CAPITAL AND GROWTH WITH OLIGARCHIC PROPERTY RIGHTS

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Abstract. To analyze effects of imperfect property rights on economic growth, we consider economies where some fraction of capital can be owned only by local oligarchs, whose status is subject to political risk. Political risk decreases local capital and wages. Risk-averse oligarchs acquire safe foreign assets for insurance, thus increasing wages in other countries that protect outside investors. Reforms to decrease political risk or to protect more outsiders' investments can decrease local oligarchs' welfare by increasing wages. A severe depression occurs when a closed country opens to let its oligarchs invest abroad without protecting outside investors, as in 1990s Russia.

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1. Introduction: a need for new models with property rights

The importance of imperfect property rights is widely recognized by observers of developing economies and economies in transition. But this basic insight has been difficult to apply in systematic economic analysis, because most standard economic models still assume perfect enforcement of property rights. Property rights may also be imperfect in many ways, and so many different assumptions about the nature of these imperfections need to be explored.

In this paper, we offer a model where an individual's ability to get property protected depends on his status in society. More specifically, we assume that protection of valuable property rights is limited to a small privileged subset of society ("the oligarchy"), and that each member of this privileged class faces some risk of losing his privileged status. We then introduce this assumption into the framework of a Ramsey-type growth model, and we develop mathematical results that make such a model analytically tractable. This is a paper of pure theory, but the model easily yields some stark features of the experience of many developing economies and economies in transition: the flow of capital from poor countries to rich countries, the dissipation of economic rents in unproductive political activity, and the presence of powerful vested interests for maintaining an inefficient status quo.

The basic idea here, that property rights are protected only for members of a small privileged elite, is a simplification that does not describe the real situation anywhere in the world. But the standard economic assumption, that all individuals have equally perfect protection of property, is also a simplification that does not apply fully anywhere. Economists often speak of transactions costs, but rarely speak of ownership costs. Even models with imperfect property rights have regularly assumed that all individuals have equal opportunities to own assets and to participate in economic transactions. The assumption that an individual's economic options depend only on his or her wealth, and not any other aspect of social status, has been a pervasive characteristic of most economics analysis.

But economists should recognize that the fundamental dynamics of political competition can create a system where property-rights protection is restricted to a privileged class of politically connected individuals. Protection of property rights is a service provided by political leaders. Once we admit that this service might not be fully provided to everyone, it becomes a scarce resource to be allocated by those leaders. Under any political system, leaders need active supporters to maintain their position, so contenders for power may rationally offer such scarce protection as a reward to their most active supporters. But both protection and political support require costly efforts that parties may not observe perfectly, limiting the circle of trust to a group of members small enough to actively monitor each other.[1] Moreover, promises to exchange political support for economic protection often cannot be disclosed, and compliance with them cannot be verified without exposing confidential information. Hence, such promises are likely to be credible only among individuals who have reputations for honoring confidential agreements.

Thus, fundamental agency problems in transactions of economic protection and political support can naturally lead to a political-economic equilibrium that is characterized by oligarchic property rights, where certain kinds of property are protected only for a limited group of people who have privileged relationships with local political leaders.[2] Because lack of trust can be a self-enforcing equilibrium, people who lack such a privileged relationship of trust with the ruling elite may find that this trust is difficult or impossible to buy for any price. An outsider who tried to buy the oligarchs' acceptance could simply find himself cheated.

Such systems of imperfect property rights may have different degrees of imperfection. Here we consider systems of oligarchic property rights that differ on two parametric dimensions.

The first dimension measures the risk that individual oligarchs may lose their good reputation and oligarchic status in the near future. To maintain trust among members of the oligarchy, it is essential that anyone who appears to have violated the terms of trust must lose his good status. With imperfect monitoring, appearances of such violations could occur with positive probability, even in an equilibrium where nobody actually chooses to violate any political agreements. Also, if political connections are established through a personal relationship between a political leader and a member of an oligarchic family, events such as the death of the family member or the downfall of the leader can cause the loss of oligarchic

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[1] Studies of the Sicilian Mafia agree that it could survive only when the chief “tendentiously maintained one-on-one relationships with the other members. … The Mafia therefore consists of a network of two-man relationships based on kinship, patronage, and friendship” (Catanzaro, 1988, pp. 42-43). See also Gambetta (1993).

[2] Varese (2001) reports stark evidence of such exclusive protection, in his study of the Russian mafia in Perm. One interviewed businessman, a former colonel in the militia, relied on his connections with the police when he was threatened by an extortion racketeer. The police made no attempt to arrest the racketeer. Instead, they summoned him to the police office where he was told in a “civilized” manner that he “had knocked on the wrong door” in approaching a connected businessman. The racketeer acknowledged his mistake and departed amicably (p. 94).

privileges by the rest of the family. Thus, members of the privileged oligarchy must always face some risk that they may lose their privileged status. Our model includes a parameter λ to measure this risk.

The second dimension on which oligarchic systems may differ is the fraction of capital that may be owned or financed by individuals outside of the oligarchy. When property protection depends on personal trust, it may be hard to credibly pledge tangible assets as collateral for loans from outside creditors, or to credibly promise a system of corporate governance that refrains from expropriating outside shareholders. Thus, there is strong reason to expect capital-market failure. De Soto (2000) has particularly attributed the failure of capitalism in poor countries to a weak collateral system that prevents property owners from inviting outside partners to finance some fraction of their capital. But even systems based on oligarchic property rights might protect some limited financial obligations to outsiders. Some forms of capital might be easier for outsiders to securely hold, or oligarchs may be able to finance some portion of their capital by selling secured debt to outside investors, to the extent that the legal system supports reliable corporate governance. So our model includes a parameter β that measures the fraction of capital that may securely be owned or financed by people who are outside the oligarchic elite.

The analytical framework is developed in Sections 2 through 5. Section 2 characterizes the optimal investment strategies for an individual oligarch, whose ownership of local assets is subject to a given political risk of expropriation. Section 3 develops a growth model of an economy where a given fraction of local capital must be owned by local oligarchs. The profits from distributing expropriated assets accrue to government offices, which are also controlled by local oligarchs. Section 4 analyzes the steady-state equilibrium of such an oligarchic economy. In Section 5, the analysis is extended to a multi-national general equilibrium model, where countries differ according to how well they protect property of oligarchs and of outside investors.

To probe the implications of our general model, we consider a series of simple examples in Sections 6 through 9. In Section 6, we analyze the steady state of a two-country model, and we show how a relatively small political risk in one country may seriously decrease capital and wages there, but may actually increase the wage in another country with better property rights. Sections 7 and 8 compare the dynamic equilibria that would follow from various political reforms in one country, starting from a given steady state. In Section 7 we consider reforms that change the degree of political risk, and in Section 8 we consider reforms that change the fraction of capital that outsiders can own. The results of these two sections illustrate how the oligarchs may prefer to maintain a system of imperfect property rights, even though it limits the security of their own holdings and their access to loanable funds. In Section 9, we analyze the effects of reforming a closed oligarchic economy where the oligarchs had been unable to acquire personal assets abroad, and we show how this model can account for some of the unanticipated problems that Russia experienced after its transition from Communism.

To simplify exposition, the basic model assumes that the oligarchs who lose their privileged status are expelled and are not replaced. In Section 10, we relax this assumption and consider the recruitment of new individuals into the oligarchy. We show that such recruitment may have only minor effects on the results of our analysis when the oligarchy is a small fraction of the overall population. In Section 11 we discuss related literature and conclusions.

2. The investment problem of an insecure oligarch

We are considering a country where, in each region, there is a small group of oligarchs, each of whom is connected to the local government by a relationship of personal trust. Our fundamental assumption is that only these oligarchs can own the capital in each region, or at least some essential fraction of this capital, because the local government will not protect outsiders' ownership claims. Thus, being an oligarch allows an individual to hold valuable local capital. We let $\pi(t)$ denote the net rate of profit that this local capital yields at any time t.

Individual oligarchs are not perfectly secure in their privileged position. As already discussed, they run a risk of losing their oligarchic status because, for example, of a sudden breaking of personal trust caused by perceived violation of the political support agreement,[3] or by the downfall of the leader (or mafia chief) through whom the political trust relationship had been established. Also, if we interpret members of the oligarchy as dynastic families, then death or departure of the family member who had formed a close personal relationship with the political leader could also cause a loss of privileged oligarchic status.

These political risks are captured in our model by making each oligarch face a small independent probability of losing his oligarchic status over any short interval of time. When this

[3] In 1999 the Russian tycoon Boris Berezovsky played a key role in bringing the previously politically obscure Mr. Vladimir Putin to power. In the next year, Mr. Berezovsky publicly complained about some of Mr. Putin's policies that appeared to violate their political support agreement. Mr. Berezovsky was ostracized, and lives in exile.

happens, all his local assets are confiscated.[4] The time until such ostracism is assumed to be an independent exponential random variable with mean $1/\lambda$ for each oligarch. That is, for anyone who is an oligarch at time 0, the probability of his still being an oligarch in good standing at time t>0 is $e^{-\lambda t}$. For simplicity we assume that this political risk is the only risk that an oligarchic investor faces, and the net profit rate $\pi(t)$ is assumed to be perfectly predictable.

We consider a simple economy in which there is a single consumption good that serves as numeraire. We assume that each investor gets logarithmic utility from his consumption over time, and future utility (which may be dynastic utility) is discounted at a rate ρ. Oligarchs can also invest in foreign bank accounts which yield a risk-free rate of interest r that is assumed to be constant over time and less than or equal to the utility-discount rate ρ. (The inequality $r \le \rho$ is justified in Section 5.) Foreign bank accounts are located in countries where the property-rights system makes them safe against political risk. Thus, if an oligarch were ostracized at a time t when he holds a safe foreign bank account worth x(t), then he (or his family) could move abroad and live off the principal and interest from this account. We may assume that the local profit rate $\pi(t)$ never falls below $r+\lambda$, because otherwise the risk-averse oligarchs would not hold any local assets. With this structure, we can now characterize the optimal solution to an oligarch's dynamic investment problem.

Let us begin by considering the situation of a former oligarch after he has been ostracized. Let t be the time when the oligarch was ostracized and let x(t) denote the amount he had in safe foreign accounts at that time. After time t, the oligarch's problem is to plan his consumption rate c(s) and his wealth $\hat{\theta}(s)$ for every time s ≥ t, so as to maximize his t-discounted expected utility

$$\int_t^{\infty} e^{-\rho(s-t)}\,\mathrm{LN}(c(s))\,ds$$

subject to the initial condition $\hat{\theta}(t)=x(t)$ and the dynamic constraint

$$\hat{\theta}'(s)=r\hat{\theta}(s)-c(s), \forall s \ge t\,.$$

Standard optimization procedure yields the optimal consumption rate equal to current wealth times the discount factor ρ, so that $c(s)=\rho\hat{\theta}(s)$ (see the proof of equation (3) in Proposition 1).

[4] Accommodating assassinations and other forms of physical ostracism is straightforward by interpreting the oligarchs' utility functions as dynastic utility functions.

So the former oligarch's wealth decays at rate $(r-\rho)\hat{\theta}(s)$, yielding $\hat{\theta}(s) = x(t)e^{(r-\rho)(s-t)}$. Thus, a former oligarch who goes into exile at time t with safe foreign assets x(t) can anticipate the t–discounted future utility

(1) $$V(x(t)) = \int_t^\infty e^{-\rho(s-t)} LN(\rho x(t)e^{(r-\rho)(s-t)})ds = LN(\rho x(t))/\rho + (r-\rho)/\rho^2 .$$

Now let us consider the first part of the oligarch's problem, before ostracism, assuming that he has some given initial wealth θ_0 at time 0. The oligarch's optimal consumption and investment plan must determine his total wealth $\theta(t)$, his consumption rate c(t), and his safe foreign assets x(t) for every time t as long as he maintains his privileged status in the local oligarchy. The oligarch's local assets at any time t are $\theta(t) - x(t)$. The quantities $\theta(t)$, c(t), x(t), and $\theta(t) - x(t)$ must always be nonnegative and must satisfy the initial condition $\theta(0) = \theta_0$. With his foreign assets earning rx(t) and his local assets earning $\pi(t)(\theta(t) - x(t))$, the oligarch's wealth will evolve according to the differential equation

$$\theta'(t) = \pi(t)(\theta(t) - x(t)) + r\,x(t) - c(t).$$

Planning to consume c(t) as an oligarch at a future time t generates utility LN(c(t)), which must be multiplied by the discount factor $e^{-\rho t}$ and by the probability $e^{-\lambda t}$ of retaining oligarchic status at time t. In the event of his losing oligarchic status at time t, the safe foreign assets x(t) would contribute the t-discounted future utility V(x(t)), given by equation (1), which must be multiplied by the time-t discount factor $e^{-\rho t}$ and by the probability density $\lambda e^{-\lambda t}$ of t being the random expropriation time. Thus, the oligarch's plan (x,c) should be chosen to maximize

(2) $$\int_0^\infty \left[LN(c(t)) + \lambda LN(\rho x(t))/\rho + \lambda(r-\rho)/\rho^2\right] e^{-(\rho+\lambda)t} dt .$$

<u>Proposition 1.</u> *The optimal solution to an oligarch's consumption and investment problem is characterized by the following conditions (3)-(5) at every time t, as long as he retains his oligarchic status.*

(3) $$c(t) = \rho\theta(t),$$

(4) $$(\pi(t) - r)/\theta(t) = \lambda/x(t),$$

(5) $$\theta'(t) = (\pi(t) - \rho - \lambda)\theta(t).$$

Proof. Notice first that any individual's optimal strategy for consumption and investment will be linearly homogeneous in his wealth. Multiplying an individual's wealth now by some constant m would cause his consumption at any future time to be multiplied by the same constant m, which would add LN(m) to his utility at that time, and so would add $\mathrm{LN}(m)/\rho$ to his expected present discounted value of all future utility. So at time t, the expected discounted utility of an individual with wealth $\theta(t)$ must be $\mathrm{LN}(\theta(t))/\rho$ plus some term that depends on his social status (oligarch or not) but is independent of his wealth. Thus, for any individual with wealth $\theta(t)$ at any time t, the marginal discounted utility of additional wealth would be $1/(\rho\theta(t))$.

Consider the effect of reducing wealth at time t by a small amount ε, by slightly increasing consumption above c during a short period just before t. The first-order effect of this ε change is to increase current logarithmic utility by $\varepsilon/c(t)$ and to decrease future discounted utility by $\varepsilon/(\rho\theta(t))$. Equation (3) says that these marginal effects must be equal.

Equation (4) is derived from the condition that, in an optimal plan, the oligarch would never want to make any small additional short-term transfer of wealth between his holdings of foreign and local assets. If an oligarch were to withdraw a small unit of wealth from foreign investments and invest one more unit locally instead over a short period from t to $t+\varepsilon$, his wealth would increase by $\varepsilon(\pi(t)-r)$. This yields a marginal expected-utility benefit of $\varepsilon(\pi(t)-r)/(\rho\theta(t))$ if he is not expropriated. But there is a small probability $\lambda\varepsilon$ that he *will* be expropriated between times t and $t+\varepsilon$. In this $\lambda\varepsilon$-probability event, the transfer would decrease his wealth in exile by one unit, yielding a marginal expected-utility cost of $\varepsilon\lambda/(\rho x(t))$. Equation (4) then says that these marginal expected-utility benefits and costs must be in balance.

If an oligarch invested only in local capital he could get income $\pi(t)\theta(t)$. The oligarch's foreign assets x(t) decrease his current income below this by the amount $(\pi(t)-r)x(t)$, which by equation (4) is equal to $\lambda\theta(t)$. Equations (3) and (4) then imply that $\theta'(t)$ satisfies (5). *Q.E.D.*

In welfare analysis, the numerical value of expected discounted logarithmic utility from formula (2) is difficult to interpret. We can more intuitively measure oligarchs' welfare by their <u>constant-equivalent consumption</u>, the guaranteed permanent consumption rate which would yield the same expected discounted utility. An oligarch's constant-equivalent consumption is proportional to his initial wealth θ_0. For an oligarch who begins at time 0 with one unit of wealth ($\theta_0 = 1$), the constant-equivalent consumption $\hat{c}$ is defined by the equation

(6) $$\mathrm{LN}(\hat{c})/\rho = \int_0^{\infty}\left\{\mathrm{LN}\left(\rho e^{\varphi(t)}\right)+\lambda \mathrm{LN}\left[\rho e^{\varphi(t)}\lambda/(\pi(t)-r)\right]/\rho + \lambda(r-\rho)/\rho^2\right\}e^{-(\rho+\lambda)t}\,dt\,,$$

where $\varphi(t) = \int_0^t (\pi(s)-\lambda-\rho)ds$.

Here we use the fact that, when logarithmic utility is discounted at rate ρ, a constant consumption rate c forever after time 0 would yield the expected discounted utility $\mathrm{LN}(c)/\rho$. If the oligarch with $\theta_0 = 1$ has not been ostracized before time t, then at time t his wealth would be $\theta(t) = e^{\varphi(t)}$, by equation (5), and his safe foreign assets would be $x(t) = e^{\varphi(t)}\lambda/(\pi(t)-r)$, by (4)

3. Equilibrium in a dynamic economy

We now develop a dynamic general equilibrium model of the local region where the oligarchs can invest. We assume that there are two kinds of assets in this region: local capital and government offices (protection rings). Both are subject to the same λ political risk.

We assume that the consumption good is produced from capital and labor according to the standard Cobb-Douglas production function:

(7) $$Y = AL^{\alpha}K^{1-\alpha},$$

where Y is the flow of output and A>0 and $\alpha \in (0,1)$ are some given constants. For simplicity, the supply of labor L is assumed constant and inelastic. The total supply of local capital at any time t is denoted by K(t). Assuming labor mobility within a country, workers must be paid a wage rate w(t) that is equal to the marginal product of labor

(8) $$w(t) = \partial Y/\partial L = (1-\alpha)A(K(t)/L)^{1-\alpha},$$

and so the gross profit rate R(t) that can be earned by each unit of capital at time t is

(9) $$R(t) = (Y(t)-w(t)L)/K(t) = (1-\alpha)A(L/K(t))^{\alpha}.$$

We assume that new capital can be made directly from the consumption good on a unit-per-unit basis, and capital depreciates at some given rate δ. Capital is mobile and can be sold abroad, so that its equilibrium price is always 1 in terms of the consumption-good numeraire.

We want to discuss two different dimensions on which imperfect property rights might vary: in the degree of political risk faced by oligarchs, and in the fraction of capital that must be owned and financed by local oligarchs. The first of these dimensions is represented in our model by the political-risk parameter λ that has already been introduced. The second dimension can be introduced by allowing oligarchs to invite outside partners to finance some fraction of their local capital. To be specific, suppose that an oligarch may finance part of his local capital holding by borrowing from outside creditors, and offering his capital as collateral, but only up to a given fraction β of the local capital. This fraction β represents the portion of local capital to which people outside the local oligarchy can be given some secure rights.[5] An oligarch who defaulted on his debts to outside creditors could conceal a fraction $1-\beta$ of his local capital from them, but the creditors could take at least temporary control of the fraction β and sell it to other oligarchs.

We assume that, when an oligarch's assets are expropriated, his creditors' claims to the β fraction are protected. Given a competitive market for such secure debts on global markets, the interest rate that oligarchs must pay on such borrowed funds would be r. Since the rate of net profit on local assets is always greater than $r+\lambda$ in equilibrium, each local oligarch will always choose to mortgage the maximal β fraction of his local capital investments. That is, every unit of local capital will take an investment $1-\beta$ from its owner and will return him the net income stream $R(t)-\delta-\beta r$. Thus, the net profit rate on oligarchs' investments in local capital is

$$\pi(t)=\frac{R(t)-\delta-\beta r}{1-\beta}=\frac{(1-\alpha)A(L/K(t))^{\alpha}-\delta-\beta r}{1-\beta}\,. \tag{10}$$

Expropriated capital that has been taken from former oligarchs is reallocated through the political sector in our model. We assume that government officials sell the newly expropriated capital to other oligarchs. This income stream from expropriated capital gives a value to government offices, and oligarchs can buy or sell these offices like capital. But also like local capital, government offices would be expropriated from an individual who loses his oligarchic status. The profits from reselling these expropriated offices accrue to other government officials.

Let G(t) denote the total value of all government offices at any time t. Then the aggregate income for government officials from their offices is $\lambda[(1-\beta)K(t)+G(t)]$. We think of the number of oligarchs as being a small fraction of the population, but large in numerical terms. Thus, the flow of expropriated wealth to government officials can be considered as a continuous income flow, subject only to the personal political risk of the recipients.

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[5] Collateralized debts need not be necessarily owed to foreign investors. It is essential, however, that collateralized loans are made through safe bank accounts, because otherwise an oligarch would not be able to collect on the loans he is making to other oligarchs in case he is ostracized.

Because an oligarch's investment in a government office involves the same personal expropriation risk as his investment in local capital, these political and economic investments must be perfect substitutes for each other. So the net rate of return from investments in government offices must always be exactly the same as the rate $\pi(t)$ for investments in local capital. In contrast to capital, however, government offices cannot be sold abroad, and so their value may change over time. Thus, for oligarchs to be indifferent between investing in local capital and government office at any time t, the following condition must hold

(11) $$\pi(t)G(t) = \lambda\left[(1-\beta)K(t) + G(t)\right] + G'(t),$$

where $G'(t)$ is rate of capital gain in the value of government offices.

At any time t, let X(t) denote the total safe foreign bank deposits held by oligarchs from this country.[6] Let $\Theta(t)$ denote the total wealth of all the oligarchs, so that

(12) $$\Theta(t) = X(t) + (1-\beta)K(t) + G(t).$$

From equation (4) in Proposition 1, we know that each oligarch holds the same fraction of wealth in safe deposits $x(t)/\theta(t) = \lambda/(\pi(t) - r)$. Thus, aggregating over all oligarchs, we get

(13) $$X(t) = \lambda\Theta(t)/(\pi(t) - r).$$

At any time t, the total oligarchic wealth $\Theta(t)$ is just the sum of the wealths $\theta(t)$ of all individual oligarchs. In equation (5), we saw that the growth rate of any individual oligarch's wealth is $(\pi(t) - \rho - \lambda)\theta(t)$ at any time t, as long as he retains his status in the oligarchy. But individuals are losing oligarchic status over time at the rate λ, and so $\lambda\Theta(t)$ must be subtracted from each individual's expected contribution to the aggregate $\Theta'(t)$. (When an oligarch is ostracized, his personal loss is only $\theta(t) - x(t)$, but he takes his remaining wealth x(t) with him out of the aggregate wealth of all oligarchs.) That is, when an oligarch has wealth $\theta(t)$, his expected individual contribution to total oligarchic wealth grows at the rate

$$(\pi(t) - \rho - \lambda)\theta(t) - \lambda\theta(t) = (\pi(t) - \rho - 2\lambda)\theta(t).$$

Aggregating the expected contribution of all individuals, the growth of total oligarchic wealth is

(14) $$\Theta'(t) = (\pi(t) - \rho - 2\lambda)\Theta(t).$$

At time 0, the oligarchs have some initial endowment of economic assets $(1-\beta)K$ and X, which have an exogenous value in the global market, but the value of their political assets G is

[6] Here X(t) includes any part of the mortgaged loans $\beta K(t)$ that may be owed to other oligarchs.

determined endogenously by transactions within the oligarchy. So our model's initial conditions must specify the aggregate value of the oligarchs' economic assets, which we denote by W_0

(15) $$W_0 = (1-\beta)K(0) + X(0).$$

G(0), the remaining component of $\Theta(0)$, is determined in equilibrium from equation (11).

The results of this analysis are summarized in the following proposition.

<u>Proposition 2.</u> *The dynamic behavior of* (Θ, K, X, G, π) *in this economy is characterized by equations (10)-(15), given the parameters* $(L, A, \alpha, \delta, \rho, r, \lambda, \beta, W_0)$.

The authors have provided a spreadsheet file that numerically solves this model.[7] A sketch of its computational approach may be instructive.

Given the model's parameters, we can define a function $\kappa(\Theta,G)$ that solves equations (10), (12), and (13) for K. This $\kappa(\Theta,G)$ satisfies

$$\Theta - G = (1-\beta)\kappa(\Theta,G) + \frac{\lambda(1-\beta)\Theta}{(1-\alpha)A\left(L/\kappa(\Theta,G)\right)^{\alpha} - \delta - r}.$$

With any $\Theta > G$, a unique solution $\kappa(\Theta,G)$ can be found between 0 and $(\Theta - G)/(1-\beta)$.

Our computational algorithm begins with an estimate of G(t) for all t (which could initially be G(t)=0). With this estimate, $\Theta(t)$ can be computed for all t from 0 to some distant time T by the differential equation (14), with $K(t) = \kappa(\Theta(t), G(t))$, and with $\pi(t)$ computed from K(t) by equation (10). If T is large enough, then K and G should be approximately constant after T, in which case equation (11) yields the boundary conditions

$$G(T) = \lambda(1-\beta)K(T)/(\pi(T) - \lambda).$$

Then we can compute a new estimate of G(t) for all t between 0 and T by solving the differential equation (11) for G' backwards from time T. The algorithm can now be repeated using the new estimate of G. For reasonable parameter values, this algorithm converges quite rapidly.

4. The long-run steady state

In a long-run steady state where the rate of return to capital is a stable constant, the capital/labor ratio K(t)/L must also be a constant, by equation (9). With a constant labor supply,

[7] Available at http://home.uchicago.edu/~rmyerson/research/oligarch.xls

capital K must be constant too. The value of government offices is based on their expropriation of capital, and so G must be constant in the steady state. With a constant net profit rate π, the fraction of deposits in oligarchs' wealth X/Θ must also be constant, by equation (13), and so total oligarchic wealth Θ must be constant, by equation (12). Thus, the growth equation (14) implies that the steady-state net profit rate for oligarchs' local investments must be[8]

(16) $$\pi^* = 2\lambda + \rho .$$

By equation (10), the gross profit rate or rental rate for local capital in the steady state must be

(17) $$R^* = (1-\beta)\pi^* + \beta r + \delta = (1-\beta)(2\lambda+\rho) + \beta r + \delta .$$

The steady-state supply of local capital can then be determined from equation (9)

(18) $$K^* = L\left(A(1-\alpha)/R^*\right)^{1/\alpha} ,$$

and the corresponding wage rate is

(19) $$w^* = \alpha A\left(K^*/L\right)^{1-\alpha} .$$

From equation (11) with $G' = 0$, the steady-state value of government offices must be

(20) $$G^* = \lambda(1-\beta)K^*/(\pi^* - \lambda) = \lambda(1-\beta)K^*/(\lambda+\rho).$$

From equations (12) and (13), the safe foreign bank accounts held by local oligarchs in the steady state must be

(21) $$X^* = \frac{\lambda[(1-\beta)K^* + G^*]}{(\pi^* - r - \lambda)} = \frac{(2\lambda+\rho)\lambda(1-\beta)K^*}{(\lambda+\rho-r)(\lambda+\rho)} .$$

Finally, the total wealth of all oligarchs in the steady state is

(22) $$\Theta^* = X^* + (1-\beta)K^* + G^* = \left(\frac{2\lambda+\rho-r}{\lambda+\rho-r}\right)\left(\frac{2\lambda+\rho}{\lambda+\rho}\right)(1-\beta)K^* ,$$

so that substituting into (21), we get $X^* = \lambda\Theta^*/(2\lambda+\rho-r)$. Since $r \le \rho$, the oligarchs in steady state will hold up to half of their wealth in safe foreign assets X*.

By equations (17)-(19), a decrease in the political risk parameter λ would cause a decrease in the returns to capital R*, which in turn will imply an increase in the capital/labor ratio K^*/L and an increase in the wage rate w*. Thus, workers would benefit from better protection of oligarchic property rights. Wealth flows from owners of capital to government officials at the expected rate $(1-\beta)\lambda K$, and so it might seem that the effects of this expropriation

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[8] If the labor supply L(t) grew at some given exponential rate n, then π^* would be $2\lambda + \rho + n$.

are just the same as a tax of rate $(1-\beta)\lambda$ on capital. In a world with perfect property rights, such a capital tax would increase the steady-state equilibrium value of R by just the amount of the tax $(1-\beta)\lambda$. But the factor of 2 in equation (17) means that the expropriation risk in our model has an adverse impact on capital and wage rates like a tax of twice the rate. This difference is caused by the fact that oligarchic investors are risk averse and cannot diversify their personal political risks.

Increasing the fraction β of capital that can be financed by outside investors would decrease the steady-state gross profit rate R* in proportion to the quantity $2\lambda+\rho-r$. So in the general case when λ is positive and π^* is strictly greater than r, a relaxation of the borrowing constraint (increasing β) causes a decrease in R*, which in turn increases the capital/labor ratio K*/L and increases the wage rate w*. Thus, workers would benefit from increasing the local capitalists' ability to borrow against their capital. Only in the case where local capital ownership is perfectly secure ($\lambda=0$) and the global risk-free interest rate is equal to the investors' personal discount rates ($r=\rho$), would the steady-state returns to local capital R* be equal to $\delta+\rho$ regardless of β, making local capital K* and wages w* independent of the ability of outside investors to securely finance local capital. The independence result in this special case of $\lambda=0$ and $r=\rho$ should not lead anyone to underestimate the general importance of creating strong corporate governance structures to protect outside investors.

We can also evaluate the welfare of the oligarchs in the steady state. Substituting $\pi^*=2\lambda+\rho$ into equation (5), we find that each individual oligarch's wealth must grow at rate $\theta'=\lambda\theta$ in the steady state, with his local and foreign assets growing at the same λ rate. This positive growth rate for individual oligarchs is just what is needed to compensate for the flow of others exiting from the local oligarchy. So an individual oligarch with the initial wealth θ_0 would at any time t consume $\rho\theta_0 e^{\lambda t}$ and hold safe deposits in the amount of $x_0 e^{\lambda t}$, where

$$x_0 = \lambda\theta_0/(\pi^*-r) = \theta_0\lambda/(2\lambda+\rho-r).$$

From (6), oligarchs' constant-equivalent consumption per unit of initial wealth is $\hat{c}^*$ such that

$$\begin{aligned}\mathrm{LN}(\hat{c}^*\theta_0)/\rho &= \int_0^\infty \left\{\mathrm{LN}\left(\rho\theta_0 e^{\lambda t}\right)+\lambda\mathrm{LN}\left(\rho x_0 e^{\lambda t}\right)/\rho+\lambda(r-\rho)/\rho^2\right\}e^{-(\rho+\lambda)t}\,dt\\ &= \mathrm{LN}(\rho\theta_0)/\rho+[\lambda+\lambda\mathrm{LN}(\lambda/(2\lambda+\rho-r))+\lambda(r-\rho)/\rho]/(\rho(\rho+\lambda)).\end{aligned}$$

So in the steady state, an oligarch's constant-equivalent consumption per unit of initial wealth is

$$(23)\qquad \hat{c}^* = \rho\exp\left(\frac{\lambda + \lambda \mathrm{LN}\big(\lambda/(2\lambda+\rho-r)\big) + \lambda(r-\rho)/\rho}{\rho+\lambda}\right) = \rho\left(\frac{\lambda e^{r/\rho}}{2\lambda+\rho-r}\right)^{\lambda/(\rho+\lambda)}.$$

Multiplying $\hat{c}^*$ by the wealth of any group of oligarchs yields the guaranteed permanent constant-equivalent consumption that would be needed to make them as well off as they expect to be in their privileged oligarchic position. In the steady state, the constant-equivalent consumption for the class of all current oligarchs is

$$(24)\qquad C^* = \hat{c}^*\,\Theta^*.$$

This quantity C* is our basic measure of aggregate oligarchic welfare in the steady state. Then the results of this section may be summarized as Proposition 3.

Proposition 3. *Equation (16)-(24) characterize the steady state of an economy with oligarchic property rights, given the political risk* λ*, collateralizability* β*, utility-discounting* ρ*, depreciation* δ*, labor supply* L*, production parameters* (A,α)*, and risk-free interest rate* r.

5. Global general equilibrium with oligarchic property rights

Property rights imperfections that impoverish one country may enrich another. To analyze the redistributive consequences of political risk, let us now consider a multi-national extension, including both the sources and recipients of capital flight.

Let J denote the set of countries in the world. For simplicity, let us assume that the basic technological and personal-preference parameters of our model $(\alpha, A, \delta, \rho)$ are the same in all countries. Let L_j denote the given fixed labor supply in country j. The openness and security of property rights in each country j are measured by the parameters β_j and λ_j where β_j is the fraction of local capital that can be owned or financed by outside investors, and λ_j measures the political risk of the privileged insiders who must own the balance of the local capital stock. The risk-free interest rate r in global capital markets now becomes an endogenous variable and must be determined in equilibrium. For any given r, however, the steady-state prices and assets in each country j are characterized by equations (16)-(22) in the previous section. In particular, the gross profit rate, capital stock, and safe foreign holdings of local oligarchs in country j are

$$R_j = (1-\beta_j)(2\lambda_j+\rho) + \beta_j r + \delta\,,$$

$$K_j = L_j\left(A(1-\alpha)/R_j\right)^{1/\alpha},$$

$$X_j = K_j(1-\beta_j)\lambda_j(2\lambda_j+\rho)/\left((\lambda_j+\rho)(\lambda_j+\rho-r)\right).$$

Let Ω_j denote the total assets held in global financial markets by people who are ostracized former oligarchs of country j. These expatriates' assets earn interest at rate r, but they consume out of their assets at rate ρ, and so their assets will decay at the rate $(\rho-r)\Omega_j$. At the same time, newly ostracized oligarchs bring their safe holdings into Ω_j at the rate $\lambda_j X_j$, where X_j denotes the safe financial assets held by oligarchs of country j. In the steady state, we must have

$$0 = \Omega_j' = \lambda_j X_j - (\rho-r)\Omega_j, \text{ and so}$$

(25) $$\Omega_j = \lambda_j X_j/(\rho-r).$$

The market-clearing global interest rate r can now be determined from the equation

(26) $$\textstyle\sum_{j\in J}\left(X_j+\Omega_j\right) = \sum_{j\in J}\beta_j K_j .$$

The left-hand side of (26) is the global demand for safe financial securities, while the right-hand side is the global supply. The gross profit rate on local capital in each country j must be positive $R_j > 0$. So the equilibrium world interest rate r must satisfy[9]

(27) $$r \geq -\left[(1-\beta_j)(2\lambda_j+\rho)+\delta\right]/\beta_j$$

for every country j where $\beta_j > 0$. Indeed, as r approaches this lower bound, R_j approaches zero, so that the steady-state demand for capital and the steady-state supply of global securities $\beta_j K_j$ from this country become infinite. The interest rate r must also be less than ρ, because the Ω_j demand for global securities by former oligarchs goes to infinity as r approaches this upper bound. We can always find an equilibrium interest rate r somewhere between these bounds.

<u>*Proposition 4.*</u> *A multinational general equilibrium satisfying the market-clearing condition (26) exists for some* r *such that* $\rho \geq r \geq -\min\left\{\left((1-\beta_j)(2\lambda_j+\rho)+\delta\right)/\beta_j \,\middle|\, \beta_j > 0\right\}$.

[9] The equilibrium real interest rate in our model may well turn out to be negative, for example if there are legal restrictions against investing in safe foreign assets for oligarchs in some large countries. See also Section 9 below.

6. A Simple Two-Country Example

In the next four sections, we show the power of our model by analyzing some numerical examples. We begin in this section by computing the steady state for a simple general equilibrium model with two countries.

In our examples here, we consider a standard set of values for the basic parameters $(\alpha, \delta, \rho, A)$. To be specific, let $\alpha = 0.6$ be labor's share of income, let $\delta = 0.03$ be the depreciation rate, and let $\rho = 0.04$ be the personal discount rate. We let A = 0.6766 be the production rate with unit inputs of capital and labor, because this constant with the other parameters would make the equilibrium wage rate be equal to 1 if all countries had perfect enforcement of property rights. That is, with these parameters, if the world had no political risk anywhere, so that $\lambda = 0$, then a steady-state equilibrium would have interest rate $\bar{r} = \rho = 0.04$, gross profit rate $\bar{R} = \rho + \delta = 0.07$, capital/labor ratio $\bar{K}/L = \left(A(1-\alpha)/\bar{R}\right)^{1/\alpha} = 9.52$, and wage rate $\bar{w} = \alpha A(\bar{K}/L)^{1-\alpha} = 1$.

Instead of this ideal world, let us now consider a simple world that is divided in two countries with equal population $L_1 = L_2 = 1$. Country 1 has perfect enforcement of property rights, so $\lambda_1 = 0$ and $\beta_1 = 1$. But country 2 is run by an oligarchy with a political risk rate $\lambda_2 = 0.02$. This expropriation rate should seem small, in that it implies that any oligarch has an expected time until expropriation of $1/\lambda_2$ = 50 years. We assume that local capital in country 2 must be owned and financed entirely by its local oligarchs, and so $\beta_2 = 0$.

For this two-country world, the global risk-free interest rate in a steady-state equilibrium is r = 0.0302. With this interest rate, the equilibrium capital and wages in countries 1 and 2 are

$$K_1 = 12.24,\ \ w_1 = 1.11,\ \ K_2 = 4.48,\ \ w_2 = 0.74.$$

So 73% of all capital is in country 1, but this capital has been financed by the current and former elite of country 2 (X_2 = 4.02 and Ω_2 = 8.22). The wage rate in country 1 is about 50% higher than in country 2. Notice that w_1 is also 11% higher than the equilibrium wage that workers would get in an ideal world without any political risk ($\bar{w}$ = 1). Thus, the political risk in country 2 has actually increased the welfare of workers in country 1.

The oligarchs' welfare in this equilibrium can be measured by their constant-equivalent consumption. By equation (23), given r and λ_2, the constant-equivalent consumption per unit wealth is $\hat{c}_2$ = 0.0380. Multiplying this $\hat{c}_2$ by the total wealth of all oligarchs, we find that the total constant-equivalent consumption for all oligarchs is

$$C_2 = \hat{c}_2[(1-\beta_2)K_2 + X_2 + G_2] = 0.0380 \times [(1-0)\times 4.48 + 4.02 + 1.49] = 0.380.$$

This constant-equivalent consumption is somewhat less than the aggregate consumption of all oligarchs ($\rho\Theta_2$ = 0.400) because it takes account of the oligarchs' political risks. For comparison, notice that the wage rate in each country is also equal to the aggregate rate of consumption of all workers, because we have $L_j = 1$. Also, the assets held by ostracized former oligarchs and their heirs support their aggregate consumption $\rho\Omega_2$ = 0.329 in this steady state.

Figure 1 shows the steady-state effects of a change in λ_2, the political risk rate in country 2. Greater political risks in country 2 obviously hurt workers in country 2, but it can be seen from Figure 1 that the effects on workers in country 1 are ambiguous. As long as λ_2 is not too high (λ_2 < 0.08 in this example), a small increase in λ_2 would increase capital flight from country 2, which would decrease world interest rates (down to r = 0.027 when λ_2 = 0.08), and thus would increase steady-state capital and wages in country 1. But when λ_2 becomes very high (λ_2 > 0.08), the principal effect would be to further impoverish country 2, decreasing the funds that its oligarchs invest abroad. This leads to less, not more capital flight (in absolute terms) from country 2 to country 1, increasing world interest rates, and *decreasing* steady-state capital and wages in country 1.

[Insert Figures 1 and 2 about here]

Effects of a change in β_2, the degree of protection for outside investors in country 2 are, on the other hand, unambiguous. Figure 2 shows those effects, given the fixed political risk rate $\lambda_2 = 0.02$. Greater protection for outside investors in country 2 would yield higher wages in country 2, but it would also increase world interest rates in the steady state and thus would decrease capital and wages in country 1. As β_2 approaches 1, so that outsiders can safely own almost everything in country 2, the world interest rate approaches the personal discount rate 0.04, the steady-state wealth of the oligarchs becomes small, and the wage rates in both countries approach the ideal wage $\overline{w} = 1$. Thus, globalization *would help workers in the poor country*, but it would *reduce the steady-state wealth of the oligarchs who dominate the poor country*, and it would also be *against the interests of workers in the rich country*.

Oligarchs’ preferences over different systems of property rights are not necessarily fully revealed by aggregate consumption plotted in Figures 1 and 2. First, aggregate consumption is not a complete measure of welfare for the oligarchs, because it does not take account of their political risks. But for the cases considered in Figures 1 and 2, the oligarchs' constant-equivalent consumption would differ only slightly from the aggregate consumption rates shown in those figures. A more serious problem comes from the fact that changing the system of property rights would not make the economy jump from one steady state to another, so that the analysis of a change must consider its full dynamic effects.

For example, suppose that the world economy was in the steady state for our basic example, with $\lambda_2 = 0.02$ and $\beta_2 = 0$, and then a political reform was proposed in country 2 that would decrease the expropriation rate to $\lambda_2 = 0.01$. Figure 1 shows that this reform would eventually lead to a new steady state in which the oligarchs have higher aggregate consumption. Furthermore, because the oligarchs would have less political risk in this new steady state, their constant-equivalent consumption would be increased even more. But this change would not occur instantly. In the steady state with $\lambda_2 = 0.01$, the oligarchs' total economic assets would be K_2+X_2 = 6.26+4.51 = 10.77, which is 27% larger than their total economic assets in the old steady state with $\lambda_2 = 0.02$ (where K_2+X_2 = 4.48+4.02 = 8.50). Thus, to reach the new steady state, the oligarchs would need to save over many years, to accumulate this increased wealth.

In Section 3, we saw how to analyze such a dynamically evolving economy, for the case of a small country whose changing economic aggregates would not affect the global interest rate. To apply this dynamic analysis here, we must revise the above example by subdividing "country 2" into many small countries, each of which has the same property-rights parameters. Then the methods of Section 3 can be applied to analyze a change of the property-rights parameters (λ, β) in any one of these small countries. In the next two sections, we consider the dynamic effects of such local property-rights changes in one small country.

7. Dynamic effects of changing political risk in a country

To examine the dynamic effects of changing the property-rights system in one country, let us consider an example with the standard parameter values from the previous section:[10]

(28) $\alpha = 0.6$, $\delta = 0.03$, $\rho = 0.04$, A = 0.6766, L = 1, r = 0.03.

In this environment, we can show that the steady state with $\lambda = 0.02$, $\beta = 0$ has a political stability in the following sense: Starting from this steady state, any political reform that

[10] We let the world interest rate be r = 0.03 here, which is close to the equilibrium value in our two-country example.

permanently changes λ or β would lead to a new dynamic equilibrium in which the oligarchs' total welfare C would be smaller than before the change. Also, from a steady state with the standard parameter values (28) and any other (λ,β), a small change toward $\lambda = 0.02$ and $\beta = 0$ would increase the oligarchs' welfare.[11] This result can be verified by a first-order perturbation analysis of the dynamic process following a small change of the (λ,β) parameters, as we discuss in the Appendix below.

For example, suppose that at time t=0, there is an unanticipated political reform that permanently decreases the political risk to $\lambda = 0.01$. For the stable steady state with $\lambda = 0.02$ and $\beta = 0$, equations (16)-(24) yield the following steady state values:

$$K^* = 4.48,\ \ X^* = 3.99,\ \ G^* = 1.49,\ \ w^* = 0.74,\ \ \pi^* = 0.08,$$

and the aggregate constant-equivalent consumption for all oligarchs is

$$C^* = \hat{c}^*(K^* + X^* + G^*) = 0.0378 \times (4.48 + 3.99 + 1.49) = 0.377.$$

As soon as they recognize the change to $\lambda = 0.01$, the oligarchs will want to invest more in local capital K. In the dynamic equilibrium, as shown in Figures 3a and 3b, this initial investment at time 0 leads to a jump in the local capital stock K by about 28%, which is paid for by an equal decrease in the oligarchs' foreign bank accounts X. The competitive wage at time 0 jumps by about 10%, to w(0) = 0.816, and the net profit rate on local investments drops from $\pi^* = 0.08$ to $\pi(0) = 0.065$. The decreased expropriation rate also lowers the value of government offices G(0) by 25%. Then, in the decades after the reform, local capital stock gradually increases towards its new steady-state value of $K(\infty) = 6.26$, which yields a competitive wage rate of $w(\infty) = 0.846$ and a local net profit rate of $\pi(\infty) = 0.06$.

[Insert Figures 3a and 3b about here]

When these dynamic effects are taken into account, it turns out that the total constant-equivalent consumption for all oligarchs at time 0 is lower than it was in the stable steady state. Specifically, with the new political risk $\lambda = 0.01$ and the anticipated net profit rates $\pi(t)$ for all t>0 in this dynamic equilibrium, the oligarchs' constant-equivalent consumption per unit wealth at time 0 is $\hat{c}$ = 0.0389 by equation (6), which is greater than in the old steady state (where $\hat{c}^*$ was 0.0378). The decline in the value of government offices at time 0, however, has decreased

[11] To state the result with more accuracy, the stable value is λ = 0.0198363. See the Appendix for derivation.

the oligarchs' total wealth, so that the new aggregate C at time 0 in this dynamic equilibrium is

$$C = \hat{c}\left(K(0) + X(0) + G(0)\right) = 0.0389 \times (5.73 + 2.74 + 1.12) = 0.373,$$

and this is less than the oligarchs' total constant-equivalent consumption in the old steady state. Thus, in this example, we find that the oligarchs would oppose a political reform that reduces their political risks and increases the security of their property rights. Of course any one oligarch would prefer that his own political risk should be reduced. But the equilibrium with systematically lower political risk for all oligarchs can actually make them worse off.

Figures 4a and 4b show the results of an opposite change starting from the stable steady state: an increase of political risk to $\lambda = 0.03$. In the new riskier political regime, the oligarchs want to increase their safe foreign assets X, which they do by exporting local capital K. The sudden capital flight decreases wages w and increases the local net profit rate π. The value of government offices G increases slightly because, although there is less local capital, the rate at which government officials can expropriate it has increased.

[Insert Figures 4a and 4b about here.]

After this change, the oligarchs' total wealth gradually declines, leading to further decreases in local capital, so that wages decline and net profit rates rise toward the new steady-state value $\pi(\infty) = 0.10$. The oligarchs' constant-equivalent consumption per unit wealth is $\hat{c}$ = 0.0371 at time 0 after the political change, and the total constant-equivalent consumption for all oligarchs at time 0 in this dynamic equilibrium is

$$C = \hat{c}\left(K(0) + X(0) + G(0)\right) = 0.0371 \times (3.70 + 4.77 + 1.64) = 0.375.$$

Again, this amount is less than their total constant-equivalent consumption in the old $\lambda = 0.02$ steady state, and so the aggregate welfare of the oligarchs is lower.

However, there may be some oligarchs who would benefit from this increase of political risk to $\lambda = 0.03$, if the oligarchs' local assets are not all equally distributed between government offices and local capital. In our model, we cannot determine how any individual oligarch should allocate his investments between these two local assets, because they are perfect substitutes as long as the parameters are held fixed. Going beyond the model, there might be some advantage for each individual oligarch to specialize as either an industrialist or a politician, concentrating his local holdings either in local capital or in government offices. Then an increase in political risk would be better for the politicians than it is for the industrialists, because an increase of the expropriation rate λ can create a windfall increase in the value of government offices.

In our example, in the steady state with $\lambda = 0.02$, the value of government offices was G* = 1.49, and each oligarch held 0.667 units of wealth abroad for each unit of local wealth. So in this steady state, politicians would hold X_{gov} = 1.49×0.667 = 1.00 units of wealth abroad, and their total constant-equivalent consumption would be

$$\hat{c}^*\left(G^* + X_{gov}\right) = 0.378 \times (1.49 + 1.00) = 0.0943.$$

Although the political change to $\lambda = 0.03$ has the effect of reducing the oligarchs' consumption-per-unit-wealth $\hat{c}$ by about 2% (relative to the old steady state), it also increases the value of government offices by about 9%. So after the political change, the politicians' total constant-equivalent consumption would become

$$\hat{c}\left(G(0) + X_{gov}\right) = 0.371 \times (1.64 + 1.00) = 0.0976.$$

Thus, the political specialists in the oligarchy would prefer this increase in political risk, but their welfare gains here are less than the other oligarchs' losses.

A lower risk-free interest rate r in international capital markets would tend to make the oligarchs more favorable to better enforcement of property rights. For example, we may consider changes in the political risk λ from a steady state with r = 0.025, $\beta = 0$, and the standard values (28) for all other parameters. In this case, starting from a steady state with any λ greater than 0.0134, the oligarchs' total welfare in the dynamic equilibrium could be improved by a small permanent decrease of the political risk λ. From the steady state with $\lambda = 0.0134$, a small change of λ in either direction would reduce the oligarchs' total welfare. But in this case, the political stability that we find at $\lambda = 0.0134$ is only local, because a larger jump down to $\lambda = 0$ would actually increase the oligarchs' total welfare.

8. Dynamic effects of opening to allow outside investors

Now let us consider the effects of an increase in the collateralizability parameter β, which measures the fraction of local capital that can be owned or financed by outsiders. Any individual oligarch would generally prefer to extend his own local investments by leverage from global financial markets. But in equilibrium, the credit from an increase of β may cause a growth of the local capital stock that increases wages and decreases local profits, and the oligarchs' total welfare may actually be decreased as a result. In fact, we find that this result holds with some generality, as long as the local political risk λ is not too high.

Consider a steady state where the capital stock is K and the gross rate of return to capital is $R = (1-\alpha)(L/K)^{\alpha}$, from equation (9). Notice that $\partial R/\partial K = -\alpha R/K$. If we allow one oligarch to finance a small new investment ε in local capital by borrowing at the interest rate r, then his profits will increase by $\varepsilon(R-\delta-r)$, but the effect on other oligarchs' profits will be $\varepsilon K(\partial R/\partial K) = -\varepsilon\alpha R$. So a small relaxation of an oligarch's credit constraint may increase or decrease total oligarchic income, depending on whether the quantity $(1-\alpha)R-\delta-r$ is positive or negative. By equation (10), $((1-\alpha)R-\delta-r) = (B(\pi,r)-\beta)(1-\alpha)(\pi-r)$, where

(29) $$B(\pi,r) = \frac{(1-\alpha)\pi-\alpha\delta-r}{(1-\alpha)(\pi-r)}.$$

So when $\beta < B(\pi,r)$, the oligarchs would collectively benefit from increasing β to relax their credit constraints. When $\beta > B(\pi,r)$, the oligarchs would similarly benefit from decreasing β. Thus $B(\pi,r)$ can be interpreted as the oligarchs' collective demand for foreign credit, as a fraction of their local capital, because they would always want to shift β towards $B(\pi,r)$.

<u>Proposition 5.</u> *Starting from a given steady state, a small change in collateralizability* β *would increase the oligarchs' welfare iff the change of* β *is toward* $B(\pi^*,r)$.

This collateralizability demand $B(\pi,r)$ is a decreasing function of the interest rate r. With $\pi^* = 2\lambda+\rho$ in the steady state, $B(\pi^*,r)$ becomes negative when

(30) $$\lambda < (r+\alpha\delta-(1-\alpha)\rho)/(2(1-\alpha)).$$

So when λ satisfies condition (30), the oligarchs would prefer to keep $\beta = 0$ in the steady state, excluding all outside investors. For a simple example, consider again the steady state with our standard parameter values (28). Then condition (30) holds as long as $\lambda < 0.04$, and so any increase of β would be harmful to the oligarchs' interests.

Figures 5a and 5b illustrate the effects of an unanticipated political reform that increases the collateralizability of local capital to $\beta = 0.5$, starting from the stable steady state with $\lambda = 0.02$, $\beta = 0$, and our other standard parameter values (28). Right after this reform, the oligarchs borrow massively abroad to finance a 67% increase in local capital, which increases wages by 23% and decreases the local net profit rate to $\pi(0) = 0.0715$ (from the prior $\pi^* = 0.08$).

The reform also causes the total value of government offices to decrease slightly. Then, in the decades after the reform, the oligarchs' total wealth and the local capital stock gradually decline, and the oligarchs' net profit rate on their leveraged local investments slowly rises back toward the original steady-state $\pi^* = 0.08$. So the post-reform depression of net profit rates is transient, but it decreases the oligarchs' total constant-equivalent consumption after the reform to

$$C = \hat{c}\left[(1-\beta)K(0) + X(0) + G(0)\right] = 0.0359 \times [(1–0.5)\times 7.50 + 4.72 + 1.33] = 0.352$$

Since the prior steady state had C* = 0.377, this reform to let outsiders finance 50% of local capital decreases the local oligarchs' welfare, as would any increase in β from $\beta = 0$.

[Insert Figures 5a and 5b about here]

9. Dynamic effects of opening to allow capital flight

The end of Communism opened new opportunities for individuals in the former Soviet Union to make personal investments abroad. In this section, we show how our model can be used to analyze the economic consequences of such a political change.

We begin by formulating a simple model of an oligarchic country that is closed to both import and export of capital. That is, only oligarchs can own local capital here, but these oligarchs cannot invest abroad. The restriction against foreigners acquiring local capital is modeled by letting collateralizability β be equal to 0. The restriction against oligarchs investing in safe foreign assets can be modeled in our framework by letting the risk-free interest rate r be very negative. When $\beta = 0$, our steady-state equations (16)-(20) yield values of π^*, R*, K*, G*, and w* that do not depend on the interest rate r. So in the steady state, the only effect of taking r to $-\infty$ is that the oligarchs' safe foreign assets X* go to 0. More intuitively, if the oligarchs have no way to hedge against their risk of expropriation, then the political risk rate will be effectively added into their rate of discounting the future, and so their consumption rate per unit wealth will increase from ρ to $\rho + \lambda$. So for the oligarchs to maintain constant aggregate wealth after consumption and expropriation, the steady-state net return to local capital must still be $\pi^* = 2\lambda + \rho$. The oligarchs' welfare is decreased by their inability to hold safe foreign assets, but workers' steady-state wages are not affected by restrictions against saving abroad.

However, comparing only long-run steady-state values does not reveal the full extent of the problem. The short-run effects of opening a closed oligarchic economy can be significant.

To illustrate those, consider, once again, an example with our standard parameter values (28) and $\lambda = 0.02$. Suppose that the country has been closed and has reached its steady state with $\beta = 0$ and $r = -\infty$. From this steady state, at time t = 0, there is an unexpected political reform that allows local oligarchs to freely invest abroad at the world interest rate r = 0.03.

For simplicity, suppose that the political risk $\lambda = 0.02$ remains the same after this political reform. Suppose also that the reform does not provide an effective legal framework to protect outside investors, so that collateralizability β remains equal to zero. This situation might be a reasonable approximation to what happened in the former Soviet Union after the collapse of communism. The dynamic equilibrium results for this model are shown in Figures 6a and 6b.

[Insert Figures 6a and 6b about here]

From the old closed steady state the economy has inherited the same steady-state capital stock that it should have under the new regime, given that λ is unchanged. But the oligarchs initially have no wealth abroad, and so their urge to acquire safe foreign assets drives them to export capital. In the dynamic equilibrium, this capital flight causes a 31% drop in the local capital stock, from K* = 4.48 to K(0)=3.09, so that the oligarchs acquire X(0) = 1.40 in safe foreign deposits. Wages fall 14%, from w* = 0.740 to w(0) = 0.637 immediately after the transition, and the net profit rate for local capital jumps to $\pi(0) = 0.108$. With these high profit rates, the oligarchs' total wealth slowly increases during the decades after the reform, so that the local capital stock and wage rate gradually climb back to their steady-state values. But a decade after the reform, wages are still more than 9% below their pre-transition level. Anticipating the $\pi(t)$ path shown in Figure 6b, the oligarchs' total constant-equivalent consumption at time 0 is

$$C = \hat{c}\left(K(0) + X(0) + G(0)\right) = 0.0446 \times (3.09 + 1.40 + 0.95) = 0.242.$$

It is not surprising that a nation's economic performance may suffer from an opening that allows its capital to leave but does not allow foreign capital to enter. In our model, allowing foreigners to invest means raising the borrowing parameter β. From the steady state of the given closed economy in this example, the post-transition loss of local capital could be avoided if the post-transition borrowing parameter were increased to $\beta = 0.33$. With the ability to get financing for 33% of their local capital, oligarchs could immediately acquire enough safe foreign assets to be willing to continue holding their shares of the local capital stock inherited from the old regime. Thereafter, as shown in Figures 7a and 7b, the capital stock would gradually increase above the pre-reform level. So after the transition with $\beta = 0.33$, wages would actually begin to rise above the closed steady state, 5% higher after the first decade, and 11% higher in the long run. But the oligarchs' total constant-equivalent consumption in this dynamic equilibrium would be

$$C = \hat{c}\left[(1-\beta)K(0) + X(0) + G(0)\right] = 0.0434 \times [(1-0.33)\times 4.51 + 1.46 + 0.93] = 0.235,$$

which is lower than in the dynamic equilibrium with $\beta = 0$.

[Insert Figures 7a and 7b about here]

Thus, as we already saw in Section 8, a systematic failure of corporate governance that prevents outsiders from acquiring local capital after the transition may actually be in the collective interests of the oligarchs. The oligarchs here prefer a reformed system that is open to capital flight but lacks any protection for outside investors. As shown in Figure 6b, the result for the workers is a depression of their already-low wages which may last for decades. This post-reform impoverishment of the workers must be ascribed to the effects of free capital flight to the global market, and not merely to the problems of local political risk. For the local workers to benefit from globalization, the local capital markets must be open to investment from outsiders.

10. Recruitment into the oligarchy

We have been considering a model where oligarchs lose their special privileges and become common citizens at random times. In the long run, the flow of people out of the oligarchy should be balanced by an opposite flow of common citizens being recruited into the oligarchy. In this section we consider a simple extension of our model with recruitment into the oligarchic class and show that it leaves the basic results of our analysis intact.

We assume that a person's initial entry into the oligarchic circle of trust requires chance personal connections that cannot be bought or hastened in any way. For simplicity, let any common citizen's waiting time to gain entry into the oligarchy be an exponential random variable with mean $1/\mu$, where μ is some small number. That is, in any short time interval of length ε, a common citizen's probability of gaining admission into the oligarchy is approximately $\mu\varepsilon$. With oligarchs exiting at rate λ, the steady-state ratio of common citizens per oligarch in the overall population would be λ/μ. So we should think of μ as being much smaller than λ, and we will be interested in the limit as $\mu \to 0$.

The main results of Proposition 1 carry over to this more general model. In an investment-consumption strategy that maximizes an individual's expected discounted logarithmic utility of consumption, the optimal consumption rate is always equal to $\rho\theta(t)$ when $\theta(t)$ is his current wealth. When the individual has oligarchic status, his optimal investment in the safe asset is $\lambda\theta(t)/(\pi(t)-r)$, when $\pi(t)$ is the local net profit rate. Thus, any individual oligarch's wealth grows at the rate $\theta'(t)=(\pi(t)-\rho-\lambda)\theta(t)$ as long as he retains his oligarchic status. On the other hand, a common citizen's wealth $\hat{\theta}(t)$ has negative growth rate $\hat{\theta}'(t)=(r-\rho)\hat{\theta}(t)$.

Now consider the dynamics of the aggregates $(\Theta(t), K(t), G(t), X(t), \Omega(t))$, where $\Omega(t)$ is the total wealth of all common citizens in the country, and the other quantities are as defined in Section 3. The equation (14) for the growth of oligarchic wealth must be revised to include the effects of commoners being randomly recruited into the oligarchy. With such recruitment at rate μ, the growth of aggregate oligarchic wealth becomes

(31) $$\Theta'(t)=(\pi(t)-2\lambda-\rho)\Theta(t)+\mu\Omega(t).$$

The commoners' aggregate wealth $\Omega(t)$ grows at rate

(32) $$\Omega'(t)=(r-\rho-\mu)\Omega(t)+\lambda X(t)=(r-\rho-\mu)\Omega(t)+\Theta(t)\lambda^2/(\pi(t)-r).$$

Here $(r-\rho)\Omega(t)$ is the change of wealth of common citizens whose status stays the same continuously at time t, while $\mu\Omega(t)$ is the outflow of wealth taken into the oligarchic class by citizens who get promoted at time t, and $\lambda X(t)=\Theta(t)\lambda^2/(\pi(t)-r)$ is the inflow of wealth that ostracized oligarchs take with them as they become commoners. All other economic variables can be characterized by the same equations (10)-(13) that we found in Section 3. So we can now characterize the dynamic behavior of this general economy in which citizens are promoted into the oligarchy at rate μ.

<u>Proposition 6.</u> *The dynamic behavior of* $(\Theta, K, X, G, \Omega, \pi)$ *in this economy with recruitment is characterized by equations (31)-(32) together with equations (10)-(13). The initial conditions at time 0 include the initial wealth of commoners* $\Omega(0)$ *as well as the initial economic wealth of the oligarchs* $W_0=(1-\beta)K(0)+X(0)$.

To evaluate welfare in this extended model, let the expected t-discounted future utility of an individual with wealth 1 at time t be denoted by u(t) if he is an oligarch and v(t) if he is a commoner. Because the optimal consumption and investment plan is always proportional to wealth, the expected t-discounted future utility of an individual with wealth $\theta(t)$ at time t is $u(t)+LN(\theta(t))/\rho$ if he is currently an oligarch, $v(t)+LN(\theta(t))/\rho$ if he is a commoner. Then u(t) and v(t) can be computed by the differential equations

$$(33) \qquad -u'(t)=\max_{c,x} LN(c)+[(1-x)\pi(t)+xr-c]/\rho+\lambda[v(t)+\lambda LN(x)-u(t)]-\rho u(t)$$

$$= LN(\rho)+[\pi(t)-\lambda-\rho+\lambda LN(\lambda/(\pi(t)-r))]/\rho+\lambda v(t)-(\rho+\lambda)u(t)\,,$$

$$(34) \qquad -v'(t)=\max_{c} LN(c)+(r-c)/\rho+\mu(u(t)-v(t))-\rho v(t)$$

$$= LN(\rho)+(r-\rho)/\rho+\mu u(t)-(\mu+\rho)v(t)\,.$$

In the long-run steady state where $\pi(t)=\pi^*$, the constant values of u(t) and v(t) are

$$(35) \qquad u^* = LN(\rho)/\rho+\{(\mu+\rho)[\pi^*-\lambda-\rho+\lambda LN(\lambda/(\pi^*-r))]+\lambda(r-\rho)\}/\left[\rho^2(\mu+\rho+\lambda)\right]\,,$$

$$(36) \qquad v^* = LN(\rho)/\rho+\{\mu[\pi^*-\lambda-\rho+\lambda LN(\lambda/(\pi^*-r))]+(\rho+\lambda)(r-\rho)\}/\left[\rho^2(\mu+\rho+\lambda)\right]\,.$$

The oligarchs' constant-equivalent consumption per unit wealth satisfies $LN(\hat{c}^*)/\rho = u^*$.

The steady-state condition $\Omega'=0$ implies that common citizens' aggregate wealth is

$$(37) \qquad \Omega^* = \Theta^*\lambda^2/[(\pi^*-r)(\mu+\rho-r)]\,.$$

Then the steady-state condition $\Theta'=0$ implies that

$$\pi^*-2\lambda-\rho+\mu\lambda^2/[(\pi^*-r)(\mu+\rho-r)]=0\,.$$

Thus, with $\pi^*>\rho+\lambda$, the local net profit rate in the steady state must be

$$(38) \qquad \pi^*=\lambda+(\rho+r)/2+0.5\sqrt{(\rho-r)[\rho-r+4\lambda+4\lambda^2/(\mu+\rho-r)]}\,.$$

The steady-state values of other economic variables (R*, K*, w*, G*, X*, Θ*) can then be computed from this π^* as in Section 4, by equations (17)-(22).

In the steady state, new recruits add to oligarchic wealth at rate

$$(39) \qquad \mu\Omega^*=\Theta^*\mu\lambda^2/[(\pi^*-r)(\mu+\rho-r)]=(2\lambda+\rho-\pi^*)\Theta^*\,.$$

But if $\mu/(\rho-r)$ goes to 0 then equation (38) becomes

$$\pi^*=\lambda+(\rho+r)/2+0.5\sqrt{(\rho-r)^2+4\lambda(\rho-r)+4\lambda^2}=2\lambda+\rho\,,$$

which is the steady-state net profit rate that we found in Section 4, and the inflow of wealth from new recruits (39) goes to 0. Thus, when μ is much smaller than $\rho-r$, the steady-state outcomes in this extended model look like our simpler model without recruitment.

For example, consider again our baseline example with the standard parameter values (28), $\lambda = 0.02$, and $\beta = 0$. Now let us consider an extended version of this model with recruitment at rate $\mu = 0.0004$, so that there are $\lambda/\mu = 50$ commoners per oligarch. In the steady state, of this extended model, we get

$$K^* = 4.50,\ X^* = 4.05,\ G^* = 1.51,\ w^* = 0.741,\ \pi^* = 0.0797,\ \hat{c}^* = 0.0378.$$

These quantities are all within 2% of their values in the simpler model without recruitment. The aggregate wealth held by commoners in this steady state is $\Omega^* = 7.79$, which may seem large, but its effect on the other aggregates is small because the total flow of wealth that new recruits bring into the oligarchy is merely $\mu\Omega^* = 0.003$. The effects of recruitment at this rate μ on the dynamic-equilibrium examples that we considered above would be similarly small.

11. Discussion and related literature

We have studied oligarchic property rights that have two dimensions of imperfection: the degree of exclusion of outside investors, and the insiders' level of political risk. Both imperfections are natural consequences of a system of property protection based on insider trust, and they negatively affect growth, the capital stock, and wages. But in a global general equilibrium where such imperfections differ across countries, a country that has better protection of property rights can become a safe haven for oligarchic investments, and so its workers' welfare could actually be higher than if property were perfectly protected everywhere.

When a closed oligarchic country suddenly opens up to let its oligarchs invest abroad, without changing its political risk and borrowing constraints, we find that short-term effects of capital flight can lead to a severe depression with a long and slow recovery. Such effects may account for much of the economic decline in the former Soviet Union during the first decade of transition. But our analysis suggests that such a transitional depression could be avoided if the oligarchic economy also opened in the other direction, to admit some investment from outsiders.

We have shown how imperfect property rights can be politically stable because they benefit the oligarchs. For reasonable economic parameters, we found that oligarchs may prefer not to reduce their political risk below a certain level, and they may prefer to minimize the protection of outside investors. Thus, inefficient oligarchic property rights may persist unless democratic institutions become strong enough to challenge the system of oligarchic privilege.

In our framework, local oligarchies are exclusive clubs, each dominating its local government, and each constituted from within by a network of trust that connects its members. Oligarchic property rights hamper capital accumulation and economic growth by restricting ownership rights to this privileged elite, who bear political risk as a cost of owning local capital.

Other economists have recognized the importance of extending economic analysis to such problems of oligarchy. Acemoglu (2004) has developed a model for comparing the fiscal and regulatory distortions of democratic and oligarchic societies. Where we have viewed oligarchic connections as a prerequisite for being able to hold local capital, Acemoglu assumes that oligarchic status follows from owning capital. But he argues that, when such oligarchs control the government, they will favor public policies that create barriers to entry, so that the oligarchy will effectively become the kind of closed club that we have assumed. Similarly, in a model of imperfect property rights, Glaeser, Scheinkman, and Shleifer (2003) assume an exogenous difference among people's abilities to punish a judge who violates a corrupt transaction. If most people have no ability to punish corrupt judges, then the few people who can effectively punish judges would act like our local oligarchs, with an exclusive ability to hold valuable local investments.

Polishchuk and Savvateev (2004) and Sonin (2003) have developed other theoretical frameworks to explain how the wealthier elite of a society might prefer imperfect protection of local property rights. In these models, individuals allocate their resources among production activities and private-protection activities, and the rich find a comparative advantage in private protection because the returns to scale in pure production are smaller. So the rich may gain from poor public protection which increases the benefits of their private-protection activities, but these benefits come from stealing the less-protected property of the poor. In our model, the imperfectly protected property is owned only by oligarchs, and the oligarchs' benefits from imperfect public protection are derived instead from its effect on the equilibrium wage rate.

The costs of imperfect property rights have been emphasized in many recent models. Murphy, Shleifer, and Vishny (1991) analyze the impact of political rent-seeking on innovation and growth, while Ehrlich and Lui (1999) examine the trade-off between political capital and human capital accumulation. Tornell and Velasco (1991) and Tornell and Lane (1999) analyze imperfect property rights as a common-pool problem, where individuals are discouraged from investing by the prospect of being expropriated by others. Their one-factor model suggests that investors should get positive externalities from each other's investments, but such a conclusion would ignore the wage effects that are central to our equilibrium analysis.

In his search for the forces that drive capital from poor to rich countries, Lucas (1990) has suggested that governments of poorer countries might be acting as local capital monopolists, holding the supply of capital down to a level that maximizes total profits less the interest cost of capital. This idea is echoed in our Section 8, where we find that the oligarchs would prefer to exclude outside investors as long as the local capital stock is above this monopolistic level.

The adverse effects of political risk and private protection on investment and capital flight are widely recognized as fundamental forces that affect the wealth and poverty of nations, and yet these effects often seem peripheral in economic analysis. The theoretical model that we have developed here is an attempt to put these effects where they belong, in the center of the economic analysis of growth and development.

APPENDIX: Computing the effects of small deviations from the steady state

In this Appendix, we show how to calculate first-order approximations of the effects of small deviations from steady state in our dynamic model from Section 3.

Throughout this Appendix, we suppose that the given initial conditions are very close to the steady state for the given parameter values $(A,\alpha,\delta,\rho,r,L,\lambda,\beta)$. It will be convenient to think of the dynamic model in Section 3 as a two-dimensional dynamic system where the state variables are total oligarchic wealth $\Theta(t)$ and the total value of government offices G(t). Other variables may be viewed as functions of $(\Theta(t),G(t))$. In particular, the net profit rate π may be viewed as a function $\pi(\Theta,G)$ that is determined by the equation (from (10), (12), and (13)):

(40) $$\Theta = \frac{\lambda\Theta}{\pi - r} + (1-\beta)L\left(\frac{A(1-\alpha)}{(1-\beta)\pi + \beta r + \delta}\right)^{1/\alpha} + G\,.$$

We use here the identities (from (9), (10), (12), (13))

$$R = (1-\beta)\pi + \beta r + \delta\,,\quad X = \lambda\Theta/(\pi - r),\ \text{ and }\ K = L(A(1-\alpha)/R)^{1/\alpha}\,.$$

Equation (40) can be implicitly differentiated to yield

(41) $$\frac{\partial\pi}{\partial G} = \frac{(\pi - r)\alpha R}{(\pi - r)(1-\beta)^2 K + \alpha R X} > 0,$$

(42) $$\frac{\partial\pi}{\partial\Theta} = -\frac{(\pi - r - \lambda)\alpha R}{(\pi - r)(1-\beta)^2 K + \alpha R X} < 0 \ \text{ (since } \pi > r + \lambda \text{ always).}$$

The differential equations for our dynamical system are, by equations (11)-(13),

(43) $$G' = \pi G - \Theta\lambda(1 - \lambda/(\pi - r)),$$

(44) $$\Theta' = (\pi - 2\lambda - \rho)\Theta\,.$$

Differentiating (43)-(44) yields the linear approximation of our dynamical system:

(45) $$\frac{\partial G'}{\partial G} = \pi + \left[G - \frac{\lambda^2\Theta}{(\pi - r)^2}\right]\frac{\partial\pi}{\partial G}\,,$$

(46) $$\frac{\partial G'}{\partial\Theta} = \left[G - \frac{\lambda^2\Theta}{(\pi - r)^2}\right]\frac{\partial\pi}{\partial\Theta} - \lambda\left(1 - \frac{\lambda}{\pi - r}\right),$$

(47) $$\frac{\partial\Theta'}{\partial G} = \Theta\frac{\partial\pi}{\partial G}\,,$$

(48) $$\frac{\partial\Theta'}{\partial\Theta} = \Theta\frac{\partial\pi}{\partial\Theta} + (\pi - 2\lambda - \rho) = \Theta\frac{\partial\pi}{\partial\Theta} \ \text{ at the steady state.}$$

Now consider a dynamic equilibrium that begins at some initial condition $(\Theta(0), G(0))$ that is close to the steady state (Θ^*, G^*). For any t≥0, we may write

$$\Theta(t) = \Theta^* + \Delta\Theta(t),\quad G(t) = G^* + \Delta G(t),\quad \pi(t) = \pi^* + \Delta\pi(t).$$

Assuming that the initial conditions are close enough to the steady state, the linear approximation of our dynamical system has a stable solution that converges to the steady state when $(\Delta\Theta(0), \Delta G(0))$ is an eigenvector corresponding to its stable eigenvalue. So let ν denote the negative (stable) eigenvalue of the Jacobian of the dynamic system (45)-(48). Then ν satisfies

$$\left(\frac{\partial G'}{\partial G} - \nu\right)\left(\frac{\partial \Theta'}{\partial \Theta} - \nu\right) - \frac{\partial \Theta'}{\partial G}\frac{\partial G'}{\partial \Theta} = 0, \text{ and so}$$

(49) $$\nu = 0.5\left(\frac{\partial G'}{\partial G} + \frac{\partial \Theta'}{\partial \Theta}\right) - 0.5\sqrt{\left(\frac{\partial G'}{\partial G} + \frac{\partial \Theta'}{\partial \Theta}\right)^2 - 4\left(\frac{\partial G'}{\partial G}\frac{\partial \Theta'}{\partial \Theta} - \frac{\partial G'}{\partial \Theta}\frac{\partial \Theta'}{\partial G}\right)}.$$

From (45)-(48) and (41)-(42) we find

$$\frac{\partial G'}{\partial G}\frac{\partial \Theta'}{\partial \Theta} - \frac{\partial G'}{\partial \Theta}\frac{\partial \Theta'}{\partial G} = \Theta\left(\pi\frac{\partial \pi}{\partial \Theta} + \frac{\lambda(\pi - \lambda - r)}{\pi - r}\frac{\partial \pi}{\partial G}\right) = \Theta(\pi - \lambda)\frac{\partial \pi}{\partial \Theta} < 0,$$

and so $\nu < 0$ is the unique negative eigenvalue of the dynamical system. The eigenvectors for $(\Delta\Theta(0), \Delta G(0))$ corresponding to this stable eigenvalue are along the line in the direction $(1+\gamma, \gamma)$, where γ can be computed from either of the following equations:

$$\nu(1+\gamma) = \frac{\partial \Theta'}{\partial \Theta}(1+\gamma) + \frac{\partial \Theta'}{\partial G}\gamma \quad\text{and}\quad \nu\gamma = \frac{\partial G'}{\partial \Theta}(1+\gamma) + \frac{\partial G'}{\partial G}\gamma,$$

Thus, γ is

(50) $$\gamma = \frac{\dfrac{\partial \Theta'}{\partial \Theta} - \nu}{\nu - \dfrac{\partial \Theta'}{\partial \Theta} - \dfrac{\partial \Theta'}{\partial G}} = \frac{\dfrac{\partial G'}{\partial \Theta}}{\nu - \dfrac{\partial G'}{\partial \Theta} - \dfrac{\partial G'}{\partial G}}.$$

Let $W(t) = \Theta(t) - G(t)$ denote the oligarchs' total economic wealth (excluding their government offices) at any time t. If their economic wealth at time 0 differs from its steady-state value by some small amount ΔW, then we get a stable dynamic solution with $\Delta\Theta'(t) = \nu\Delta\Theta(t)$ and $\Delta G'(t) = \nu\Delta G(t)$ when $\Delta\Theta(0) = (1+\gamma)\Delta W$ and $\Delta G(0) = \gamma\Delta W$. Thus, for small changes in the initial economic wealth W(0) from the steady state $W^* = \Theta^* - G^*$, the initial values our dynamic system will change from steady state according to

(51) $$\frac{\partial\Theta(0)}{\partial W(0)} = 1+\gamma\,,$$

(52) $$\frac{\partial G(0)}{\partial W(0)} = \gamma\,,$$

(53) $$\frac{\partial\pi(0)}{\partial W(0)} = (1+\gamma)\frac{\partial\pi}{\partial\Theta} + \gamma\frac{\partial\pi}{\partial G}\,.$$

Thereafter, the dynamic system will decay toward the steady state at the proportional rate ν. In particular, $\pi'(t) = \nu\Delta\pi(t)$, where $\nu < 0$, and we can compute profit rates by the formula

(54) $$\pi(t) = \pi^* + \Delta\pi(0)e^{\nu t} = 2\lambda + \rho + \Delta\pi(0)e^{\nu t}\,.$$

Consider an oligarch who starts with one unit of wealth $\theta(0) = 1$. His future wealth and utility depend on the path of the dynamic equilibrium entirely through the net profits $\pi(t)$ for all t>0. By equations (5) and (54), his time-t wealth $\theta(t)$ can be computed by

(55) $$\ln(\theta(t)) = \int_0^t [\pi(t) - \lambda - \rho]dt = \int_0^t \left(\lambda + \Delta\pi(0)e^{\nu t}\right)dt = \lambda t + \Delta\pi(0)\left(e^{\nu t} - 1\right)/\nu\,.$$

The expected utility accumulation rate at time t (the integrand in (2)) is

(56) $$U(t) = \ln(\rho\theta(t)) + \frac{\lambda}{\rho}\ln\left(\frac{\lambda\rho\theta(t)}{\pi(t) - r}\right) + \frac{\lambda(r-\rho)}{\rho^2}\,.$$

When we differentiate at the steady state ($\Delta\pi(0) = 0$), we get

(57) $$\frac{\partial U(t)}{\partial\pi(0)} = \left(1 + \frac{\lambda}{\rho}\right)\left(\frac{e^{\nu t} - 1}{\nu}\right) - \frac{\lambda e^{\nu t}}{\rho(2\lambda + \rho - r)}\,.$$

Integrating (57) over t, we obtain the effect of a small change in $\pi(0)$ near steady state on the overall expected discounted utility of the oligarch

(58) $$\frac{\partial EU}{\partial\pi(0)} = \int_0^\infty \frac{\partial U(t)}{\partial\pi(0)} e^{-(\rho+\lambda)t}dt =$$

$$= \left(1 + \frac{\lambda}{\rho}\right)\int_0^\infty \frac{e^{\nu t} - 1}{\nu} e^{-(\rho+\lambda)t}dt - \int_0^\infty \frac{\lambda e^{\nu t}}{\rho(2\lambda + \rho - r)} e^{-(\rho+\lambda)t}dt$$

$$= \frac{(\lambda + \rho - r)}{\rho(\lambda + \rho - \nu)(2\lambda + \rho - r)}\,.$$

From the basic equation $LN(\hat{c})/\rho = EU$, we find that small deviations from steady state affect the oligarchs' constant-equivalent consumption per unit wealth according to the formula

$$\text{(59)} \qquad \frac{\partial \hat{c}}{\partial \pi(0)} = \frac{(\lambda+\rho-r)\hat{c}^*}{(\lambda+\rho-\nu)(2\lambda+\rho-r)}.$$

From the steady state, a small change in the oligarchs' initial economic wealth W(0) would affect their total constant-equivalent consumption C according to

$$\text{(60)} \qquad \frac{\partial C}{\partial W(0)} = \frac{\partial \hat{c}}{\partial \pi(0)} \frac{\partial \pi(0)}{\partial W(0)} \Theta^* + \frac{\partial \Theta(0)}{\partial W(0)} \hat{c}^*$$

Now consider the effects of a small change in λ, starting from the steady state. Differentiating (23) with respect to λ, we find that the steady-state constant-equivalent consumption per unit wealth would vary with λ according to

$$\text{(61)} \qquad \frac{\partial \hat{c}^*}{\partial \lambda} = \frac{\hat{c}^*}{(\lambda+\rho)^2}\left[\rho + \rho LN\left(\frac{\lambda}{2\lambda+\rho-r}\right) + \frac{(\rho-r)(r-\lambda)}{(2\lambda+\rho-r)}\right].$$

Moreover, differentiating (16)-(22), we can see that the steady-state aggregates vary with λ by the following formulas:[12]

$$\frac{\partial \pi^*}{\partial \lambda} = 2, \quad \frac{\partial R^*}{\partial \lambda} = 2(1-\beta),$$

$$\text{(62)} \qquad \frac{\partial K^*}{\partial \lambda} = -\frac{2(1-\beta)K^*}{\alpha R^*},$$

$$\text{(63)} \qquad \frac{\partial W^*}{\partial \lambda} = \left(\frac{W^*}{K^*}\right)\frac{\partial K^*}{\partial \lambda} + \left(\frac{4\lambda+\rho}{(\lambda+\rho)(\lambda+\rho-r)} - \frac{\lambda(2\lambda+\rho)(2\lambda+2\rho-r)}{(\lambda+\rho)^2(\lambda+\rho-r)^2}\right)(1-\beta)K^*,$$

$$\text{(64)} \qquad \frac{\partial \Theta^*}{\partial \lambda} = \left(\frac{\Theta^*}{K^*}\right)\frac{\partial K^*}{\partial \lambda} + \left(\frac{8\lambda+4\rho-2r}{(\lambda+\rho)(\lambda+\rho-r)} - \frac{(2\lambda+\rho)(2\lambda+\rho-r)(2\lambda+2\rho-r)}{(\lambda+\rho)^2(\lambda+\rho-r)^2}\right)(1-\beta)K^*.$$

So if we could jump directly to a new steady state when λ changes slightly, then the total constant-equivalent consumption C* of all oligarchs in steady state would change according to

$$\text{(65)} \qquad \frac{\partial C^*}{\partial \lambda} = \hat{c}^* \frac{\partial \Theta^*}{\partial \lambda} + \frac{\partial \hat{c}^*}{\partial \lambda}\Theta^*.$$

But a reform that changes political risk by some amount $\Delta\lambda$ would not lead immediately to the new steady state, because the oligarchs' initial economic wealth W(0) would then not be

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[12] Here (63) is obtained by differentiating

$$W^* = X^* + (1-\beta)K^* = \left[(2\lambda+\rho)\lambda \big/ \big((\lambda+\rho-r)(\lambda+\rho)\big) + 1\right](1-\beta)K^*$$

where the second step uses (21). Similarly, (64) is obtained by differentiating (22).

equal to the amount W* needed for the new steady state. So the effects of changing λ on the oligarchs' welfare in (65) can be decomposed into two parts:

$$\frac{\partial C^*}{\partial \lambda} = \frac{\partial C}{\partial \lambda} + \frac{\partial C}{\partial W(0)} \frac{\partial W^*}{\partial \lambda}.$$

The first part $\partial C/\partial \lambda$ is the effect of changing λ on the oligarchs' welfare in the dynamic equilibrium, given their initial economic wealth W(0) from the old steady state. The second part is the effect on the oligarchs' welfare of changing their aggregate economic wealth to the level that is required for the new steady state (which would affect the welfare of an oligarch with any given wealth through the change in profit rates). The first part $\partial C/\partial \lambda$ is what actually interests us, but the second part can be computed from (60) and (63), and their sum is (65). So the effect of changing λ on the oligarchs' total constant-equivalent consumption in dynamic equilibrium is

(66) $$\frac{\partial C}{\partial \lambda} = \frac{\partial C^*}{\partial \lambda} - \frac{\partial C}{\partial W(0)} \frac{\partial W^*}{\partial \lambda}.$$

This system of equations for computing $\partial C/\partial \lambda$ may seem complicated, but they are easily calculated in a computational spreadsheet that is available from the authors. Thus, for any given steady state, we can say whether the oligarchs would benefit from a small increase or decrease of political risk to $\lambda + \Delta\lambda$, depending on whether $\partial C/\partial \lambda$ in (66) is positive or negative.

<u>Proposition 7.</u> *Starting from a given steady state, a small change $\Delta\lambda$ in the political risk parameter would increase the oligarchs' welfare iff $\Delta\lambda$ has the same sign as $\partial C/\partial \lambda$.*

Consider again our standard baseline parameters (28) with $\beta = 0$ but different values of λ. We find that $\partial C/\partial \lambda = 0$ when $\lambda = 0.01984$. With any $\lambda > 0.01984$, we get $\partial C/\partial \lambda < 0$, which implies that the oligarchs' total welfare could be increased by decreasing λ. With any λ in the interval $0.01984 > \lambda > 0.000325$, we find $\partial C/\partial \lambda > 0$, indicating that the oligarchs' total welfare would increase from increasing λ. In this sense, $\lambda = 0.019837$ is politically stable, because from a steady state with any λ in an interval around this value, the oligarchs could benefit by moving λ towards this value.

In the very low end of political risks below 0.000325, the oligarchs would have some incentive to decrease λ locally. But it can be shown numerically that the dynamic effects of jumping from $\lambda = 0.01984$ to $\lambda = 0$ (holding $\beta = 0$ fixed) would reduce the oligarchs' welfare.

A similar analysis can be done for changes in the collateralizability parameter β.

Differentiating the steady-state equations (16)-(23) with respect to β, we get

$$\frac{\partial \hat{c}^*}{\partial\beta} = 0\,, \quad \frac{\partial \pi^*}{\partial\beta} = 0\,,$$

$$\frac{\partial R^*}{\partial\beta} = -(\pi^* - r)\,, \quad \frac{\partial K^*}{\partial\beta} = \frac{(\pi^* - r)K^*}{\alpha R^*} = \frac{(R^* - \delta - r)K^*}{(1-\beta)\alpha R^*}\,,$$

$$\frac{\partial}{\partial\beta}\left[(1-\beta)K^*\right] = (1-\beta)K^*\left(\frac{(R^* - \delta - r)}{(1-\beta)\alpha R^*} - \frac{1}{1-\beta}\right) =$$

$$= (1-\beta)K^*\left(\frac{(1-\alpha)R^* - \delta - r}{(1-\beta)\alpha R^*}\right)$$

$$= (1-\beta)K^*\left(\frac{(1-\alpha)(\pi^* - r)}{(1-\beta)\alpha R^*}\right)\left(B(\pi^*, r) - \beta\right)$$

Because W* and Θ^* are proportional to $(1-\beta)K^*$, multiplied by factors that do not depend on β,

$$\frac{\partial \Theta^*}{\partial\beta} = \Theta^*\left(\frac{(1-\alpha)(\pi^* - r)}{(1-\beta)\alpha R^*}\right)\left(B(\pi^*, r) - \beta\right),$$

$$\frac{\partial W^*}{\partial\beta} = W^*\left(\frac{(1-\alpha)(\pi^* - r)}{(1-\beta)\alpha R^*}\right)\left(B(\pi^*, r) - \beta\right).$$

Thus, as in equation (66),

$$\text{(67)} \qquad \frac{\partial C}{\partial\beta} = \frac{\partial C^*}{\partial\beta} - \frac{\partial C}{\partial W(0)}\frac{\partial W^*}{\partial\beta} = \hat{c}^*\frac{\partial \Theta^*}{\partial\beta} - \frac{\partial C}{\partial W(0)}\frac{\partial W^*}{\partial\beta}$$

$$= \left(C^* - \frac{\partial C}{\partial W(0)}W^*\right)\left(\frac{(1-\alpha)(\pi^* - r)}{(1-\beta)\alpha R^*}\right)\left(B(\pi^*, r) - \beta\right).$$

So from a given initial steady state, the effect of a small change in β on the oligarchs' welfare C in a dynamic equilibrium is determined by the sign of $\left(B(\pi^*, r) - \beta\right)$, as we found in Section 8.[13]

[13] Here $C^* - W^*\partial C/\partial W(0) = L\,\partial C/\partial L$, because the economy is linearly homogeneous in labor L and economic wealth W(0), and $\partial C/\partial L > 0$ because adding workers cannot hurt the oligarchs. A more detailed derivation of this inequality is available from the authors.

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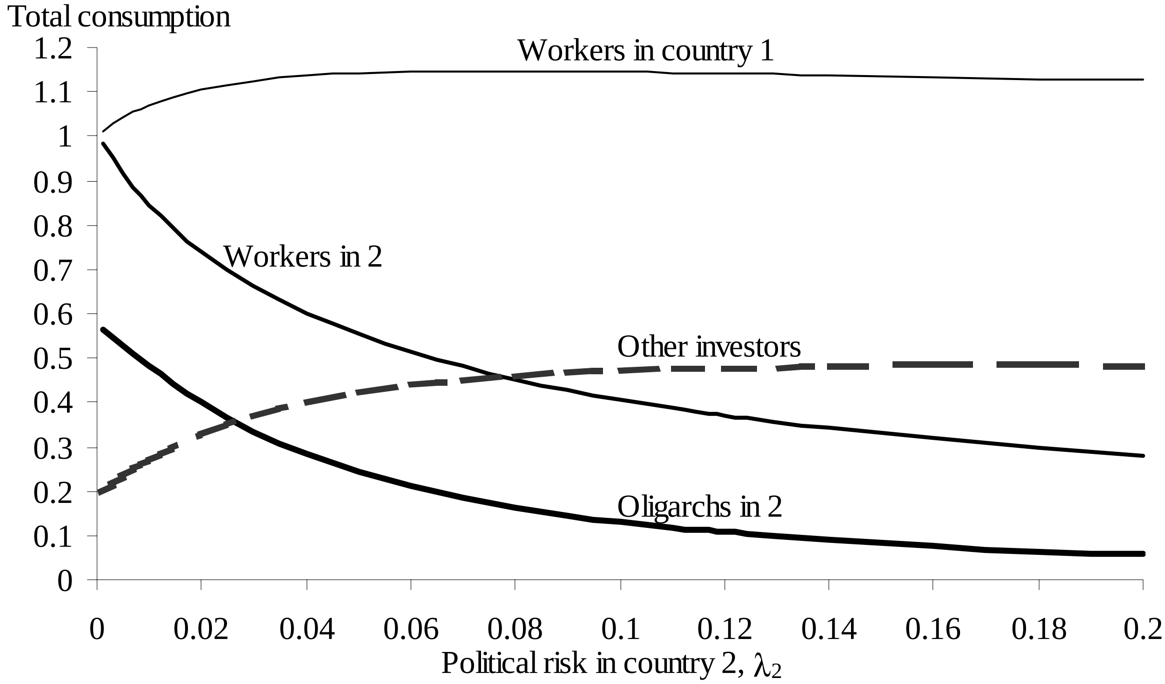

Figure 1. Aggregate steady-state consumption rates with different political risks in country 2, for the two-country model with α=0.6, δ=0.03, ρ=0.04, A=0.6766, L_1=L_2=1, λ_1=0, β_1=1, β_2=0.

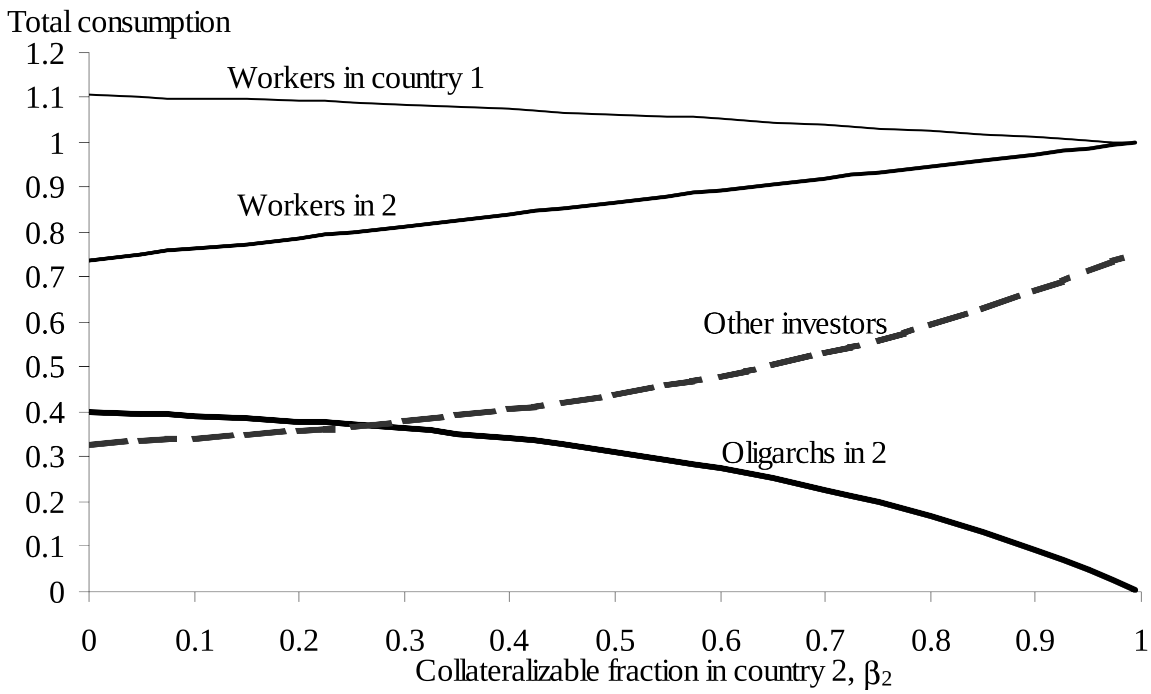

Figure 2. Aggregate steady-state consumption when outsiders can finance different fractions of local capital in country 2, for the two-country model with α=0.6, δ=0.03, ρ=0.04, A=0.6766, L_1=L_2=1, λ_1=0, β_1=1, λ_2=0.02.

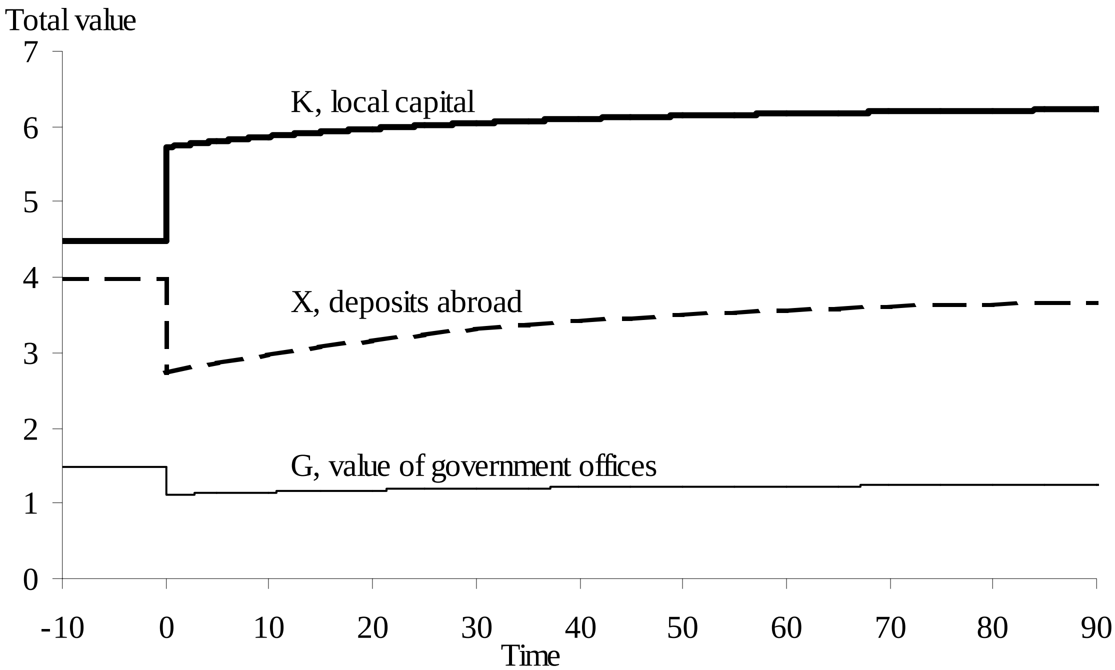

Figure 3a. Asset changes after decreasing political risk to λ = 0.01 at t = 0, starting from the steady state of λ = 0.02, holding fixed α=0.6, δ=0.03, ρ=0.04, A=0.6766, r = 0.03, L = 1, β=0.

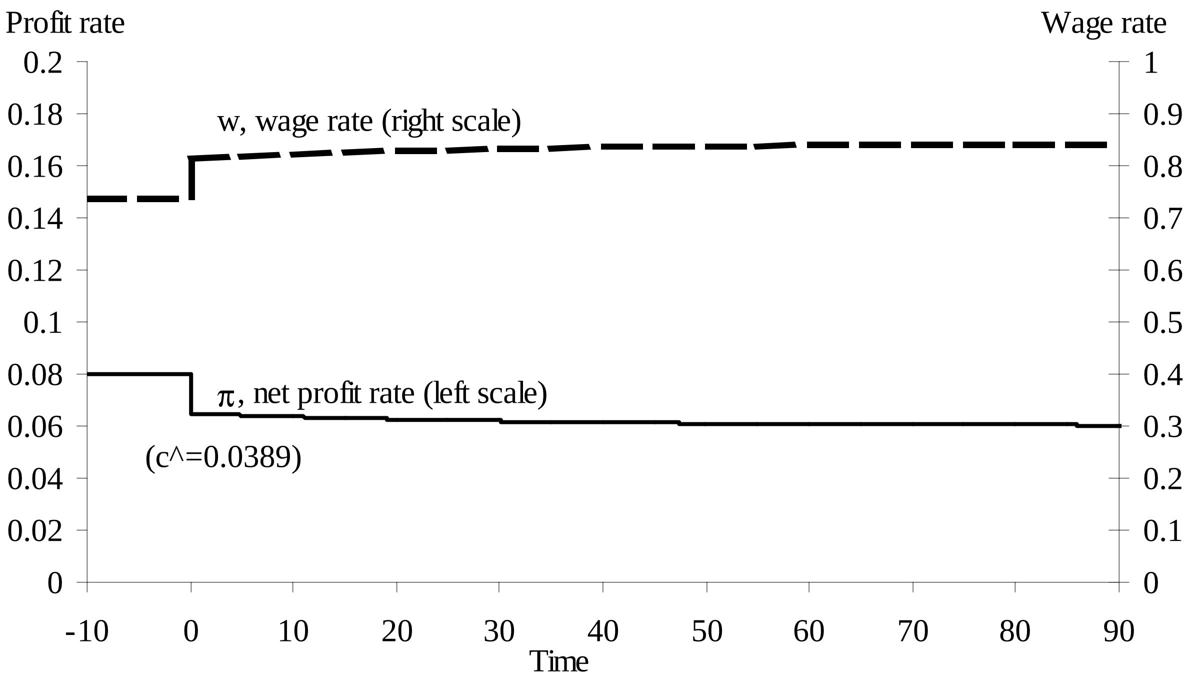

Figure 3b. Profits and wages after decreasing political risk to λ = 0.01 at t = 0, starting from the steady state of λ = 0.02, holding fixed α=0.6, δ=0.03, ρ=0.04, A=0.6766, r = 0.03, L = 1, β=0.

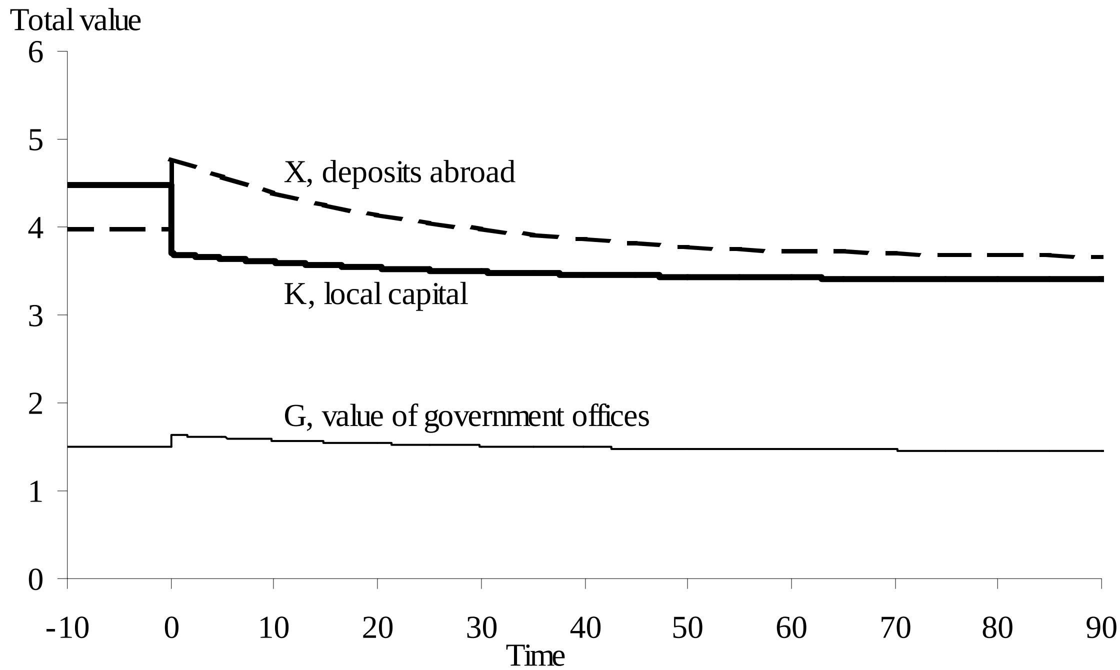

Figure 4a. Asset changes after increasing political risk to λ = 0.03 at t = 0, starting from the steady state of λ = 0.02, holding fixed α=0.6, δ=0.03, ρ=0.04, A=0.6766, r=0.03, L=1, β=0.

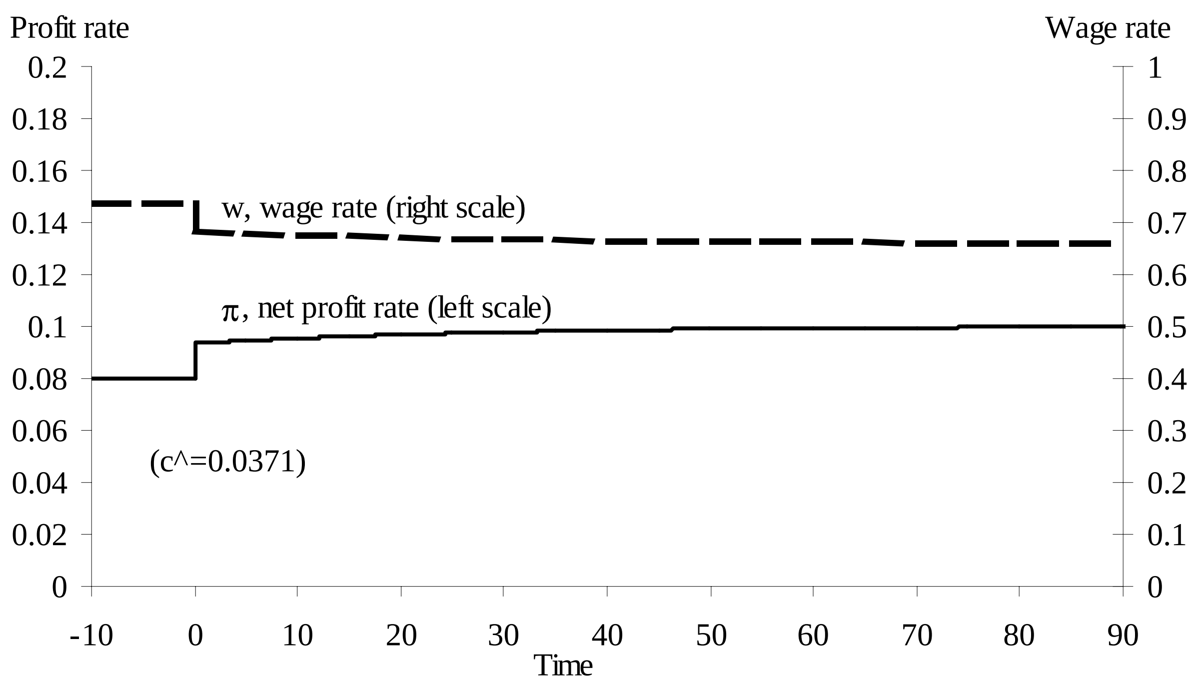

Figure 4b. Profits and wages after increasing political risk to λ = 0.03 at t = 0, starting from the steady state of λ = 0.02, holding fixed α=0.6, δ=0.03, ρ=0.04, A=0.6766, r=0.03, L=1, β=0.

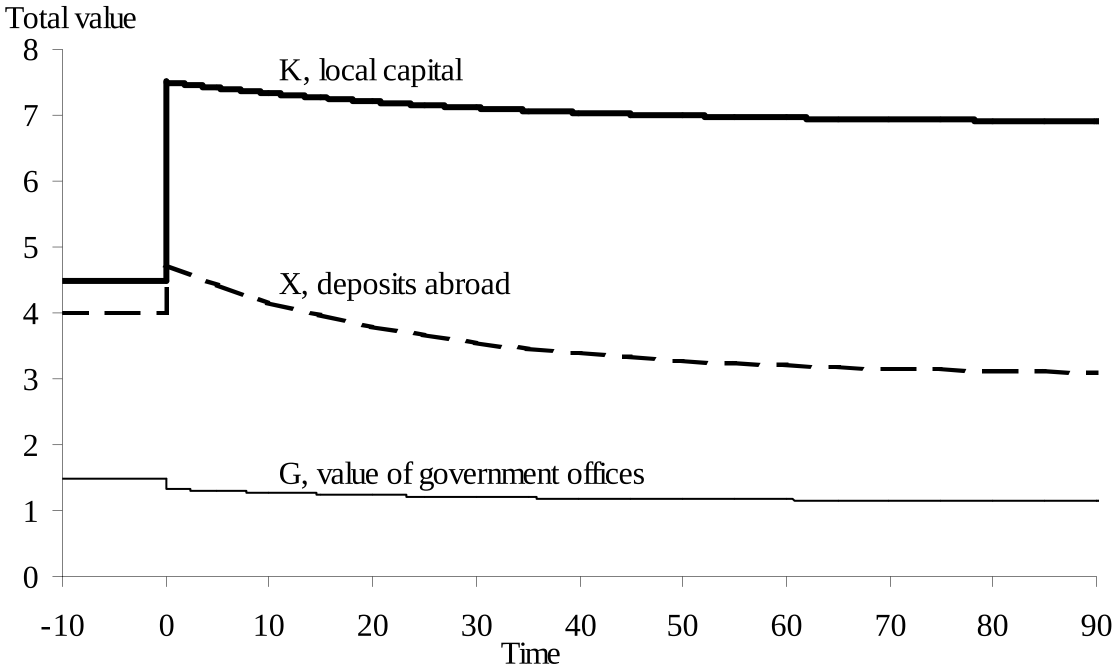

Figure 5a. Asset changes after increasing collateralizability to $\beta = 0.5$ at $t = 0$, starting from the steady state with $\beta = 0$, holding fixed α=0.6, δ=0.03, ρ=0.04, A=0.6766, r=0.03, L=1, λ=0.02.

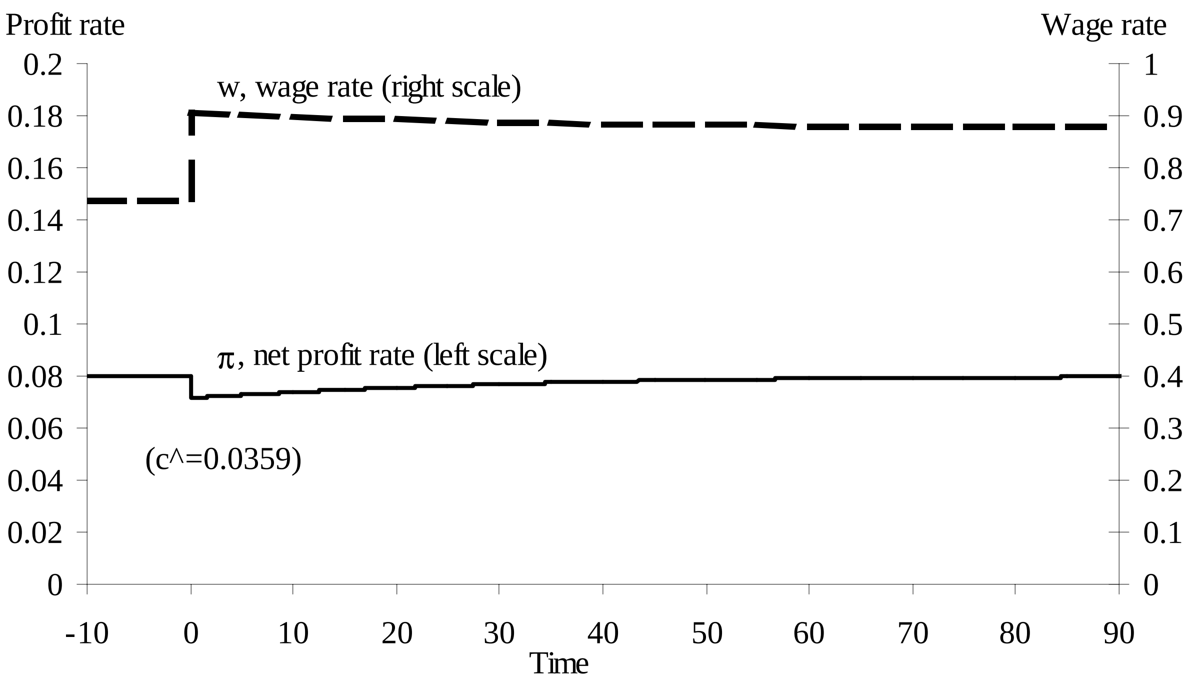

Figure 5b. Profits and wages after increasing collateralizability to $\beta = 0.5$ at $t = 0$, starting from steady state with $\beta = 0$, holding fixed α=0.6, δ=0.03, ρ=0.04, A=0.6766, r=0.03, L=1, λ=0.02.

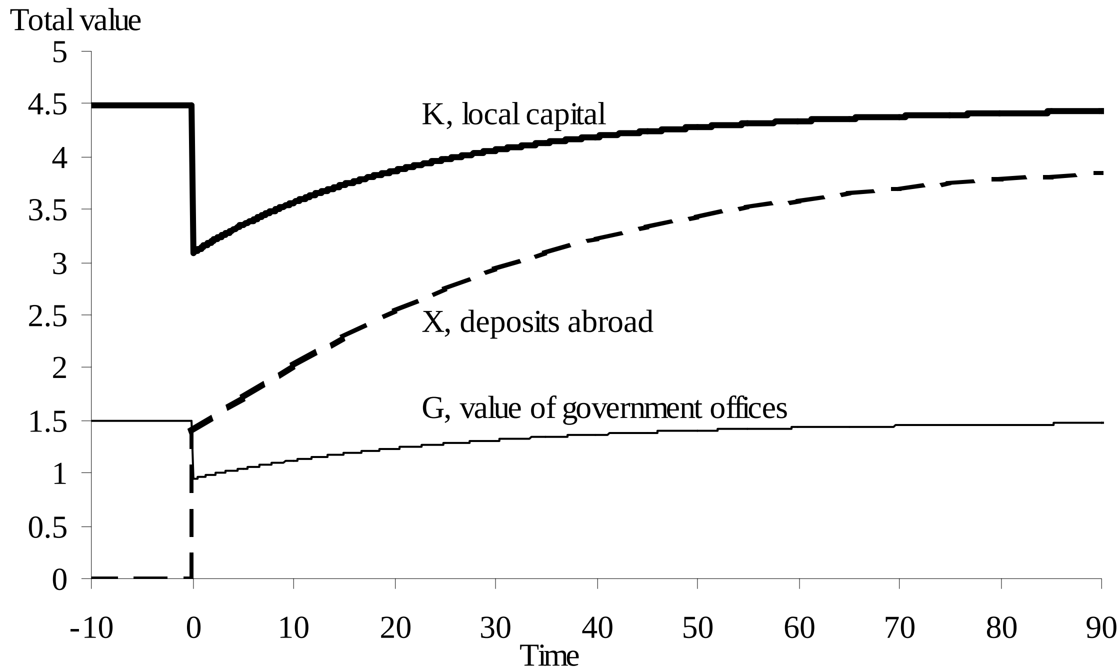

Figure 6a. Asset values after allowing r = 0.03 investment abroad at t = 0, starting from the closed (r = –∞) steady state, with α=0.6, δ=0.03, ρ=0.04, A=0.6766, L=1, λ=0.02, β =0.

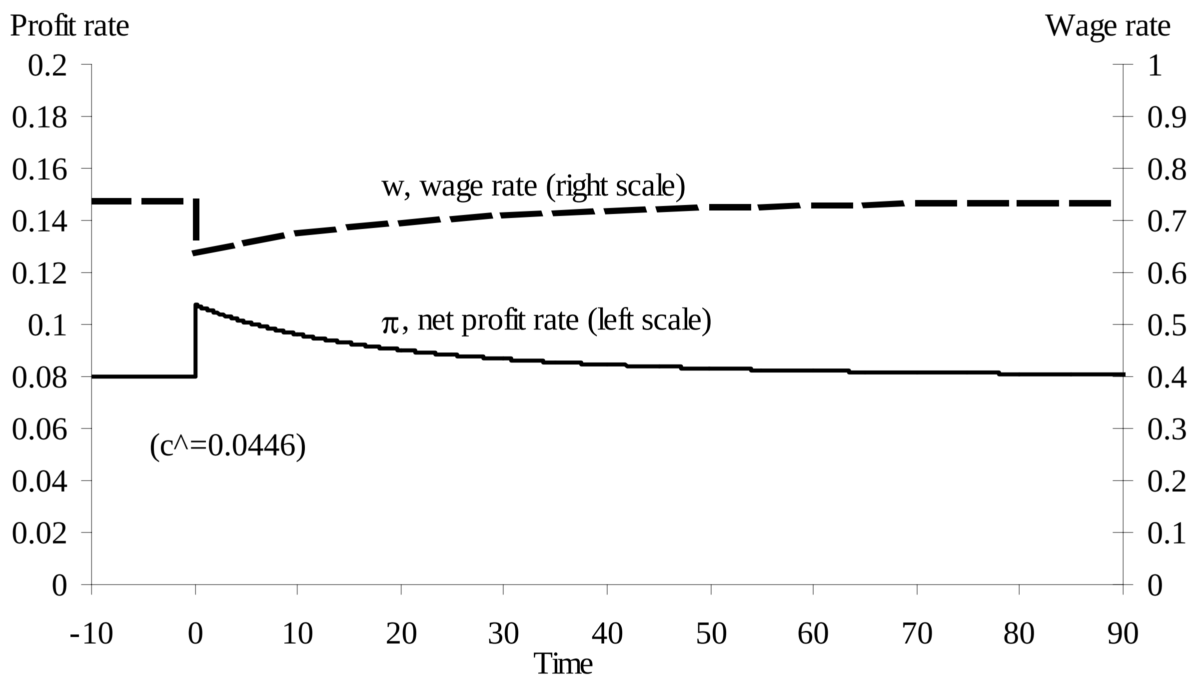

Figure 6b. Profit and wages after allowing r = 0.03 investment abroad at t=0, starting from the closed (r = –∞) steady state, with α=0.6, δ=0.03, ρ=0.04, A=0.6766, L=1, λ=0.02, β =0.

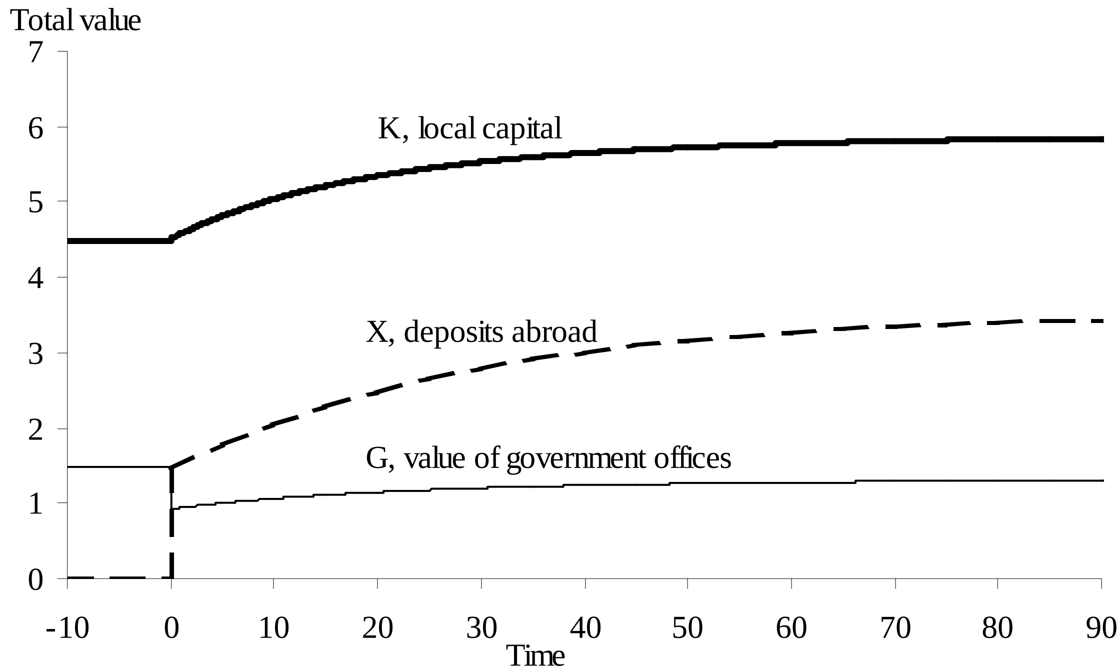

Figure 7a. Results of financial opening to r = 0.03 and β = 0.33 at t=0, starting from the closed steady state (r = –∞, β = 0), with α=0.6, δ=0.03, ρ=0.04, A=0.6766, L=1, λ=0.02 fixed.

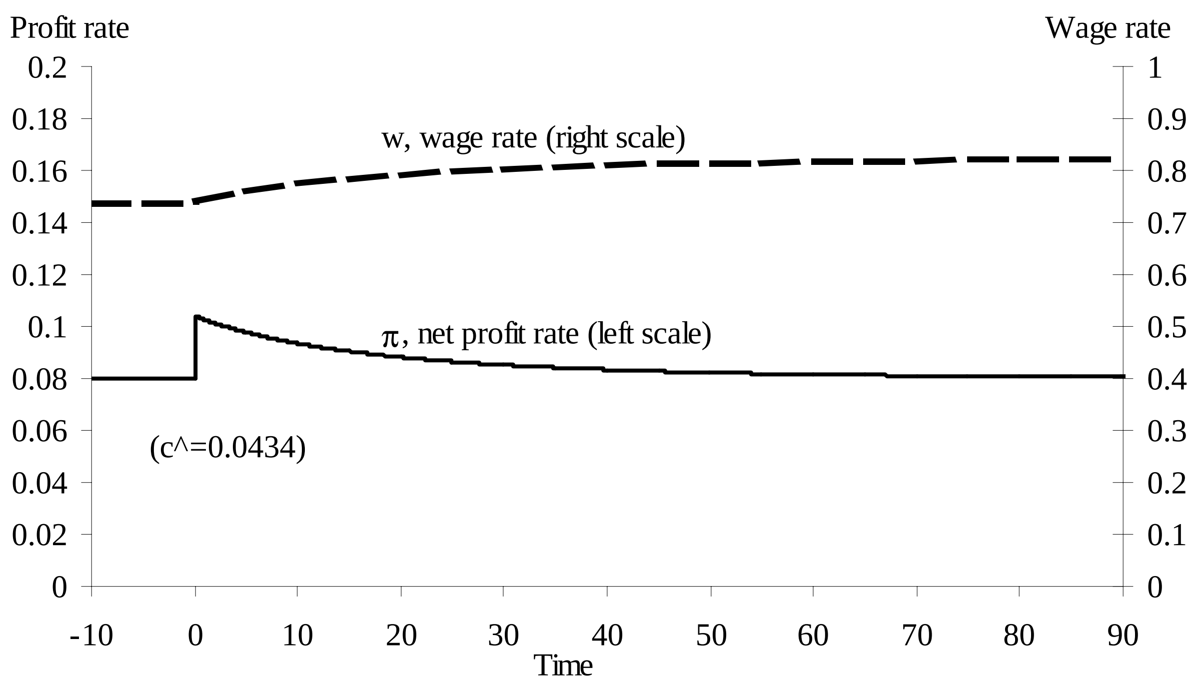

Figure 7b. Results of financial opening to r = 0.03 and β = 0.33 at t=0, starting from the closed steady state (r = –∞, β = 0), with α=0.6, δ=0.03, ρ=0.04, A=0.6766, L=1, λ=0.02 fixed.