Answer Key Problem Set 5

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1 Question 1

A small country can import a good at a world price of 10 per unit. The domestic supply curve of the good is

$$S = 50 + 5P$$

The demand curve is

$$D = 400 - 10P$$

In addition, each unit of production yields a marginal social benefit of 10.

a Calculate the total effect on welfare of a tariff of 5 per unit levied on imports

Before we are able to calculate the total effect on welfare of this policy instrument we need to compute a few things. We begin by calculating equilibrium in the domestic market, ie we set domestic demand equal to domestic supply and we solve for price.

$$\begin{aligned} S &= D \\ 50 + 5P &= 400 - 10P \\ 15P &= 350 \\ P^e &= \frac{350}{15} = \frac{70}{3} \end{aligned}$$

With a domestic price of $P^e = \frac{70}{3}$ we now substitute this value into demand or supply to obtain the domestic (autarky) quantity. Therefore, we have that

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$$
\begin{aligned}
D &= 400 - 10\left(\frac{350}{15}\right) \\
&= 400 - \frac{700}{3} \\
&= \frac{1200 - 700}{3} \\
D &= \frac{500}{3}
\end{aligned}
$$

That the quantity, price pair that make this market clear in autarky is $\left(\frac{500}{3}, \frac{70}{3}\right)$. Now, at a price of 10 per unit this country would be producing less and consuming more. Therefore, to obtain these quantities we substitute the price of 10 into domestic demand and domestic supply. We have then that quantity supplied is the following:

$$
\begin{aligned}
S &= 50 + 5\,(10) \\
Q_S &= 100
\end{aligned}
$$

and quantity demanded is then the following:

$$
\begin{aligned}
D &= 400 - 10\,(10) \\
Q_D &= 300
\end{aligned}
$$

Finally, imports are defined as the difference between quantity demanded and quantity supplied. Then, it follows:

$$
\begin{aligned}
M_0 &= Q_D - Q_S \\
&= 300 - 100 \\
&= 200
\end{aligned}
$$

Figure 1. below presents the initial situation as well as the excess demand curve.

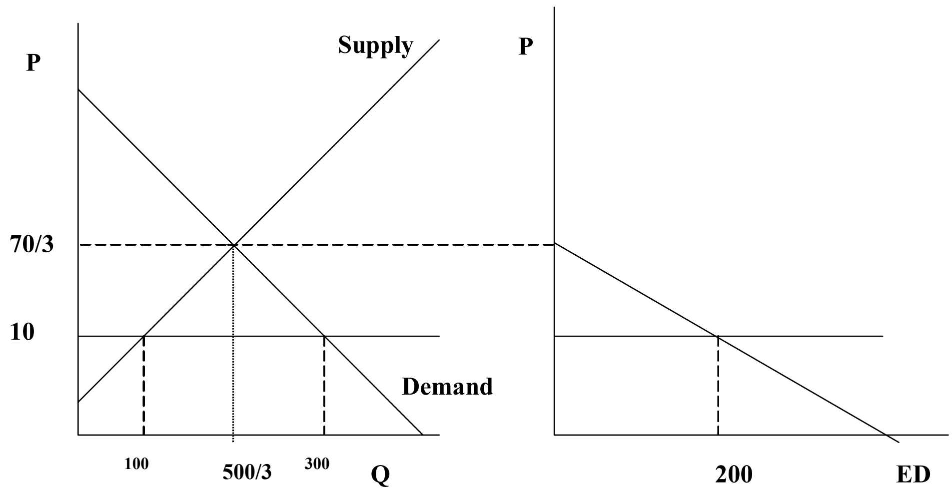

Figure 1. Small importing country

Now we are in a position to impose the \$5 per unit tariff and analyze its effects on welfare. Since it is a per unit tariff we can just add it to the international price, that is the new price domestic consumers are going to be facing is now 15 instead of 10. The new price comes about by increasing the amount of the tariff to the international price. To obtain the new quantity demanded and quantity supplied we now substitute the price of 15 into domestic demand and domestic supply. We have then the following:

$$\begin{aligned} S &= 50 + 5\,(15) \\ Q_S^t &= 125 \end{aligned}$$

and

$$\begin{aligned} D &= 400 - 10\,(15) \\ Q_D^t &= 250 \end{aligned}$$

Finally, the new amount of imports is again the difference between the quantity demanded with the tariff and the quantity supplied with the tariff, that is the new amount of imports $M_1 = 125$. Figure 2 below presents the outcome for a small country that imposes an import tariff of \$5 per unit. Note that there are distributional changes that we still need to calculate.

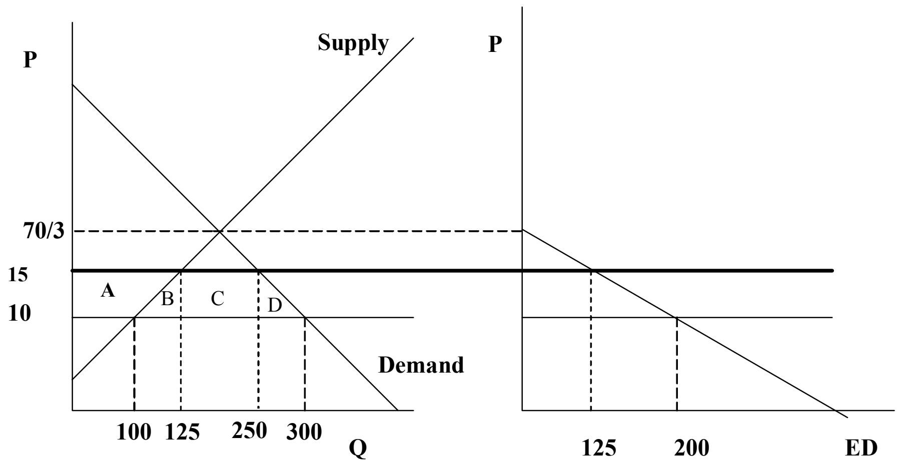

Figure 2. Small country and import tariff

Starting with consumers we know that when prices increase consumers are going to lose. In this case, consumers are going to lose areas A+B+C+D. The magnitude of this loss of consumer surplus is $(250 * 5) + \left(\frac{50*5}{2}\right) = 1375$. Producers on the other hand win area A. This gain is equal to $(100 * 5) + \left(\frac{25*5}{2}\right) = 562.5$. The government is going to win area C which is equal to $(125 * 5) = 625$. Finally, we include the Marginal Social Benefit of 10 per the extra units of production which is equal to $(10 * 25) = 250$. Putting all these changes in welfare due to the imposition of the import tariff we have that this country had a positive net welfare gain of 62.5. This figure is calculated from the algebraic sum of the following terms: $-1375 + 562.5 + 625 + 250 = 62.5$.

b Calculate the total effect of a production subsidy of 5 per unit

Intuitively, a production subsidy will only distort the prices producers face in the market place, ie the government has to incentive producers to increase their production. However, at the same time the government will let consumers keep facing a price of \$10.when they purchase this good. Therefore, there will be no welfare changes for consumers, there will be a welfare gain for producers, a welfare loss to the government, and a social welfare gain due to the increase in production. Lets look at these effects.

The first thing we need to remember is a production subsidy will create an imaginary shift of the supply curve towards the right. This is so because at every price now producers are willing to supply a larger quantity, precisely because of the subsidy. The subsidy in this case is of the same amount as the import tariff. Therefore, the rightward shift of the supply curve will cross the \$10 price line at a quantity of 125. Therefore, producers will gain area A which is equal to $(100 * 5) + \left(\frac{25*5}{2}\right) = 562.5$. The government will lose the amount of the subsidy multiplied by the amount produced which is equal to $(125 * 5) = 625$. Social benefit will then be $(10 * 25) = 250$. Remember, consumer do not observe any welfare changes since they are unaffected by the production subsidy. However, imports do change because there is more domestic production. Since consumption is constant at 300, but production has increased to 125 units then imports are reduced to 175. The balance for this policy instrument is a positive one of 187.5.This figure is calculated by the algebraic sum of the following quantities $250 + 562.5 - 625 = 187.5$. Figure 3. below presents this case.

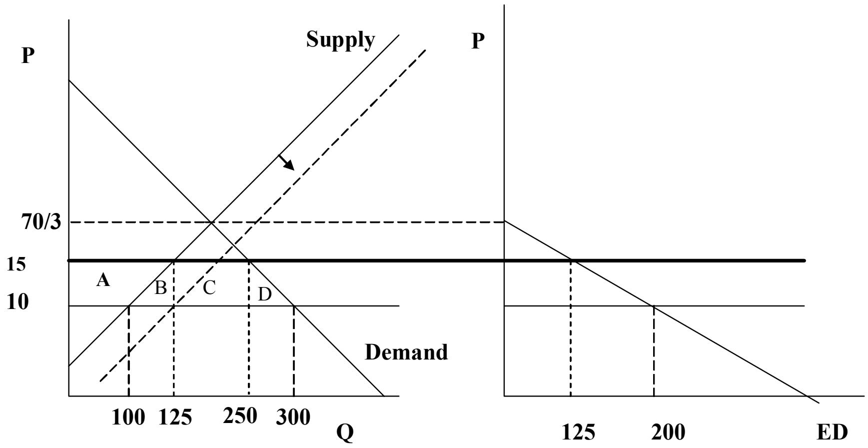

Figure 3. Small country, import tariff, and production subsidy

Remember, consumers stay at their consumption point of 300 units. Producers increase production to 125 units, the government has to pay \$5 for each of the 125 units produced, and society is also better because each of the 25 extra units produced is worth \$10.

c Why does the production subsidy produce a greater gain in welfare than the tariff?

The answer to this question lies in the fact that the production subsidy only distorts the production side. It is only producers that see their status-quo situation modified. Consumers remain at their original consumption point and observe no distortion in the final good market. On the other hand, the import tariff modifies both the behavior of consumers, they consume less, and of producers, they produce more.

d What would the optimal production subsidy be?

To calculate the optimal production subsidy we first need to identify what is it exactly we want to maximize. From the previous questions we noticed that welfare is enhanced given the combination of economic policies and the externality. Therefore, our goal would be to make that change in welfare as big as possible. To that effect we have to realize that welfare for this country is going to be the summation of consumer surplus, producer surplus, government expenditures, and social benefit. Therefore, we define Welfare as the following:

$$Welfare = CS + PS + GVT + SB$$

However, we do not want to maximize welfare per se, but the change in welfare. Therefore, our objective function is going to be the change in welfare which we define in the following manner:

$$MAX\ \Delta Welfare = \Delta CS + \Delta PS + \Delta GVT + \Delta SB$$

As already explained in question 1c the production subsidy will not affect consumers. They will remain at their original consumption point of 300 units. Therefore, we know that $\Delta CS = 0$. Our objective function then reduces to the change in producer surplus, change in government expenditures, and change in social benefit. To construct our objective we proceed in the following way:

Note that the new price producers will get will be the sum of the old price and the subsidy. Therefore, define the $P_{new} = 10 + subsidy$. Take this new price and substitute into the supply equation to obtain the new quantity as a function of the subsidy. Therefore, we have the following:

$$S = 50 + 5P$$

Substitute for the price to obtain the following:

$$\begin{aligned} S &= 50 + 5\,(10 + subsidy) \\ &= 100 + 5 * subsidy \end{aligned}$$

To calculate the ΔPS we have the following:

$$\begin{aligned} \Delta PS &= (100 * subsidy) + \left(\frac{subsidy * (100 + 5 * subsidy - 100)}{2}\right) \\ &= 100 * subsidy + \frac{5}{2} * subsidy^2 \end{aligned}$$

To calculate ΔGVT we have the following:

$$\begin{aligned} \Delta GVT &= subsidy * (100 + 5 * subsidy) \\ &= 100 * subsidy + 5 * subsidy^2 \end{aligned}$$

To calculate ΔSB we have the following:

$$\begin{aligned} \Delta SB &= 10\,(100 + 5 * subsidy - 100) \\ &= 50 * subsidy \end{aligned}$$

We substitute these expression into our objective function to obtain an expression that is only a function of the subsidy. In this way we can take a derivative with respect to the subsidy, set it equal to zero, and solve for the optimal subsidy. We have the following:

$$\begin{aligned} MAX\ \Delta Welfare &= 100 * subsidy + \frac{5}{2} * subsidy^2 - \left(100 * subsidy + 5 * subsidy^2\right) + 50 * subsidy \\ &= -\frac{5}{2} * subsidy^2 + 50 * subsidy \end{aligned}$$

Take the derivative of the objective function and set equal to zero, i.e $\frac{\partial \Delta Welfare}{\partial subsidy} = 0$. We have that

$$\begin{aligned} \frac{\partial \Delta Welfare}{\partial subsidy} &= -5 * subsidy + 50 = 0 \\ subsidy &= \frac{50}{5} \\ &= 10 \end{aligned}$$

Since this a strictly concave function we know that is has a unique maximum and it opens downward. Therefore, there is no need to check the second order conditions. Finally, the increase in welfare that a subsidy of 10 would provide is of the amount of 250.

2 Question 2

Suppose that ABC is a Taiwanese firm, and XYZ is a Korean firm. The following payoffs result, depending on the firm's decisions. What equilibrium, and what payoffs will result if there is no government intervention?

		XYZ		
ABC			Produce	Not Produce
		Produce	$-40, 25$	$60, 0$
		Not Produce	$0, 150$	$0, 0$

If there is no government intervention then the equilibrium we would observe would be that of the XYZ firm producing and the ABC firm not producing. The payoffs would be $(0, 150)$ where ABC would receive 0 and XYZ would receive 150. The process by which we arrive at this outcome is the following:

We are looking for the equilibrium where the players have no unilateral incentives to deviate from it. This equilibrium is called a Nash equilibrium. To find the Nash equilibrium we first check to see if the players have dominant strategies. If they do this would simplify the process because this would imply there is a set of strategies that a player will never use. In this case, we can see that firm XYZ has a dominant strategy of Produce because the payoffs of producing are always greater than the payoffs of Not Producing. Firm XYZ is comparing 25 to 0; and 150 to 0. Producing will always yield a greater payoff than Not producing. Conclusion, firm XYZ will always produce. On the other hand, firm ABC does not have a dominant strategy. However, since firm ABC knows that firm XYZ will always produce then the best response from firm ABC is to Not produce. This way firm ABC will obtain a payoff of 0 instead of one of -40.

3 Question 3

Strategic trade. The cost for production is 3,000 per car in the US and 2,000 per car in Japan. Suppose automobile prices are described by the following inverse demand function:

$$P = 60,000 - 20\left(X_{US} + X_J\right)$$

a Derive the reaction function for US producers, ie $X_{US}\left(X_J\right)$, express the number of US cars produced as a function of Japanese production.

To derive reaction functions we first need to maximize profits taking the other country's production as given. So, if we want to obtain the US reaction function then we maximize US profits taking Japanese production as given:

$$\begin{aligned} Max\ \pi &= P * X_{US} - 3,000X_{US} \\ &= \left(60,000 - 20\left(X_{US} + X_J\right)\right)X_{US} - 3,000X_{US} \end{aligned}$$

Now take the derivative of profits with respect to the quantity produced in the United States. We have the following:

$$\begin{aligned} \frac{\partial \pi}{\partial X_{US}} &= 60,000 - 40X_{US} - 20X_J - 3,000 \\ &= 57,000 - 40X_{US} - 20X_J \end{aligned}$$

Set $\frac{\partial \pi}{\partial X_{US}} = 0$ and solve for X_{US}. We have the following:

$$\begin{aligned} 57,000 - 40X_{US} - 20X_J &= 0 \\ 40X_{US} &= 57,000 - 20X_J \\ X_{US} &= \frac{57,000 - 20X_J}{40} \end{aligned}$$

If we wanted to obtain the Japanese reaction function we would proceed in the same manner as we just did. Note that these two reaction function are almost symmetric except for marginal cost which is lower in Japan. Taking this difference into account we obtain the Japanese reaction function:

$$\begin{aligned} Max\ \pi &= P * X_J - 2,000X_J \\ &= \left(60,000 - 20\left(X_{US} + X_J\right)\right)X_J - 2,000X_J \end{aligned}$$

Now take the derivative of profits with respect to the quantity produced in Japan. We have the following:

$$\begin{aligned}\frac{\partial\pi}{\partial X_J} &= 60,000 - 40X_J - 20X_{US} - 2,000 \\ &= 58,000 - 40X_J - 20X_{US}\end{aligned}$$

Set $\frac{\partial\pi}{\partial X_J} = 0$ and solve for X_J. We have the following:

$$\begin{aligned}58,000 - 40X_J - 20X_{US} &= 0 \\ 40X_J &= 58,000 - 20X_{US} \\ X_J &= \frac{58,000 - 20X_{US}}{40}\end{aligned}$$

Now consider the reaction functions graphed below and use them to answer the questions b and c

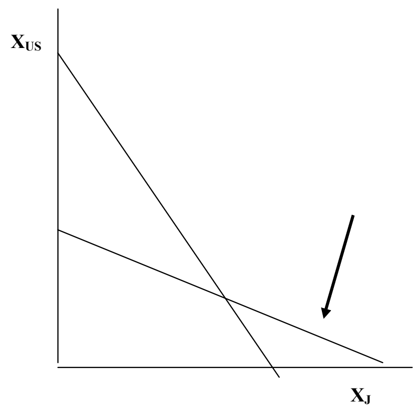

Figure 4. Reaction Functions

b Which reaction function does the arrow point to? $X_{US}(X_J)$ or $X_J(X_{US})$?

The arrow points towards the United States reaction function, that is $X_{US}(X_J)$.

c What factors may cause any US strategic trade policy attempts to fail? List 2

The reasons that might make US strategic trade policy fail are many, but among them are the following:

- The US is facing another large country and there is a possibility of retaliation;
- The US makes a non-credible threat and the other country calls the "bluff" of the United States;
- Rent-seeking