}{1 - \alpha\rho_v}\nu_t + \alpha\gamma\tilde{\sigma}^2\xi_t,
$$

so it follows by equation (B.41) that the vector $\tilde{A}$ must satisfy

$$
\tilde{A} = \sum_{n=0}^{\infty} \alpha^{n+1}h'_1(\tilde{M}\tilde{H})^n.
$$

Note that this equation matches equation (4.40) exactly, so that all that remains of this proof is to characterize the state transition matrices $\tilde{M}$ and $\tilde{N}$ and the steady-state variance $\tilde{\sigma}^2$.

Let $\tilde{e}_t = \tilde{e}_t - \frac{\alpha\gamma\tilde{\sigma}^2}{1 - \alpha\rho_v}\nu_t$. If the foreign central bank announces the value of $\nu_t$ publicly, the relevant observations for each investor $i$ in each period $t$ are given by

$$
\tilde{z}_{it} = \begin{pmatrix} x_{it} \\ \tilde{e}_t \\ \tilde{e}_t \end{pmatrix} = D\tilde{\Pi}_t(k) + R \begin{pmatrix} \sigma_{\epsilon}^{-1}\epsilon_{it} \\ \sigma_{\xi}^{-1}\xi_{it} \\ \sigma_{\chi}^{-1}\chi_{it} \end{pmatrix},
$$

where

$$
D = \begin{pmatrix} 1 & 0_{2\times k} \\ 1 & \tilde{A} \end{pmatrix},
$$

and $R = (R_1 \ R_2)$, with

$$
R_1 = \begin{pmatrix} \sigma_{\epsilon} \\ 0 \\ 0 \end{pmatrix}, \quad R_2 = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_{\chi} \\ 0 & \alpha\gamma\tilde{\sigma}^2\sigma_{\xi} \end{pmatrix}.
$$

If the state vector of higher-order expectations evolves according to equation (B.42), then Bayesian updating implies both that the exchange rate in period $t + 1$ is conditionally normally distributed
(this justifies the assumption of conditional normality) and that
\[ E_{it}[\tilde{\Pi}_t(k)] = \tilde{M}E_{it-1}[\tilde{\Pi}_t-1(k)] + K \left( \tilde{z}_{it} - D\tilde{M}E_{it-1}[\tilde{\Pi}_t-1(k)] \right), \]
where \( K \) is the Kalman gain matrix. Averaging this equation over all investors yields
\[ E_t[\tilde{\Pi}_t(k)] = \tilde{M}E_{t-1}[\tilde{\Pi}_t-1(k)] + K \left( D\tilde{M}\tilde{\Pi}_{t-1}(k) + (D\tilde{N} + R_2)\tilde{w}_t - D\tilde{M}E_{t-1}[\tilde{\Pi}_t-1(k)] \right) \]
\[ = (\tilde{M} - KD\tilde{M})E_{t-1}[\tilde{\Pi}_t-1(k)] + KD\tilde{M}\tilde{\Pi}_{t-1}(k) + K(D\tilde{N} + R_2)\tilde{w}_t. \]
Equation (B.43) implies that
\[ \tilde{M}^\prime\pi_{it} = \tilde{M}\pi_{(k-1)t} + \tilde{N}\tilde{w}_t, \]
where
\[ \tilde{M} = \begin{pmatrix} \tilde{m}_{01} & 0_{1\times k} \\ 0_{k\times 1} & 0_{k\times k+1} \end{pmatrix} + \begin{pmatrix} 0_{1\times k+1} & 0_{1\times k+1} \\ [M - KD\tilde{M}]_{-} & [KD\tilde{M}]_{-} \end{pmatrix}, \]
\[ \tilde{N} = \begin{pmatrix} \sigma_{\xi} \chi \xi_{t+1} \\ 0 \end{pmatrix}, \]
and \([\tilde{M} - KD\tilde{M}]_{-}\) is the matrix \( \tilde{M} - KD\tilde{M} \) with the last row and column removed and \([KD\tilde{M}]_{-}\) and \([K(D\tilde{N} + R_2)]_{-}\) are, respectively, the matrices \( KD\tilde{M} \) and \( K(D\tilde{N} + R_2) \) with the last row removed. The Kalman gain matrix \( K \) is given by
\[ K = (PD' + \tilde{N}R_2' - R')(PD' + \tilde{N}R_2')^{-1}, \]
where \( P \) satisfies the matrix Riccati equation
\[ P = \tilde{M} \left( P - (PD' + \tilde{N}R_2')(PD' + RR')^{-1}(PD' + \tilde{N}R_2')' \right) \tilde{M}' + \tilde{N}\tilde{N}'. \]

As in the proof of Theorems 4.6 and 4.7, the final step is to solve for the steady-state variance of the exchange rate \( \tilde{\sigma}^2 \). In order to do this, it is necessary to compute the variance-covariance matrix
\[ \tilde{P} = \text{Var}_{it} \left[ \tilde{\Pi}_{t+1}(k) \right] = \text{Var}_{t} \left[ \tilde{\Pi}_{t+1}(k) \right], \]
which depends on the steady-state dynamics of a system slightly more general than the system from equation (B.42). Note that
\[ \begin{pmatrix} \tilde{\Pi}_t(k) \\ \tilde{\xi}_t \end{pmatrix} = \begin{pmatrix} \tilde{M} & 0_{k+1\times 1} \\ 0_{1\times k+2} & \tilde{M} \end{pmatrix} \begin{pmatrix} \tilde{\Pi}_t-1(k) \\ \tilde{\xi}_t-1 \end{pmatrix} + \begin{pmatrix} \tilde{N}_1 & \tilde{N}_2 \end{pmatrix} \begin{pmatrix} \sigma_{\xi}^{-1}\chi \xi_{t+1} \\ \sigma_{\xi}^{-1}\chi \xi_{t} \end{pmatrix} \]
where \( \tilde{N}_1 \) and \( \tilde{N}_2 \) consist, respectively, of the first two columns and the last column of the matrix.
\( \tilde{N} \) from equation (B.46) above, and that
\[
\tilde{z}_{it} = \begin{pmatrix} x_{it} \\ \tilde{y}_t \\ \tilde{e}_t \end{pmatrix} = \begin{pmatrix} 1 & 0_{2 \times k+1} \\ A & \tilde{\alpha} \tilde{\gamma} \tilde{\sigma}^2 \end{pmatrix} \begin{pmatrix} \Pi_t(k) \\ \xi_t \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \epsilon_{it} \\ \chi_t \end{pmatrix}.
\]

This system of equations both justifies the assumption that the conditional variance is equal for all investors \( i \) and implies that the matrix \( \hat{P} \) is given by the solution to the Riccati equation
\[
\hat{P} = \hat{M} \left( \hat{P} - (\hat{P} \hat{D}' + \tilde{N} \hat{R}'_2)(\hat{D} \hat{P} \hat{D}' + \hat{R} \hat{R}')^{-1}(\hat{P} \hat{D}' + \tilde{N} \hat{R}'_2)' \right) \hat{M}' + \tilde{N} \tilde{N}', \tag{B.49}
\]
where
\[
\hat{M} = \begin{pmatrix} \tilde{M} & 0_{k+1 \times 1} \\ 0_{1 \times k+2} & 1 \end{pmatrix}, \quad \tilde{N} = \begin{pmatrix} \tilde{N}_1 & \tilde{N}_2 \\ 0 & \sigma_\xi \end{pmatrix}, \quad \hat{D} = \begin{pmatrix} 1 & 0_{2 \times k+1} \\ A & \tilde{\alpha} \tilde{\gamma} \tilde{\sigma}^2 \end{pmatrix},
\]
\[
\hat{R} = \begin{pmatrix} \hat{R}_1 & \hat{R}_2 \end{pmatrix}, \quad \hat{R}_1 = \begin{pmatrix} \sigma_\epsilon \\ 0 \\ 0 \end{pmatrix}, \quad \hat{R}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_\chi & 0 \\ 0 & 0 & 0 \end{pmatrix}.
\]

Because \( \tilde{e}_{t+1} = \tilde{A} \tilde{\Pi}_t + (\tilde{\alpha} \tilde{\gamma} \tilde{\sigma}^2) \nu_t + \alpha \gamma \hat{\sigma}^2 \xi_{t+1} \), it follows that
\[
\hat{\sigma}^2 = (\tilde{A} \quad \alpha \gamma \hat{\sigma}^2 \quad \hat{P} \quad \tilde{A} \quad \alpha \gamma \hat{\sigma}^2)' + \left( \frac{\alpha \gamma \hat{\sigma}^2}{1 - \alpha \rho_\nu} \right)^2 \hat{\sigma}_e^2. \tag{B.50}
\]

I conclude that the matrices \( \hat{M} \) and \( \tilde{N} \) and the steady-state variance \( \hat{\sigma}^2 \) from the approximate equilibrium of Theorem 4.8 are given by the joint solution to equations (B.45), (B.46), (B.47), (B.48), (B.49), and (B.50). The fact that this approximation converges to the true steady-state equilibrium of this model is shown by Nimark (2010a). \( \square \)
References


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