Income Taxation in a Life Cycle Model with Human Capital

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Abstract: I examine the effect of labor income taxation in a simple life-cycle model where work experience builds human capital. There are four key findings: First, contrary to conventional wisdom, in such a model, permanent tax changes can have larger effects on labor supply than temporary tax changes. This is because permanent tax changes affect the future return to human capital investment, not just the current wage. Second, even with small returns to work experience, conventional methods of estimating the intertemporal elasticity of substitution will be seriously biased towards zero. (This includes methods that rely on exogenous changes in tax regimes). Third, both compensated and uncompensated labor supply elasticities are also likely to be larger than (conventional) estimates (that ignore human capital) would suggest. Fourth, for plausible parameter values, welfare losses from proportional income taxation are likely to be much larger than conventional wisdom suggests.

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I. Introduction

This paper examines the effects of income taxation in a life-cycle model where work experience builds human capital. In such a model, the wage no longer equals the opportunity cost of time. This has important implications for how workers respond to tax changes, and for estimation of wage elasticities of labor supply. For instance, I show permanent tax changes can have larger effects on current labor supply than transitory changes. This contradicts the conventional wisdom that transitory tax changes should have larger (short run) effects.

Of course, Imai and Keane (2004) already studied introduction of human capital into the standard life-cycle model of MaCurdy (1981). They showed that ignoring human capital leads to severe downward bias in estimates of the intertemporal elasticity of substitution. Unfortunately, however, Imai and Keane (2004) did not consider the effects of human capital on estimates of Marshall and Hicks elasticities with respect to permanent tax changes. These are more relevant for tax policy. Here, I show that elasticities with respect to permanent tax changes can also be seriously biased downward by failure to account for human capital.

One motivation for this paper is that the Imai-Keane (2004) model is quite elaborate, allowing for a complex and flexible human capital production function, wage uncertainty, taste shocks, a 45-period working life, a bequest motive, etc.. Here I focus on a simple two-period version of their model. The two-period model has the virtue that it delivers simple and intuitive analytic expressions for (i) the magnitude of bias in estimates of the elasticity of substitution that ignore human capital, and (ii) the conditions under which permanent tax changes have larger current effects than transitory ones.

Using the simple model, I show that bias in estimates of the intertemporal elasticity of substitution will be large even if returns to experience are "small" (in a sense made precise below). I also show that permanent tax changes will have larger current effects than transitory changes under a condition that requires the returns to work experience to be sufficiently large relative to the size of the income effect. Using a calibrated version of the simple two-period model, I show this does in fact occur for plausible parameter values.

I also provide new simulations of the Imai-Keane (2004) model to see what it implies about effects of permanent tax changes. Averaged over ages from 20 to 65, the model implies a large compensated elasticity of 1.3. It also implies that permanent tax changes do have larger effects on current labor supply than transitory tax changes, at least for younger workers. Furthermore, the elasticity with respect to permanent tax changes is not a single number. Instead, I find that the effect of permanent tax changes grows over time: higher taxes reduce labor supply, which in turn leads to yet lower wages in the next period, etc..

These findings are in sharp contrast to the consensus of the existing literature, which is based mostly on either static models or dynamic models that include savings but not human capital. The consensus is summed up nicely in a recent survey by Saez, Slemrod and Giertz (2009), who state: "… optimal progressivity of the tax-transfer system, as well as the optimal size of the public sector, depend (inversely) on the compensated elasticity of labor supply …. With some exceptions, the profession has settled on a value for this elasticity close to zero… In models with only a labor-leisure choice, this implies that the efficiency cost of taxing labor income … is bound to be low as well."[1, 2] The results presented here challenge this consensus by showing that, once we consider human capital, the data appear consistent with higher labor supply elasticities, and larger welfare losses from taxation, than is widely supposed.

To proceed, Section II presents a simple two-period version of the basic life-cycle model of labor supply and savings (MaCurdy (1981)). Section III discusses extension of the model to include human capital. Section IV presents simulations that show how the introduction of human capital alters the behavior of the model, particularly with regard to effects of tax changes. Section V presents welfare calculations. Section VI concludes.

II. A Simple Life-Cycle Model without Human Capital

I start by presenting a simple model of life-cycle labor supply of the type that has strongly influenced economists' thinking on the subject since the pioneering work by MaCurdy (1981). In order to clarify the key points, it is useful to consider only two periods, and to abstract from wage uncertainty. The period utility function is given by:

$$(1) \qquad U_t = \frac{C_t^{1+\eta}}{1+\eta} - \beta \frac{h_t^{1+\gamma}}{1+\gamma} \qquad t = 1,2 \qquad \eta \le 0,\ \gamma \ge 0$$

Here C_t and is consumption in period t and h_t is hours of labor supplied in period t. The present value of lifetime utility is given by:

$$(2) \qquad V = \frac{[w_1 h_1(1-\tau_1)+b]^{1+\eta}}{1+\eta} - \beta \frac{h_1^{1+\gamma}}{1+\gamma} + \rho \left\{ \frac{[w_2 h_2(1-\tau_2)-b(1+r)]^{1+\eta}}{1+\eta} - \beta \frac{h_2^{1+\gamma}}{1+\gamma} \right\}$$

Here w_1 and w_2 are wage rates in periods 1 and 2, while τ_1 and τ_2 are tax rates on earnings.

[1] Inclusion of this quote is not meant a criticism of Saez, Slemrod and Giertz (2009). They are simply making a statement of fact. I quote them only because they state the consensus and its implications so succinctly.

[2] As Ballard and Fullerton (1992) note, if a wage tax is used to finance compensating lump sum transfers (as in the Harberger approach), the welfare cost depends only on the compensated elasticity. But if it is used to finance a public good (that has no impact on labor supply) it is the uncompensated elasticity that matters. Saez (2001) presents optimal tax rate formulas for a Mirrlees (1971) model (with both transfers and government spending on a public good) and shows that, in general, both elasticities matter for optimal tax rates (see, e.g., his equation 9).

Agents can borrow/lend across periods at interest rate r. The quantity b is net borrowing at t=1, while $b(1+r)$ is the net repayment at t=2, and ρ is the discount factor. (I assume there is no exogenous non-labor income. This simplifies the analysis while not changing the results).

In the standard life cycle model, there is no human capital accumulation via returns to work experience. That is, hours of work in period 1 do not affect the wage rate in period 2, and the consumer simply treats the wage path $\{w_1, w_2\}$ as exogenously given. Thus, the first order conditions for his/her optimization problem are simply:

$$(3) \qquad \frac{\partial V}{\partial h_1} = \left[w_1 h_1(1-\tau_1)+b\right]^{\eta} w_1(1-\tau_1) - \beta h_1^{\gamma} = 0$$

$$(4) \qquad \frac{\partial V}{\partial h_2} = \left[w_2 h_2(1-\tau_2)-b(1+r)\right]^{\eta} w_2(1-\tau_2) - \beta h_2^{\gamma} = 0$$

$$(5) \qquad \frac{\partial V}{\partial b} = \left[w_1 h_1(1-\tau_1)+b\right]^{\eta} - \rho\left[w_2 h_2(1-\tau_2)-b(1+r)\right]^{\eta}(1+r) = 0$$

Equation (5) can be simplified to read $\left[C_1\right]^{\eta} \big/ [C_2]^{\eta} = \rho(1+r)$, the classic inter-temporal optimality condition that sets b to equate the ratio of the marginal utilities of consumption across the two periods to $\rho(1+r)$. Utilizing this condition, we can divide (4) by (3) obtain:

$$(6) \qquad \left(\frac{h_2}{h_1}\right)^{\gamma} = \frac{w_2(1-\tau_2)}{w_1(1-\tau_1)} \frac{1}{\rho(1+r)}$$

Taking logs we obtain MaCurdy's equation for hours changes as a function of wage changes:

$$(7) \qquad \ln\left(\frac{h_2}{h_1}\right) = \frac{1}{\gamma}\left\{\ln\frac{w_2}{w_1} + \ln\frac{(1-\tau_2)}{(1-\tau_1)} - \ln\rho(1+r)\right\}$$

From (7) we obtain:

$$(8) \qquad \frac{\partial \ln(h_2/h_1)}{\partial \ln(w_2/w_1)} = \frac{1}{\gamma}$$

Thus, the intertemporal (or Frisch) elasticity of substitution, the rate at which a worker shifts hours of work from period 1 to period 2 as the relative wage increases in period 2, is simply $1/\gamma$. The elasticity with respect to a change in the tax ratio $(1-\tau_2)/(1-\tau_1)$ is identical.

Before solving (3)-(5) to obtain the labor supply functions for h_1 and h_2, it is useful to first look at the static case, which can arise in three ways: (i) there is only one period, or (ii) there is no borrowing and lending across periods, or (iii) people are myopic. Then the utility function in (1) generates the labor supply function:

(9) $$\ln h = \frac{1+\eta}{\gamma-\eta}\ln w - \frac{1}{\gamma-\eta}\ln\beta$$

Thus, $\frac{1+\eta}{\gamma-\eta}$ is the Marshallian (or uncompensated) labor supply elasticity. As η<0, the Frisch elasticity ($1/\gamma$) must exceed the Marshallian. The two approach each other as $\eta \to 0$ (the case of utility linear in consumption, so there are no income effects).

For future reference we will also need the income and compensated substitution effects in the static model. Writing the Slutsky equation in elasticity form we have:

(10) $$\frac{w}{h}\frac{\partial h}{\partial w} = \frac{w}{h}\frac{\partial h}{\partial w}\bigg|_u + \frac{wh}{N}\frac{N}{h}\frac{\partial h}{\partial N}$$

where N represents non-labor income. The two terms on the right side are the compensated (or Hicks) elasticity and the income effect. Using (9), we can easily verify that the income effect (evaluated at N=0) is $\frac{\eta}{\gamma-\eta}$. Thus, the Hicks elasticity is simply $\frac{1}{\gamma-\eta}$. As $\eta < 0$, this is smaller than the Frisch elasticity but larger than the Marshallian.

Now return to the dynamic model with saving. In what follows I assume $\rho(1+r)=1$, so that (5) requires the consumer to equate the marginal utility of consumption in both periods. Furthermore, as the simple model in (1) has time invariant preferences, this is equivalent to equalizing consumption across periods. None of the points I wish to make hinge on this assumption, and it simplifies the analysis considerably.

From (3) we have that:

(11) $$\frac{\beta h_1^{\gamma}}{C_1^{\eta}} = w_1(1-\tau_1)$$

where $C_1=w_1h_1+b$ is consumption in period 1. This is the familiar within-period optimality condition equating the marginal rate of substitution (MRS) between consumption and leisure to the opportunity cost of time, which is just the after tax wage rate. Given $\rho(1+r)=1$, we have $C_1=C_2=C$, and C is just the present value of earnings times the factor $(1+r)/(2+r)$:

(12) $$C = \{w_1(1-\tau_1)h_1(1+r) + w_2(1-\tau_2)h_2\}/(2+r)$$

Now we use equation (6), with $\rho(1+r)=1$, to substitute out for h_2 in (12), obtaining:

$$C = h_1 C^* = h_1\{w_1(1-\tau_1)(1+r) + w_2(1-\tau_2)\left[\frac{w_2(1-\tau_2)}{w_1(1-\tau_1)}\right]^{1/\gamma}\}/(2+r) \tag{13}$$

Here C^* contains all the factors that govern lifetime wealth. We can now write (11) as:

$$\ln h_1 = \frac{1}{\gamma-\eta}\left\{\ln w_1(1-\tau_1) - \ln\beta + \eta\ln C^*\right\} \tag{14}$$

Notice that $\partial\ln h_1/\partial\ln w_1$, holding C^* fixed, is $1/(\gamma-\eta)$, the compensated (or Hicks) elasticity, while $\partial\ln h_1/\partial\ln C^* = \eta/(\gamma-\eta)$ is the income effect.

We are now in a position to consider effects of permanent vs. temporary changes in tax rates. Via some tedious algebra we can obtain the effect of a tax reduction in period 1:

$$\frac{\partial\ln h_1}{\partial\ln(1-\tau_1)} = \left[\frac{1+\eta}{\gamma-\eta}\right] - \left[\frac{\eta}{\gamma-\eta}\frac{1+\gamma}{\gamma}\frac{1}{1+x}\right] \quad \text{where} \quad x \equiv \left[\frac{w_1(1-\tau_1)}{w_2(1-\tau_2)}\right]^{(1+\gamma)/\gamma}(1+r) \tag{15}$$

Note that the first term on the right is the Marshallian elasticity. The second term is positive because $\eta<0$, so the elasticity with respect to a temporary tax change exceeds the Marshallian. If $w_1=w_2$ and $\tau_1=\tau_2$ then the second term in (15) takes on a simple form:

$$\frac{\partial\ln h_1}{\partial\ln(1-\tau_1)} = \left[\frac{1+\eta}{\gamma-\eta}\right] - \left[\frac{\eta}{\gamma-\eta}\frac{1+\gamma}{\gamma}\frac{1}{2+r}\right] \tag{16}$$

If $(1+\gamma)/\gamma(2+r) > 1$ the elasticity with respect to a temporary tax exceeds the Hicks elasticity.[3]

Now consider a <u>permanent</u> tax change. We assume that $\tau_1 = \tau_2 = \tau$, and look at the effect of a change in $(1-\tau)$. With $\tau_1 = \tau_2 = \tau$ equation (13) becomes:

$$C = h_1(1-\tau)C^{**} = h_1(1-\tau)\left\{w_1(1+r) + w_2\left[\frac{w_2}{w_1}\right]^{1/\gamma}\right\}/(2+r) \tag{13'}$$

And we can rewrite (14) as:

$$\ln h_1 = \frac{1}{\gamma-\eta}\left\{\ln w_1(1-\tau) - \ln\beta + \eta\ln(1-\tau)C^{**}\right\} \tag{17}$$

It is then clear that:

$$\frac{\partial\ln h_1}{\partial\ln(1-\tau)} = \frac{1+\eta}{\gamma-\eta} \tag{18}$$

[3] This condition will hold if $0 < \gamma < (1+r)^{-1}$. In a 2 period model where each period corresponds to roughly 20 years of a working life, a plausible value for $1+r$ is about $(1+.03)^{20} \approx 1.806$, or $(1+r)^{-1} \approx 0.554$. So (16) will exceed the Hicks elasticity if the Frisch elasticity $(1/\gamma)$ is at least $(.554)^{-1}=1.8$.

which is just the Marshallian elasticity. So, comparing (16) and (18), we have the well known result that the labor supply elasticity with respect to a temporary tax change is greater than that with respect to a permanent change in the standard life-cycle model.

Anticipating the next section, it is useful to stress that transitory tax changes have larger effects than permanent changes because they have smaller income effects. As $\eta \rightarrow -\infty$, so the income effect becomes stronger, the wedge between the two elasticities, captured by the extra term $-[\eta/(\gamma-\eta)][(1+\gamma)/\gamma][1/(2+r)]$ in equation (16), grows larger.

That transitory changes in taxes or wages should have greater effects on labor supply than permanent changes is firmly entrenched as the conventional wisdom in the profession. As Saez et al (2009) state: "The labor supply literature … developed a dynamic framework to distinguish between responses to temporary changes vs. permanent changes in wage rates.… Because of inter-temporal substitution, and barring adjustment costs, responses to temporary changes will be larger than responses to permanent changes."

In the next two sections I show how introduction of human capital into the standard labor supply model undermines this conventional wisdom, such that permanent tax changes can have larger effects than temporary changes (for a wide range of reasonable parameter values). I begin in Section III.A by introducing human capital into a simple model with no borrowing or lending. This makes the impact of human capital clear. Then in Section III.B I present a model that includes both human capital and borrowing/lending.

III. Incorporating Human Capital in the Life-Cycle Model

III.A. A Life-Cycle Model with Human Capital and Borrowing Constraints

Next I assume that the wage in period 2, rather than being exogenously fixed, is an increasing function of hours of work in period 1. Specifically, I assume that:

(19) $$w_2 = w_1(1+\alpha h_1)$$

where α is the percentage growth in the wage per unit of work. Given a two period model with each period corresponding to 20 years, it is plausible in light of existing estimates that αh_1, the percent growth in the wage over 20 years, is on the order of 1/3 to 1/2.[4] Note that we could approximate (19) by $\ln W_2 \approx \ln W_1 + \alpha h_1$. Thus, it is similar to a conventional log wage function, but without the usual quadratic in hours. I introduce that in the simulation section, but for purposes of obtaining analytical results (19) is much more convenient.

[4] For instance, using the PSID, Geweke and Keane (2000) estimate that for men with a high school degree, average earnings growth from age 25 to 45 is 33% (most of which is due to wage growth). For men with a college degree the estimate is 52%. They also find that earnings growth essentially ceases after about age 45.

In a model with human capital but no borrowing/lending, equation (2) is replaced by:

$$(20)\qquad V=\frac{[w_1h_1(1-\tau_1)]^{1+\eta}}{1+\eta}-\beta\frac{h_1^{1+\gamma}}{1+\gamma}+\rho\left\{\frac{[w_1(1+\alpha h_1)h_2(1-\tau_2)]^{1+\eta}}{1+\eta}-\beta\frac{h_2^{1+\gamma}}{1+\gamma}\right\}$$

and the first order conditions (3)-(5) are replaced by:

$$(21)\qquad \frac{\partial V}{\partial h_1}=\left[w_1h_1(1-\tau_1)\right]^{\eta}w_1(1-\tau_1)-\beta h_1^{\gamma}+\rho\left[w_1(1+\alpha h_1)h_2(1-\tau_2)\right]^{\eta}w_1\alpha h_2(1-\tau_2)=0$$

$$(22)\qquad \frac{\partial V}{\partial h_2}=\left[w_1(1+\alpha h_1)h_2(1-\tau_2)\right]^{\eta}w_1(1+\alpha h_1)(1-\tau_2)-\beta h_2^{\gamma}=0$$

It is useful to rewrite (21) in the form:

$$(23)\qquad \frac{\beta h_1^{\gamma}}{C_1^{\eta}}=w_1(1-\tau_1)+\rho\left[\frac{C_2^{\eta}}{C_1^{\eta}}\right]\left\{w_1\alpha h_2(1-\tau_2)\right\}$$

where $C_1=w_1h_1(1-\tau_1)$ and $C_2=w_1(1+\alpha h_1)h_2(1-\tau_2)$ are consumption at t=1 and 2. The main point of this paper can be seen simply by comparing (11) and (23). Each equates the MRS to the opportunity cost of time. But in the standard life-cycle model (11) this is just the after tax wage rate $w_1(1-\tau_1)$. The human capital model adds the term $\rho[C_2^{\eta}/C_1^{\eta}]\{w_1\alpha h_2(1-\tau_2)\}$, which is the human capital investment component of the opportunity cost of time.

To understand this extra term, note that $dw_2/dw_1 = w_1\alpha$ is the increment to the time t=2 wage for each additional unit of hours worked at time t=1. This is multiplied by $h_2(1-\tau_2)$ to obtain the increment to after-tax *earnings*. It is also discounted back to t=1, and multiplied by the ratio of marginal utilities of consumption in each period (recall there is no borrowing, so these may differ).

Now, a key point is that a temporary tax change in period 1 affects only $(1-\tau_1)$, and hence it only affects the first component of the opportunity cost of time (the current wage rate). In contrast, a permanent tax change affects <u>both</u> $(1-\tau_1)$ and $(1-\tau_2)$, thus shifting both components of the opportunity cost of time. As we'll see, this means that in the model with human capital and no borrowing/lending, a permanent tax change must have a larger impact on time t labor supply than a temporary tax change (that is only in effect at time t).

To solve the model for h_1 we use (22) to solve for h_2 and substitute this into (21). This gives the following implicit function for h_1:

$$(24)\quad \beta h_1^{\gamma}=\left[w_1(1-\tau_1)\right]^{1+\eta}h_1^{\eta}+\rho\alpha\beta^{-(1+\eta)/(\gamma-\eta)}\left[w_1(1-\tau_2)\right]^{1+\eta+(1+\eta)^2/(\gamma-\eta)}(1+\alpha h_1)^{(1+2\eta+\gamma\eta)/(\gamma-\eta)}$$

As it is not possible to isolate h_1 in (24), we must totally differentiate to obtain the elasticity of hours in period 1 with respect to $(1-\tau_1)$:

$$(25)\quad \frac{\partial \ln h_1}{\partial \ln(1-\tau_1)} = \frac{(1+\eta)[w_1(1-\tau_1)]^{1+\eta} h_1^{\eta}}{\gamma\beta h_1^{\gamma} - \eta[w_1(1-\tau_1)]^{1+\eta} h_1^{\eta} - \rho\alpha^2[w_1(1-\tau_2)]^{\Gamma_0}\beta^{-\Gamma_3}\Gamma_1(1+\alpha h_1)^{\Gamma_2}}$$

where $\Gamma_0 \equiv (1+\eta)(1+\gamma)/(\gamma-\eta)$, $\Gamma_1 \equiv (1+2\eta+\gamma\eta)/(\gamma-\eta)$, $\Gamma_2 \equiv (1+3\eta+\gamma\eta-\gamma)/(\gamma-\eta)$, $\Gamma_3 \equiv (1+\eta)/(\gamma-\eta)$. Obviously this expression simplifies to the Marshallian elasticity $(1+\eta)/(\gamma-\eta)$ if α=0 (i.e., the case of no human capital accumulation), because the third term in the denominator vanishes.

This third term results because changes in hours at t=1 alter the wage at t=2. Thus, if a t=1 tax cut increases work hours at t=1, it will increase the wage at t=2 (substitution effect). But this also increases income at t=2 (income effect). Thus, the sign of the third term in the denominator of (25) is ambiguous. It is determined by the sign of $\Gamma_1 = (1+2\eta+\gamma\eta)$.

Note that if η = -1 (that case of log (C) utility) income effects are sufficiently strong to balance substitution effects, rendering the Marshallian elasticity zero. Then $\Gamma_1 = -1-\gamma < 0$, so the third term *increases* the denominator. Of course this is irrelevant as the numerator is zero, but for somewhat larger values of η we see that the human capital effect will render the elasticity with respect to temporary tax/wage changes smaller than the Marshallian![5]

Indeed, for any value of η in the -1 to -.5 range the elasticity in (25) must be less than the Marshallian. Only if $-.5 < \eta < 0$ is it possible to find values of γ small enough that the substitution effect dominates and (25) is larger than the Marshallian elasticity.[6]

Now consider the effect of a permanent tax increase. To simplify the analysis I will assume that $\tau_1 = \tau_2 = \tau$. This modifies (24) so that τ replaces that τ_1 and τ_2. As a result, when we totally differentiate (24) with respect to $(1-\tau)$ we get:

$$(26)\quad \frac{\partial \ln h_1}{\partial \ln(1-\tau)} = \frac{(1+\eta)[w_1(1-\tau)]^{1+\eta} h_1^{\eta} + \rho\alpha\frac{(1+\eta)(1+\gamma)}{\gamma-\eta}[w_1(1-\tau)]^{\frac{(1+\eta)(1+\gamma)}{\gamma-\eta}}(1+\alpha h_1)^{\Gamma_1}\beta^{-\Gamma_3}}{\gamma\beta h_1^{\gamma} - \eta[w_1(1-\tau)]^{1+\eta} h_1^{\eta} - \rho\alpha^2[w_1(1-\tau)]^{\Gamma_0}\Gamma_1(1+\alpha h_1)^{\Gamma_2}\beta^{-\Gamma_3}}$$

Note that the denominators of (25) and (26) are identical. The only difference is an additional term in the numerator that captures human capital effects. Specifically, it captures the fact that a tax cut at t=2, by increasing the fraction of his/her earnings a worker can keep at t=2, increases the return to human capital investment (and so the opportunity cost of time) at t=1.

[5] At the other extreme is the η = 0 case (utility linear in C, no income effects). Then Γ_1 = 1, and the third term reduces the denominator. Thus, the elasticity with respect to temporary tax changes will *exceed* the Marshallian.
[6] Strikingly, the change occurs radically. For η slightly larger than -.5 a nearly infinite Frisch elasticity of substitution ($1/\gamma$) is necessary for the substitution effect to dominate. But for η = -0.40 all we need is $1/\gamma > 2$. These are the sort of values typically used in calibrating real business cycle models (see Prescott (1986, 2006)).

The sign of the new second term in the numerator of (26) depends on the term $(1+\eta)(1+\gamma)/(\gamma-\eta)$. Note that $(1+\gamma)$ must be positive, as $\gamma>0$. Thus, the sign depends on that of $(1+\eta)/(\gamma-\eta)$, the Marshallian elasticity itself. As long as the Marshallian elasticity is positive (i.e., the income effect does not dominate), the labor supply elasticity with respect to a permanent tax change (26) will exceed that with respect to a temporary tax change (25).

In summary, in the model with borrowing but no human capital, the income effect tends to make the response to a temporary tax change greater than that to a permanent tax change. In a model with human capital and no borrowing, the human capital effect leads to the opposite outcome. In the next Section I present a model with both human capital and borrowing/saving. Not surprisingly, we will find that whether permanent or temporary tax cuts have a larger effect depends on the relative strength of the human capital and income effects.

III.B. A Life-Cycle Model with both Human Capital and Saving/Borrowing

In a model with both human capital and borrowing/saving equation (2) is replaced by:

$$V=\frac{[w_1h_1(1-\tau_1)+b]^{1+\eta}}{1+\eta}-\beta\frac{h_1^{1+\gamma}}{1+\gamma}+\rho\left\{\frac{[w_1(1+\alpha h_1)h_2(1-\tau_2)-b(1+r)]^{1+\eta}}{1+\eta}-\beta\frac{h_2^{1+\gamma}}{1+\gamma}\right\} \tag{27}$$

and the first order conditions for the problem are:

$$\begin{aligned}\frac{\partial V}{\partial h_1}&=\left[w_1h_1(1-\tau_1)+b\right]^{\eta}w_1(1-\tau_1)-\beta h_1^{\gamma}\\&\quad+\rho\left[w_1(1+\alpha h_1)h_2(1-\tau_2)-b(1+r)\right]^{\eta}w_1\alpha h_2(1-\tau_2)=0\end{aligned} \tag{28}$$

$$\frac{\partial V}{\partial h_2}=\left[w_1(1+\alpha h_1)h_2(1-\tau_2)-b(1+r)\right]^{\eta}w_1(1+\alpha h_1)(1-\tau_2)-\beta h_2^{\gamma}=0 \tag{29}$$

$$\frac{\partial V}{\partial b}=\left[w_1h_1(1-\tau_1)+b\right]^{\eta}-\rho\left[w_1(1+\alpha h_1)h_2(1-\tau_2)-b(1+r)\right]^{\eta}(1+r)=0 \tag{30}$$

As before, I assume $\rho(1+r)=1$ to simplify the analysis. In that case (28) can be rewritten:

$$\frac{\beta h_1^{\gamma}}{C^{\eta}}=w_1(1-\tau_1)+\rho\alpha w_1h_2(1-\tau_2) \tag{31}$$

It is useful to compare this to (11), the MRS condition for the model without human capital. Here the opportunity cost of time is augmented by the term $\rho\alpha w_1h_2(1-\tau_2)$, which captures the effect of an hour of work at $t=1$ on the present value of earnings at $t=2$.

Now, continuing to assume $\rho(1+r)=1$, we can divide (29) by (28) to obtain:

$$(32)\qquad \left(\frac{h_2}{h_1}\right)^{\gamma} = \frac{w_1(1+\alpha h_1)(1-\tau_2)}{w_1(1-\tau_1)+\rho\alpha w_1 h_2(1-\tau_2)} = \frac{w_2(1-\tau_2)}{w_1(1-\tau_1)+\rho\alpha w_1 h_2(1-\tau_2)}$$

Taking logs we obtain:

$$(33)\qquad \ln\left(\frac{h_2}{h_1}\right) = \frac{1}{\gamma}\ln\left[\frac{w_2(1-\tau_2)}{w_1(1-\tau_1)+\rho\alpha w_1 h_2(1-\tau_2)}\right]$$

This equation illustrates clearly why the conventional procedure of regressing hours growth on wage growth leads to underestimates of the Frisch elasticity ($1/\gamma$), and overestimates of the key utility function parameter γ. The effective wage rate at t=1 is understated by failure to account for the term $\rho\alpha w_1 h_2(1\text{-}\tau_2)$ that appears in the denominator.

We can get a better sense of the magnitude of the problem if we simplify by assuming $\tau_1 = \tau_2 = \tau$. Then we can rewrite (33) as:

$$(34)\qquad \ln\left(\frac{h_2}{h_1}\right) = \frac{1}{\gamma}\left\{\ln\frac{w_2}{w_1}\frac{1}{(1+\rho\alpha h_2)}\right\}$$

If we solve this for $1/\gamma$ we obtain:

$$(35)\qquad \frac{1}{\gamma} = \ln\left(\frac{h_2}{h_1}\right) \div \ln\left(\frac{w_2}{w_1(1+\rho\alpha h_2)}\right) = \ln\left(\frac{h_2}{h_1}\right) \div \left[\ln\left(\frac{w_2}{w_1}\right) - \ln(1+\rho\alpha h_2)\right]$$

Thus, wage growth from t=1 to t=2 must be adjusted downward by a factor of roughly $\rho\alpha h_2$ percent in order to correct for the missing human capital term. This adjustment gives a valid estimate of the growth of the opportunity cost of time (OCT).

As I noted earlier, a reasonable estimate of αh_1 is about 33%. A reasonable figure for hours growth over the first 20 years of the working life is roughly 20% (see, e.g., Imai and Keane (2004) or the descriptive regressions in Pencavel (1986)). So assume that h_2 is 20% greater than h_1. Then αh_2 is roughly 40%. Let $\rho = 1/(1.03)^{20} = 0.554$. Then we obtain $\rho\alpha h_2$ = 22%. Thus, while wage growth is 33%, the growth in the OCT is only 33% –22% = 11%.

Hence, if we used observed wage growth to calculate the Frisch elasticity we would obtain $(1/\gamma) = ln(1.20)/ln(1.33) \approx .64$. But the correct value based on equation (35) is $(1/\gamma)= ln(1.20)/ln[1.33/1.22] \approx 2.1$. Thus, for reasonable parameter values, the downward bias in estimates of the Frisch elasticity due to ignoring human capital will tend to be substantial.[7]

[7] The bias here is about a factor of 3 for a range of plausible growth rates in hours. For example, if hours grow by only 10% instead of 20%, the conventional method gives $(1/\gamma) \approx .33$ while the correct calculation is 0.93.

Now consider the impact of permanent vs. temporary wage/tax changes in this model. First, solve (29) for h_2 to obtain:

$$(36) \qquad h_2 = \beta^{-1/\gamma}\left[w_1(1+\alpha h_1)(1-\tau_2)\right]^{1/\gamma} C^{\eta/\gamma}$$

Substituting this into (28) we obtain:

$$(37) \qquad \beta h_1^{\gamma} = w_1(1-\tau_1)C^{\eta} + \rho\alpha\beta^{-1/\gamma} w_1^{(1+\gamma)/\gamma}(1+\alpha h_1)^{1/\gamma}(1-\tau_2)^{(1+\gamma)/\gamma} C^{\eta(1+\gamma)/\gamma}$$

Unfortunately (37) involves C. Recall that C is given by equation (12). In the model without human capital we substituted for h_2 in (12) using the intertemporal optimization condition (6), obtaining an equation for C in terms of h_1 only (equation (13)). We then substituted (13) into the first order condition for h_1 to obtain an explicit function for h_1 (equation (14)) that was fairly easy to differentiate. Things are much more difficult here, because the intertemporal optimization condition (32) cannot be solved explicitly for h_2 in terms of h_1. Instead, we use (36) to substitute for h_2. However, this only delivers an implicit function for C:

$$(38) \qquad C = \{w_1(1-\tau_1)h_1(1+r) + \left[w_1(1-\tau_2)(1-\tau_2)\right]^{(1+\gamma)/\gamma} \beta^{-1/\gamma} C^{\eta/\gamma}\}/(2+r)$$

We are now in a position to calculate labor supply elasticities of h_1 with respect to temporary tax changes, using the two equation system (37)-(38). First, we implicitly differentiate (38) to obtain an expression for $dC/d(1-\tau_1)$ that involves $dh_1/d(1-\tau_1)$. Then we implicitly differentiate (37) to obtain an expression for $dh_1/d(1-\tau_1)$ that involves $dC/d(1-\tau_1)$. Finally, we substitute the former expression into the latter, group terms, and convert to elasticity form to obtain:

$$\frac{\partial \ln h_1}{\partial \ln(1-\tau_1)} =$$

$$(39) \qquad \frac{A\left[D + EC^{\frac{\eta}{\gamma}}\left(\frac{\gamma-\eta}{\gamma}\right)\right] + \eta\left[A + B\frac{1+\gamma}{\gamma}\right]D}{\gamma\left[A + B\left(1 + \frac{\alpha h_1/\gamma^2}{1+\alpha h_1}\right)\right]\left[D + EC^{\frac{\eta}{\gamma}}\left(\frac{\gamma-\eta}{\gamma}\right)\right] - \eta\left[A + B\frac{1+\gamma}{\gamma}\right]\left[D + EC^{\frac{\eta}{\gamma}}\left(\frac{1+\gamma}{\gamma}\right)\right]\frac{\alpha h_1}{1+\alpha h_1}}$$

where:

$$A \equiv w_1(1-\tau_1)C^{\eta} \qquad B \equiv \rho\alpha\beta^{-1/\gamma}\left[w_1(1-\tau_2)\right]^{(1+\gamma)/\gamma}(1+\alpha h_1)^{1/\gamma} C^{\eta(1+\gamma)/\gamma}$$

$$D \equiv w_1 h_1(1-\tau_1)(1+r) \qquad E \equiv \beta^{-1/\gamma}\left[w_1(1-\tau_2)\right]^{(1+\gamma)/\gamma}(1+\alpha h_1)^{(1+\gamma)/\gamma}$$

The term B is the human capital affect that arises because an increase in h_1 increases income at t+2 (holding h_2 fixed). It is exactly the second term on the right hand side of (37). The term

$EC^{\eta/\gamma}(\gamma-\eta)/\gamma$ is the usual income effect of the higher after-tax wage in period t=1. The term $EC^{\eta/\gamma}[(1+\gamma)/\gamma][\alpha h_1/(1+\alpha h_1)]$ is a special income effect that arises because an increase in h_1 increases the wage rate at t=2.

It can be verified via cumbersome algebra that (39) reduces to (15) – the elasticity of hours with respect to a temporary tax cut in the standard life-cycle model without human capital – if we set $\alpha = 0$. The simulations in Section IV.C will reveal that (39) is strongly *decreasing* in α (for given η and γ). This is intuitive: as human capital becomes more important, a temporary tax hits a smaller part of the opportunity cost of time.

We now look at the effect of a permanent tax increase by setting $\tau_1 = \tau_2 = \tau$ in (37) and (38), and following the same solution procedure as above. This leads to the result:

$$\text{(40)}\quad \frac{\partial \ln h_1}{\partial \ln(1-\tau)} = \frac{\left[A+\left\{B\frac{1+\gamma}{\gamma}\right\}\right]\left[D+EC^{\frac{\eta}{\gamma}}\left(\frac{\gamma-\eta}{\gamma}\right)\right]+\eta\left[A+B\frac{1+\gamma}{\gamma}\right]\left[D+\left\{EC^{\frac{\eta}{\gamma}}\left(\frac{1+\gamma}{\gamma}\right)\right\}\right]}{\gamma\left[A+B\left(1+\frac{\alpha h_1/\gamma^2}{1+\alpha h_1}\right)\right]\left[D+EC^{\frac{\eta}{\gamma}}\left(\frac{\gamma-\eta}{\gamma}\right)\right]-\eta\left[A+B\frac{1+\gamma}{\gamma}\right]\left[D+EC^{\frac{\eta}{\gamma}}\left(\frac{1+\gamma}{\gamma}\right)\right]\frac{\alpha h_1}{1+\alpha h_1}}$$

This expression reduces to the Marshallian elasticity (18) if we set $\alpha = 0$. Compared to equation (39), equation (40) has two new terms, both of which appear in curly brackets in the numerator. The first is $\left\{B\frac{1+\gamma}{\gamma}\right\}$ which is an additional human capital effect. It captures that a lower tax rate in period t=2 provides an additional incentive to accumulate human capital at t=1. The second is $\left\{EC^{\frac{\eta}{\gamma}}\left(\frac{1+\gamma}{\gamma}\right)\right\}$ which captures an additional income effect (i.e., the lower tax in period 2 leads to higher lifetime income holding labor supply fixed).

Whether a permanent or a temporary tax change has a larger effect on labor supply depends on which of these two effects dominates. A permanent tax change will have the larger effect if the following condition holds:

$$\left\{B\frac{1+\gamma}{\gamma}\right\}\left[D+EC^{\frac{\eta}{\gamma}}\left(\frac{\gamma-\eta}{\gamma}\right)\right] > \eta\left[A+B\frac{1+\gamma}{\gamma}\right]\left\{EC^{\frac{\eta}{\gamma}}\left(\frac{1+\gamma}{\gamma}\right)\right\}$$

Some tedious algebra reveals that this condition is equivalent to a bound on the parameter α, which governs how work experience in period 1 affects the wage in period 2. The bound is:

$$(41) \qquad \alpha > \frac{-\eta\left(\beta h_1^{\gamma}\right)C^{-\eta}}{\rho(2+r)C + \eta h_1\left(\beta_1 h_1^{\gamma}\right)C^{-\eta}} > 0$$

Note that the numerator of (41) is obviously positive, as η<0, and the next two terms are the marginal utilities of leisure and consumption respectively, which are both positive. But the sign of the denominator appears ambiguous, as the first term is positive while the second is negative. However, we can show the denominator is positive as follows:

Utilizing the fact that $\rho(1+r)=1$, so that $\rho(2+r)=(1+\rho)$, we see that, in order for the denominator to be positive, we must have:

$$(42) \qquad C > \frac{-\eta}{1+\rho} h_1 \frac{\left(\beta h_1^{\gamma}\right)}{C^{\eta}}$$

Now recall from equation (31) that $\frac{\beta h_1^{\gamma}}{C^{\eta}} = w_1(1-\tau) + \rho\alpha w_1 h_2(1-\tau)$. Thus we have that:

$$(43) \qquad C > \frac{-\eta}{1+\rho} h_1\left[w_1(1-\tau) + \rho\alpha w_1 h_2(1-\tau)\right] = \frac{-\eta}{1+\rho}\left[w_1 h_1(1-\tau) + \frac{1}{1+r}(\alpha h_1) w_1 h_2(1-\tau)\right]$$

where in the second term on the right we have substituted $\rho(1+r)=1$. Of course we have that the present value of lifetime consumption equals that of lifetime income:

$$(44) \qquad \frac{2+r}{1+r} C = \left\{ w_1(1-\tau)h_1 + \frac{1}{1+r} w_1(1+\alpha h_2)(1-\tau)h_2 \right\}$$

Thus, the term in the square brackets in (43) is $\frac{2+r}{1+r}C - \frac{1}{1+r} w_1 h_2(1-\tau)$, which is lifetime income minus a part of period 2 earnings. So we can rewrite (43) as:

$$(45) \qquad C > \frac{-\eta}{1+1/(1+r)}\left[\frac{2+r}{1+r}C - \frac{1}{1+r} w_1 h_2(1-\tau)\right] = -\eta\left[C - \frac{1}{2+r} w_1 h_2(1-\tau)\right]$$

As long as $\eta > -1$ (i.e., substitution effects dominate income effects) this inequality must hold. The right hand side takes on its greatest value when $\eta = -1$, and then (45) just says that C is greater than a fraction of C.

Thus, equation (41) gives a positive lower bound that the human capital effect α must exceed in order for permanent tax changes to have a larger effect than temporary tax changes in the model with human capital and saving. This lower bound is greater the stronger is the income effect. To see this, note that as η approaches -1 (i.e., income effects become stronger), the numerator of (41) increases while the denominator decreases. It is also obvious that when utility is linear in consumption (no income effects) (41) reduces to $\alpha > 0$. In the simulations of Section IV.C we will see clearly how the lower bound for α increases in ($-\eta$).

If we make the approximation that $\alpha^2 \approx 0$, which is reasonable given that, as noted earlier, a plausible value for αh_1 is about .33, we can obtain the more intuitive expression:

$$(46) \qquad \alpha > -\left[\frac{\eta}{1+\eta}\right]\frac{w_1(1-\tau)}{w_1(1-\tau)h_1 + \frac{1}{1+r}w_1(1-\tau)h_2}$$

This makes clear that the bound for α gets higher as income effects grow stronger.

IV. Simulations of the Model

IV.A. Model Calibration

Given that we have a two period model, we can think of each period as 20 years of a 40 year working life (e.g., 25 to 44 and 45 to 64). I assume a real annual interest rate of 3%. Note that $1/(1+.03)^{20} = 0.554$. This implies a 20 year interest rate of $r = .806$. Thus, I assume the discount factor is $\rho = 1/(1+r) = 0.554$. I set the initial tax rates $\tau_1 = \tau_2 = .40$.

I will examine how the model behaves for a range of values of the key utility function parameters η and γ. Two studies that estimate life-cycle models that include both savings and human capital investment, and that also assume CRRA utility, are Keane and Wolpin (2001) and Imai and Keane (2004). Keane-Wolpin estimate that $\eta \approx -.5$ while Imai-Keane estimate that $\eta \approx -.75$.[8] Goeree, Holt and Palfrey (2003) present extensive experimental evidence, as well as evidence from field auction data, in favor of $\eta \approx -.4$ to $-.5$. Bajari and Hortacsu (2005) estimate $\eta \approx -.75$ from auction data. Thus, I will consider values of -.25, -.50 and -.75 for η, with most of the emphasis on the -.50 and -.75 cases.[9]

Of course, the value of γ has been the subject of great controversy. As discussed by Imai and Keane (2004), most estimates of the intertemporal elasticity of substitution ($1/\gamma$) in

[8] Shaw (1989) was the first to estimate a dynamic model that included both human capital and saving. But she used a translog utility function, so the estimates are not very useful for calibrating (1). Van der Klauuw and Wolpin (2008) is the only other study I am aware of that could be used to calibrate η. They obtain $\eta \approx -1.60$, which is much lower than the values I assume. This may be because they look at people at or near retirement, while Keane and Wolpin (2001) and Imai and Keane (2004) use data on young men.

[9] The value of $\eta \approx -.50$ obtained by Keane and Wolpin (2001) implies less curvature in consumption (i.e., higher willingness to substitute inter-temporally) than much of the prior literature. But their model includes liquidity constraints that limit the maximum amount of uncollateralized borrowing. Keane and Wolpin (2001, p. 1078) argue that the failure of prior work to accommodate liquidity constraints will have led to downward bias in η. Specifically, in the absence of constraints on uncollateralized borrowing, one needs a large negative η to rationalize why youth with steep age-earnings profiles don't borrow heavily in anticipation of higher earnings in later life. Notably, their model fits the empirical distribution of assets for young men quite well.

the literature are quite small. Two rare exceptions are French (2005), who obtains 1.33 for 60 year olds in the PSID, and Heckman and MaCurdy (1982) who obtain 2.3 for married women in the PSID. But most estimates of $(1/\gamma)$ are in the 0 to .50 range. At the same time, many macro economists have argued that values of $(1/\gamma)$ of 2 or greater are needed to explain business cycle fluctuations using standard models (see Prescott (1986, 2006)).

Imai and Keane (2004) is a major exception to the prior literature, as they estimate that $\gamma \approx .25$. They were first to estimate the intertemporal elasticity in a model that includes human capital, and they argue, for reasons similar to those discussed here, that failure to do so would have led prior work to severely underestimate $(1/\gamma)$.[10] Keane and Rogerson (2010) discuss a variety of other mechanisms that may have caused past work to understate $(1/\gamma)$.

Given the controversy over γ, I will examine the behavior of the model for a wide range of values. Specifically, I look at $\gamma = \{0, 0.25, 0.50, 1, 2, 4\}$. But I will often focus on $\gamma = 0.50$. I consider this value plausible in light of Imai and Keane (2004) and results in Section III.B that prior estimates (ignoring human capital) are likely to be severely biased upward.

Next consider β. This is just a scaling parameter that depends on the units for hours and consumption, and has no bearing on the substantive behavior of the model. Thus, in each simulation, I set β so optimal hours would be 100 in a static model (α=0). The initial wage w_1 is also set to 100. These values were chosen purely for ease of interpreting the results.

Finally, consider the wage function. In contrast to the simple function assumed for analytical convenience in Sections II-III, here I assume the more realistic function:

$$(47) \qquad w_2 = w_1 \exp(\alpha h_1 - \kappa\left(h_1^2/100\right) - \delta)$$

This corresponds more closely to a conventional Mincer log earnings specification:

$$\ln w_2 = \ln w_1 + \alpha h_1 - \phi\left(h_1^2/100\right) - \delta$$

where w_1 is the initial skill endowment, and there is a quadratic in hours (experience). But I also include a depreciation term δ, which causes earnings to fall if the person does not work sufficient hours in period one (see Keane and Wolpin (1997)).

Given that β is chosen so hours will be close to 100 in period one,[11] let's think of h=100 as corresponding (roughly) to full-time work and h=50 as part-time work. I decided to

[10] It is notable that French (2005), who also obtained a high value of $(1/\gamma)$, did so for 60 year olds. As both Shaw (1989) and Imai and Keane (2004) note, human capital investment is less important for people late in the life-cycle. For them, the wage is close to the opportunity cost of time, so the bias that results from ignoring human capital will be much less severe.

[11] Actually, agents will typically supply somewhat more than 100 units of labor when α>0, due to the incentives to acquire human capital in the dynamic model.

calibrate the model so that (i) the person must work at least part-time at t=1 for the wage not to fall at t=2, and (ii) the return to additional work falls to zero at 200 units of work. Given these constraints, the wage function reduces to:

$$(48) \qquad w_2 = w_1 \exp\left(\alpha h_1 - \frac{\alpha}{4}\left(h_1^2/100\right) - \frac{175}{4}\alpha \right)$$

Thus, the single parameter α determines how work experience maps into human capital. I will calibrate α so that it is roughly consistent with the 33% to 50% wage growth for men from age 25 to 45 discussed earlier. As we'll see below, this requires α in the .008 to .010 range. However, I will also consider a range of other α values, to learn about how the behavior of the model changes when human capital is more or less important.

IV.B. Baseline Simulation

Table 1 reports baseline simulations of models with η = -.75, η = -.50 and η = -.25. It reports units of work in periods 1 and 2, as well as the wage rate in period 2. As the wage at t=1 is normalized to 100, one can read the amount of wage growth directly from the table. Results are reported for values of α ranging from 0 to .012. Recall that β is normalized in all models so h = 100 in the static case. Thus, the overall level of hours rises as we move down the rows of the table and the return to human capital investment increases.

I first look at how the models capture wage growth. Consider models with η = -.50. With α = .007, wage growth from t=1 to t=2 ranges from 26% when γ = 4 to 37% when γ = 0, including 32% at my preferred value of γ = .50. These are plausible values, but a bit low compared to the 33% to 52% values that Geweke and Keane (2000) estimated from the PSID. At α = .008, wage growth ranges from 31% when γ = 4 to 46% when γ = 0, including 39% at my preferred value of γ = .50. These are solidly in the range of values that Geweke-Keane obtained from the PSID. At α = .010, wage growth ranges from 41% when γ = 4 to 66% when γ = 0, including 54% at my preferred value of γ = .50. This brings us to the upper end of the range of values that Geweke-Keane estimated. Based on these simulation results, I conclude that values of α in the .008 to .010 range are reasonable (at least for η = -.50).

A notable feature of Table 1 is that the rate of wage growth is not very sensitive to the setting of η, although it gets slightly greater as $\eta \rightarrow 0$ (i.e., income effects become weaker). For instance, compare η = -.75 vs. -.50 vs. -.25, and look only at γ = .50. For α = .008 we get wage growth of 35%, 39% and 44%, respectively. Thus, α = .008 is a reasonable setting regardless of the value of η. But for η = -.25, γ = .50, α = .010, we get wage growth of 64%, which is a bit high. So for η = -.25 the plausible range for α is more like .007 to .009.

Next consider what the models imply about hours of work at t=1 and t=2. McGrattan and Rogerson (1998) document that in 1990 the typical married male in the 25-44 age range worked 40 hours per week, while in the 45-64 age range he worked 34 hours per week, a 15% decline. None of the models in Table 1 match this pattern, as all imply that hours increase, albeit modestly, from t=1 to t=2. For instance, the model with α = .008, η = -.50, and γ = .50 gives an increase in work units from 121 to 133, or 10%.

There are two possible reactions to this. First, one could view it as a failure of the model. Alternatively, one could accept that this is a simple stylized model designed to clarify some issues about human capital, taxes and labor supply. McGrattan and Rogerson (1998) also document that the hours decline in the 45-64 age range is almost entirely due to a sharp decline at ages 55 to 64. To capture this decline one would obviously need to account for factors that motivate retirement, such as declining tastes for work, health issues, pensions and Social Security benefits, etc.. The simple model here abstracts from these issues entirely.

More relevant for our purposes is that hours do follow a hump shape over the life cycle; as Imai and Keane (2004) note, for men in the NLSY79 average annual hours rise from 2042 at age 25 to 2294 at age 35, a 12% increase. They then plateau before beginning to fall with retirement. Descriptive hours regressions in Pencavel (1986) show a similar pattern.[12] Thus, the model with α = .008, η = -.50, and γ = .50, which generates 10% hours growth, can be interpreted as successfully capturing the modest growth in hours that occurs over the life cycle prior to the onset of the forces that drive retirement (which I do not model).

Using this "modest" hours growth criterion (i.e., the model should not generate hours growth more than 10%-15%), we see that some specifications in Table 1 can be ruled out. In particular, if we look at α in the plausible .007 to .010 range, we see that models with γ = 0 generate implausibly large increases in labor supply (e.g., 69% in the α = .008, η = -.50 case). If η = -.25 then the γ = .25 models can be ruled out as well.

Table 2 reports the same sort of simulations, but for the model of Section III.A, where no borrowing is allowed. One striking finding here is that levels of t=1 hours, and hence t=2 wages, are almost identical to those in the model with borrowing.[13] The other notable finding is that hours growth is actually negative in the models with η = -.75 or -.50. For example, in the model with η = -.50, α = .008 and γ = .50, hours decline by 5% from period 1 to period 2. In models with η = -.25 hours increase very modestly, unless γ is very small.

[12] For the entire 1956-65 birth cohort (that used in Imai-Keane), McGrattan and Rogerson (1998) use US Census data to project typical weekly hours of 30.8, 34.5, 34.2 and 18.6 at ages 25-34, 35-44, 45-54 and 55-64 (see their Table 10). Notice the 12% hours growth between the first two age intervals, and the sharp drop after age 55.

[13] For example, in the model with η = -.50 and α = .008 (and no borrowing), wage growth ranges from 31% when γ = 4 to 48% when γ = 0. In the model *with borrowing* the range is essentially identical (31% to 46%).

There are two factors that drive negative hours growth in the models with borrowing constraints. The first is that the part of the opportunity cost of time that arises from the return to human capital investment (i.e., the second term in equation (23)) vanishes at t=2, as there is no future. This drives down the OCT as people age, which tends to reduce hours. But this factor was also present in models with borrowing, so it alone cannot explain why hours fall.

The second reason for the fall in hours is the income effect that arises because wages are higher in period two than in period one. The inability to smooth consumption over time means this income effect is much stronger at t=2 in the model with borrowing constraints. This income effect is sufficient to cause hours to fall.

In summary, the results of this section suggest that human capital effects in the α = .008 to .010 range are plausible for the η = -.75 to -.50 models, and that α in the .007 to .009 range is plausible for the η = -.25 model. The value γ = 0 does not appear plausible in the η = -.75 to -.50 models, while γ = 0 or .25 both appear implausible in the η = -.25 model.

IV.C. Simulation of Effects of Tax Rate Changes

In this Section I use the simple models of Section III to simulate effects of temporary and permanent tax changes. Tables 3-5 present the results for the models with unconstrained borrowing/lending. Table 3 presents results for η = -.75 (the Imai-Keane estimate). The left panel shows elasticities with respect to temporary tax changes at t=1. The right panel shows elasticities with respect to permanent tax changes (i.e., changes that apply in both periods). The first three rows show results for α = 0, the case of no human capital accumulation.

Consider the case with γ = .50, which is a commonly assumed value in calibrating real business cycle models. Then, the Marshallian elasticity is $(1+\eta)/(\gamma-\eta) = (1-.75)/(0.50+0.75) =$ 0.20. The compensated (or Hicks) elasticity is $1/(\gamma-\eta) = 1/(0.50+0.75) = 0.80$, and the Frisch elasticity is $1/\gamma = 2$. As we see in the first three rows of Table 3, these *theoretical* elasticities correspond almost exactly to the *simulated* values of the uncompensated and compensated elasticities for <u>permanent</u> tax changes, and to the Frisch elasticity for hours growth with respect to wage growth. The latter is calculated as the percentage increase in labor supply from t=1 to t=2 (-2%) divided by the after-tax wage increase (-1%). Simulated values for these elasticities reported in the first three rows of Table 3 differ slightly from the theoretical values only because we are taking finite difference derivatives (i.e., we increase $(1-\tau)$ by 1%).

A key point is that elasticities with respect to <u>transitory</u> wage/tax changes at t=1do <u>not</u> correspond to any of the usual Marshall, Hicks of Frisch concepts. For example, given that in the baseline (i.e., prior to the tax cut) we have w_1=w_2 and $\tau_1 = \tau_2$, we can use equation (16) to obtain the theoretical value of the (uncompensated) labor supply elasticity with respect to a temporary tax change at t=1 in the model with no human capital:

$$\frac{\partial \ln h_1}{\partial \ln(1-\tau_1)} = \left[\frac{1-.75}{0.50+.75}\right] - \left[\frac{(-.75)}{.50-(-.75)}\frac{1+.50}{.50}\frac{1}{2+.806}\right] = 0.20 + 0.64 = 0.84$$

This aligns closely with the value of 0.835 obtained in the simulation. It exceeds the Hicks elasticity for reasons discussed earlier. Finally, Table 3 also reports a compensated elasticity with respect to a temporary tax cut (for which I have no analytic expression) of 1.222.

It is necessary to take a detour to explain how the compensated elasticities in Tables 3 to 5 are calculated. There is no direct equivalent to the Slutsky equation in the dynamic case. Thus, I have defined the compensated elasticity as the effect of a wage/tax change holding the optimized value function fixed. In order to determine the amount of initial assets a consumer must be given to compensate for a tax change, I solve the equation:

(49) $$V(\tau_1,\tau_2,0) = V(\tau_1',\tau_2',A) \approx V(\tau_1',\tau_2',0) + u'(C)A \quad \Rightarrow \quad A \approx \frac{V(\tau_1,\tau_2,0) - V(\tau_1',\tau_2',0)}{u'(C)}$$

where τ_1' and τ_2' denote the tax rates after the tax change. Giving people the initial asset level defined by A in (49) equates the initial value function $V(\tau_1,\tau_2,0)$ and the post-tax change value function $V(\tau_1',\tau_2',A)$ to a high degree of accuracy. For small tax changes this procedure is approximately equal to redistributing the proceeds lump sum.

The second panel of Table 3 presents results when the human capital effect α is set at the very low level of .001. Strikingly, even this very small human capital effect renders the conventional method of estimating the Frisch elasticity – i.e., taking the ratio of hours growth to wage growth – completely unreliable.[14] With α = .001, in the baseline model, the wage rate increases by only about 3% from t=1 to t=2 (for all values of γ). For instance, if γ = 0.50 the wage increases by 3.25%. At the same time, hours increase from 101.43 to 102.16, or 0.72%. Thus, taking the ratio, we would incorrectly infer a Frisch elasticity of only $(1/\gamma)$=0.72/3.25= 0.221, compared to the true value of $(1/\gamma)$ = 2.0. [Note: Here I define the "correct" Frisch elasticity as the response of hours to changes in the *opportunity cost of time*, as in (35)].

One might surmise that the reason the conventional method of calculating the Frisch elasticity is so severely downward biased in this case is that the wage change from t=1 to t=2 in the baseline model is entirely endogenous. That is, it results entirely from human capital investment. There is no source of exogenous variation in the after-tax wage, such as an

[14] Of course, econometric studies that estimate the Frisch elasticity by regressing hours changes on wage changes use more complex IV techniques, designed to deal with measurement error in wages, heterogeneity in tastes for work, and unanticipated wage changes. We do not have any of those problems here, so the appropriate estimator boils down to just taking the ratio of the percentage hours change to the percentage wage change.

exogenous tax change or a change in the rental rate on human capital (e.g., a labor demand shift). One might further surmise that if the data contained an event such as a temporary tax cut that shifted the wage path exogenously, one could infer γ more reliably.

Surprisingly, it turns out this intuitive logic is fundamentally flawed. The left panel of Table 3 reports Frisch elasticities calculated in the conventional manner in a regime with a temporary 1% tax cut in t=1. Looking at the $\gamma = 0.50$ case, we see the estimate is -.478, which is not even the correct sign. What happens in this case is that the tax cut causes labor supply to increase in period 1, which, in turn, increases the wage in period 2. But despite this wage increase, labor supply actually *declines* in period 2. This is because (i) the tax cut is removed, and (ii) the human capital investment part of the OCT is removed. Thus, although the wage is higher at t=2, the opportunity cost of time is lower. This illustrates the important distinction between the wage and the opportunity cost of time in the model with human capital.

Another way to look at this is that a strictly exogenous shift in the wage path cannot exist in the model with human capital. For instance, a higher after-tax wage at t=1 increases hours, but this raises the wage at t=2 via the human capital effect. So a t=1 tax cut does not cause an exogenous change in the wage profile: the wage at t=2 is altered by the *behavioral response*. This has fundamental implications for estimation of wage elasticities. If experience alters wages, methods that rely on exogenous wage variation will not work. One must model the joint wage/labor supply process, and determine how labor supply responds to the OCT.

Next consider the case of $\underline{\alpha = .008}$, which we determined is a plausible value. Given this value, labor supply at t=1 is 113.94, about 14% higher than in the $\alpha = 0$ case, because the human capital effect raises the opportunity cost of time. This model generates 35.24% wage growth, which, as we noted in Section IV.B, is roughly consistent with observations. Labor supply at t=2 is now 121.04, which is a 6.23% increase over t=1. Thus, using the baseline data (i.e. no tax change), and using conventional methods, we would estimate the Frisch elasticity as only 6.23/35.25 = 0.177. If, instead of the baseline, we use the data that includes a tax cut at t=1, we would obtain 0.198.[15] Thus, in either case, the estimate is far too small.

Next we examine labor supply elasticities with respect to <u>transitory</u> (t=1) tax cuts. Consider first the η=-.75, α=.008, γ=.50 case in Table 3. The first thing to note is that both *total* and *compensated* elasticities drop substantially when human capital is included in the model. Specifically, they fall from .835 and 1.222 in the no human capital case to .312 and

[15] Recall that for $\alpha = .001$ conventional methods produced very different Frisch estimates depending on whether the data contain a tax change. But for larger values like $\alpha = .008$ the estimates are quite close (although far too small in any case). This is because at larger values of α wage growth from period t=1 to t=2 is much greater, and this insures that the OCT does increase (despite the tax rate increase and human capital return drop) at t=2.

.606 in the α=.008 case (more than a factor of two). This pattern of human capital leading to reduced labor supply elasticities holds for all values of γ, and, as we will see in Tables 4-5, it holds for other values of η. As I noted in Section III.B, this pattern is intuitive: if human capital is more important, a temporary tax hits a smaller part of the opportunity cost of time.

Next, compare elasticities with respect to temporary vs. permanent tax changes. The *uncompensated* elasticity of labor supply at t=1 with respect to a temporary tax cut at t=1 is 0.312, while that with respect to a permanent tax cut is 0.176. This seems consistent with the conventional wisdom that temporary tax changes have larger effects than permanent changes. However, the *compensated* elasticity of labor supply at t=1 with respect to a temporary tax cut is 0.606, while that with respect to a permanent tax cut is greater, 0.698. So, at least for compensated tax changes, we see it is indeed possible for permanent changes to have larger effects than transitory changes at plausible parameter values. Later, in Tables 4-5, we'll see this can be true for uncompensated elasticities as well.[16]

Finally, in Table 3 we also see that the Frisch elasticity – as conventionally measured – is 3 to 4 times smaller than compensated elasticities for both permanent and temporary tax changes. This illustrates another key point: the generally low estimates of the Frisch elasticity in the literature should not be viewed as an upper bound on compensated elasticities.[17]

Next I turn to Table 4, which presents results for models with η = -.50 (the Keane-Wolpin (2001) estimate). Focus again on the γ = 0.50 case. In the model *without* human capital in the first three rows, the uncompensated elasticity with respect to a temporary tax cut at t=1 is, as expected, almost exactly twice as large as that with respect to a permanent tax cut (1.03 vs. 0.50).[18] But in the α = .008 case, the uncompensated elasticity with respect to a permanent tax cut is *greater* than that with respect to a temporary tax cut (0.445 vs. 0.420). If we move to the α=.010 case, which is towards the higher end of the plausible range, the difference grows even larger (0.424 vs. 0.327).

The result is even clearer if we look at compensated elasticities. In the α = .008 case the compensated elasticity with respect to a permanent tax cut is 0.884, while that with respect to a temporary tax cut is only 0.661. (And recall that in Table 3, where η = -.75, we also found that the compensated elasticity was greater in the permanent case).

[16] Recall from equation (46) that the hurdle α must exceed for permanent tax cuts to have larger *uncompensated* effects is increasing in (-η). When we move to Tables 4-5 (where (-η) is smaller) we'll see this hurdle being met.
[17] In fact, the Frisch elasticities do not even give an upper bound on the uncompensated elasticities (e.g., the two methods of calculating the Frisch elasticity produce values of 0.177 and 0.198, while the uncompensated elasticity for a t=1 tax cut is 0.312).
[18] Recall from equation (16) that $\frac{\partial \ln h_1}{\partial \ln(1-\tau_1)} = \left[\frac{1-.50}{0.50+.50}\right] - \left[\frac{(-.50)}{.50-(-.50)}\frac{1+.50}{.50}\frac{1}{2+.806}\right] = 0.50 + 0.53 = 1.03$.

These results illustrate a key point: for plausible parameter values – indeed for what I have argued in Section IV.A are the preferred range of values for α, η and γ – labor supply effects of permanent tax cuts can exceed those of temporary tax cuts in the life-cycle model with borrowing/lending and human capital. Consistent with the theory section, this is more likely if income effects are weaker (and hence more likely for compensated elasticities).

Finally, note that in the α=.008 case the conventional method of calculating the Frisch elasticity produces values of .304 and .256 (if the data do or do not contain a temporary tax cut, respectively). These estimates, typical of the low values in prior empirical work, imply values of γ of 3.3 to 3.9. Yet we know the true value is γ=0.50. Strikingly, these conventional Frisch elasticity estimates are even smaller than the un compensated elasticity with respect to a permanent tax cut (.445) and much smaller than the compensated elasticity (.884).

This again illustrates one of my key points: The low estimates of the Frisch elasticity obtained in prior literature are consistent not only with large values of (1/γ), but also with quite large values for compensated and even uncompensated elasticities. Hence, existing estimates of the Frisch elasticity that ignore human capital should not be viewed as upper bounds on either compensated or uncompensated elasticities.

Table 5 reports results for η = -.25. This value implies rather weak income effects, so it magnifies the results from Tables 3-4. For example, in the α=.008 case, the uncompensated elasticity with respect to a permanent tax cut is now much greater than that with respect to a temporary tax cut in period one (0.836 vs. 0.557). And the compensated elasticity is much greater as well (1.110 vs. 0.700). In the α=.008, γ=.50 case the (conventional) estimates of the Frisch elasticity are again smaller than compensated and uncompensated elasticities. And the compensated and uncompensated elasticities with respect to temporary tax cuts are again at least a factor of two below their values in the model without human capital.

Table 6 reports results for the model with borrowing constraints. I only report results for the η = -.50 case. This is because Keane and Wolpin (2001) estimated a model with both human capital and liquidity constraints and obtained an estimate of η = -.50. Note that they estimated the extent of liquidity constraints (rather than assuming their existence) and their estimates implied rather tight limits on uncollateralized borrowing.

Of course with no borrowing or lending the inter-temporal substitution mechanism is completely shut down. If taxes are temporarily lowered in the first period it is no longer possible to "make hay while the sun the shins" (Heywood (1547)) and save part of the earnings for the second period. Hence, the Frisch elasticity properly defined does not exist.

It still makes sense, however, to ask what one would obtain for the Frisch elasticity (and what one would infer about γ) if one applied conventional methods in an environment with liquidity constraints (Domeij and Floden (2006) ask a similar question). As we see in the first three rows of Table 6, for the case of no human capital ($\alpha = 0$) one just obtains the Marshallian elasticity. But when human capital is included one typically obtains *negative* values. For example, if α=.008 and γ=0.5 the "Frisch" elasticity appears to be -.189 or -.119, depending on whether one uses the data that do or do not contain a temporary tax cut.

As we saw in Section III.A, in a model with human capital but no borrowing/lending (so the inter-temporal substitution mechanism is shut down), the labor supply response to permanent tax changes *must* exceed that to temporary changes. We see this clearly in Table 6. For instance, in the α = .008 and γ = 0.5 case, the uncompensated elasticity with respect to a temporary tax cut is 0.345 while that with respect to a permanent tax cut is 0.469.

Finally, I consider compensated elasticities. We can no longer use equation (49) to determine the (net) assets to give an agent to compensate him/her for a tax change, because now consumption, and hence the marginal utility of consumption, differs in the two periods. Thus, to compensate for a permanent tax change, I find the asset level A that solves:

$$(50) \qquad A \approx \frac{V(\tau_1,\tau_2,0)-V(\tau_1',\tau_2',0)}{\left[u'(C_1)+u'(C_2)\right]/2}$$

and give the agent $A(1+r)/(2+r)$ in each period. To compensate for a temporary tax change in period 1, I find the asset level A that solves:

$$(51) \qquad A \approx \frac{V(\tau_1,\tau_2,0)-V(\tau_1',\tau_2,0)}{u'(C_1)}$$

and give the agent A in period 1. Now, looking again at the α = .008 and γ = 0.5 case, we see that the compensated elasticity with respect to a temporary tax change is 0.687 while that with respect to a permanent tax change is 0.958.

Finally, I compare the model with borrowing constraints (Table 6) to that without (Table 4). *With borrowing*, uncompensated elasticities with respect to permanent tax cuts exceed those for temporary tax cuts once $\alpha \geq .008$, and this is true for all values of γ. And compensated elasticities with respect to permanent tax cuts are higher than for transitory tax cuts for all values of γ once $\alpha \geq .006$. As I argued in Section IV.B, α = .008 is at the low end of the plausible range for α, so cases where elasticities with respect to permanent tax cuts exceed those for temporary cuts appear to be quite likely.

With borrowing constraints, both uncompensated and compensated elasticities with respect to permanent tax changes always exceed those for temporary tax changes, and the size of the difference grows with the importance of human capital effects.

The models with and without borrowing constraints are polar cases, with the "truth" presumably somewhere in between. If borrowing constraints are in fact important, it becomes more likely that permanent tax changes have larger effects than temporary ones.

IV.D. The Case of Log Utility (Income and Substitution Effects Cancel)

An interesting question is whether my results hinge on using calibrated values for η such that substitution effects dominate income effects (η>-1). Recall that the values of η=-0.5 and -.75 were chosen based on results in Imai and Keane (2004), Keane and Wolpin (2001) and several other studies. But macro models often assume log utility to generate balanced growth paths. Thus, I have also run simulations for the log(C) utility case (η=-1). To conserve space I only give an overview of the results. If γ=0.5 than wage growth is 33%, 45% and 59% in the α=0.008, 0.010 and 0.012 cases, respectively (recall that wage growth does not differ much if we vary γ). And hours growth is 4% to 5% in these cases, so the "modest" growth criterion is met. Thus, a plausible range for α is (0.008, 0.012) based on our earlier criteria.

Of course, the uncompensated elasticity with respect to permanent tax cuts is always zero, as income and substitution effects cancel. The uncompensated elasticities of t=1 labor supply with respect to transitory t=1 tax cuts are 0.24, 0.18 and 0.13 in the α=0.008, 0.010 and 0.012 cases, respectively. So, exactly as expected, transitory tax cuts must have larger effects than permanent tax cuts in the uncompensated case.

What is surprising is that in the compensated case this is not true. The compensated elasticities of t=1 labor supply with respect to transitory t=1 tax cuts are 0.56, 0.48 and 0.42 for α=0.008, 0.010 and 0.012 cases, respectively. But for permanent tax cuts these figures are 0.57, 0.56 and 0.55. And this same pattern continues to hold for all values of γ in the 0.25 to 4 range.[19] Thus, for plausible values of the human capital effect, and for all plausible values of γ, permanent tax effects are larger than transitory tax effects.

IV.E. Multi-Period Extension of the Basic Model

A key question is whether the finding that permanent tax cuts can have larger effects than transitory is special to the simple 2-period model, or whether it holds more generally. Here I report labor supply elasticities obtained from the full model in Imai and Keane (2004).

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[19] For example, if γ=2.0, which conforms closely with conventional wisdom, the compensated elasticity of t=1 labor supply with respect to transitory t=1 tax cuts is 0.27, 0.24 and 0.22 for the α=0.008, 0.010 and 0.012 cases, respectively. But for permanent tax cuts these figures are 0.32, 0.31 and 0.31. Thus, the permanent tax effects are 20% to 40% greater than transitory effects.

This model includes wage uncertainty, a more complex human capital production function,[20] age varying tastes, and a motive for retirement savings. It includes annual periods from age 20 to 65, and generates retirement behavior (hours fall substantially in the 50s and 60s).[21]

It is important to note that this exercise compliments, but does not substitute for, the simulations using the simple model: Say we find permanent tax cuts have larger effects than transitory in the Imai-Keane model. As their model is so complex, the intuition for why this occurs would not be as transparent as in the simple model. [Indeed, as I noted at the outset, the complexity of the Imai-Keane model is a key motivation for studying the simple model.]

To proceed, Table 7 reports effects of permanent and transitory tax increases in the Imai-Keane model. The effects of transitory tax increases were already reported in Imai and Keane (2004). But they did not report effects of permanent tax changes, which are more relevant for evaluating tax policy. So the permanent tax simulations are new.[22]

Table 7 reports effects of 5% tax increases. In the column labelled "transitory," the tax increase applies for one year at the indicated age. For example, at age 20, a temporary 5% tax increase reduces hours by 1.5%. This implies an elasticity of only 0.3. This is far smaller than one might expect, given that Imai-Keane estimate $(1/\gamma) = 3.8$. But as they noted, effects of transitory taxes grow substantially with age. For instance, at age 60, a temporary 5% tax increase reduces hours by 8.6%, implying an elasticity of 1.7.

The intuition for why the labor supply elasticity increases with age is clear from our earlier discussion of the 2-period model, particularly equation (31). The transitory tax only directly affects the current after-tax wage, and not the return to human capital investment. But as workers age, the current wage makes up a larger share of the opportunity cost of time.

The last two columns of Table 7 report effects of permanent 5% tax increases, both uncompensated and compensated. The tax increase occurs (unexpectedly) at the indicated age and lasts until age 65. The Table reports only the effect on current labor supply in the year the tax increase is first implemented. A notable finding is that compensated effects are much larger than uncompensated, implying that income effects are important.

It is also notable that effects of permanent tax changes on current labor supply differ greatly depending on a worker's age when the tax is implemented. For workers in their 20s,

[20] Specifically, their production function allows for complimentarity between the stock of human capital and hours of work in the production of skill, and it lets parameters differ by four education levels.

[21] The Imai and Keane (2004) model predicts average weekly hours (for white males in the 1958-65 cohort) of 44.4, 48.9, 43.4 and 19.9 at ages 25-34, 35-44, 45-54 and 55-64. This pattern is similar to what McGrattan and Rogerson (1998) project for all men in the same cohort (see their Table 8). [Except the Imai-Keane hours figures are shifted up, presumably because of exclusion of minorities]. Notice that "retirement," in the form of greatly reduced hours, starts to take place (on average) well before age 65, both in the data and the model.

[22] I thank Susumu Imai for providing me with these new simulations.

30s and 40s, the compensated effects of a 5% permanent tax increase on current annual hours range from -2.3% to -3.2%. But for workers in their 50s and 60s the effects are much greater.

The key result in Table 7 is that, for younger workers, permanent tax increases have larger effects on current labor supply than do transitory tax increases. For instance, consider a 5% tax increase that takes place at age 25. If it is transitory, hours are reduced by 1.8%. But if it is permanent and the proceeds are distributed lump sum, hours fall by 2.7%. So at age 25, the permanent tax effect is 50% greater. By the mid-30s permanent and transitory tax effects are roughly equal. Only in the 40s do effects of transitory tax cuts become somewhat larger.

Again these results are consistent with intuition from the simple 2-period model. Permanent tax cuts may have larger effects on current hours than transitory tax cuts because they hit both the wage and human capital terms in (31), while a transitory tax only hits the current wage. But as workers age the human capital term becomes less important, so the mechanism that magnifies the effect of permanent taxes is diminished.

So far, I have only discussed the effects of tax changes on *current* period hours. But in Table 8 we see that the effects of permanent tax increases grow with time (age). The Table considers a permanent (compensated) 5% tax increase that takes effect at either age 25, 30 or 35. I report how this alters a person's labor supply at 5-year intervals from age 25 to 65. For instance, suppose the 5% tax increase goes into effect (unexpectedly) when the worker is 25. Then, at age 25, his labor supply is reduced by 2.7%. But, at age 45 his hours are reduced by 5.1%, and at age 60 the reduction is 19.3%.

The effect of a permanent tax change grows with age for two reasons: First, as I've already noted, as workers get older, the after-tax wage makes up a larger fraction of the OCT, so a given tax has a larger direct effect. Second, a permanent tax hike produces a "snowball" effect: If a worker reduces his labor supply at time t, he will have less human capital at time t+1. This causes him to work even less at time t+1, leading to a lower wage at t+2, etc..

This "snowball" effect of taxes on wages is also shown in Table 8. At first, tax effects on human capital are modest, but they grow substantially with age. For instance, if a 5% tax increase is instituted when a worker is 25, then by age 40 his wage is reduced by 1.0%, but by age 55 his wage is reduced by 3.6%, and by age 65 the reduction is 11.6%. Thus, if we focus only on current labor supply, we will understate the extent to which permanent tax changes have larger effects than transitory changes, because we miss this "snowball" effect. This suggests the 2-period model is likely to *understate* the relative impact of permanent taxes.

Finally, I examine how a permanent tax increase affects lifetime labor supply. That is, I simulate the impact of a permanent 5% tax rate hike (starting at age 20 and lasting to age

65) on labor supply over the entire working life. If the revenue is thrown away, then average hours (from ages 20 to 65) drop by 2%. But if the revenue is redistributed lump sum, hours drop 6.6%. The former figure implies an uncompensated elasticity of 0.4, while the latter figure implies a compensated elasticity with respect to permanent tax changes of 1.3. These values are quite large compared to ones typically obtained in models without human capital.

Notably, the compensated (Hicks) elasticity implied by the Imai-Keane parameter estimates in a model *without* human capital is $1/(\gamma-\eta) = 1/(.262+.736) \approx 1.0$. Thus, the human capital mechanism and the "snowball" effect on wages in the multi-period model combine to augment the compensated elasticity by 30%. (This is in sharp contrast to the intertemporal elasticity, which is dampened so as to be less than $(1/\gamma)$, for reasons discussed earlier). Given a compensated elasticity of 1.3, we would expect welfare losses from taxation to be large.

V. "Optimal" Income Tax Rates and the Welfare Losses from Taxation

In this Section I consider how introducing human capital into the life-cycle model affects the (second best) optimal proportional income tax rate, and the welfare losses from distortionary taxes on labor income. Throughout this section I assume a flat rate income tax that is equal in both periods ($\tau_1 = \tau_2 = \tau$). In order to talk about optimal taxation it is necessary to specify that the government provides a public good from which workers derive utility.[23] Let the quantity of the public good be denoted by P, and assume that the government provides the same level of P in each period. Then the government budget constraint is:

$$(52)\qquad P+\frac{1}{1+r}P=\left\{w_1h_1\tau+\frac{1}{1+r}w_2h_2\tau\right\} \qquad \Rightarrow \qquad P=\tau\left\{w_1h_1+\frac{1}{1+r}w_2h_2\right\}\frac{1+r}{2+r}$$

Next we modify the value function in equation (27) to include a public good:

$$(53)\quad V=\lambda f(P)+\frac{[w_1h_1(1-\tau)+b]^{1+\eta}}{1+\eta}-\beta\frac{h_1^{1+\gamma}}{1+\gamma}+\rho\left\{\lambda f(P)+\frac{[w_2h_2(1-\tau)-b(1+r)]^{1+\eta}}{1+\eta}-\beta\frac{h_2^{1+\gamma}}{1+\gamma}\right\}$$

where $\lambda f(P)$ is the utility that consumers derive from the public good.

Given (53), the first order conditions (28)-(30) are modified to become:

$$(54)\quad \begin{aligned}\frac{\partial V}{\partial h_1}=\lambda f'(P)\frac{dP}{dh_1}(1+\rho)+\left[w_1h_1(1-\tau)+b\right]^{\eta}w_1(1-\tau)-\beta h_1^{\gamma}\\ +\rho\left[w_2h_2(1-\tau)-b(1+r)\right]^{\eta}(dw_2/dh_1)h_2(1-\tau)=0\end{aligned}$$

[23] As we have a representative agent model, the redistributive motive for taxation that is central to work in the tradition of Mirrlees (1971), Sheshinski (1972) and Stern (1976) is not relevant here.

(55) $$\frac{\partial V}{\partial h_2} = \lambda f'(P)\frac{dP}{dh_2}(1+\rho) + \rho\left\{\left[w_2 h_2(1-\tau) - b(1+r)\right]^{\eta} w_2(1-\tau) - \beta h_2^{\gamma}\right\} = 0$$

(56) $$\frac{\partial V}{\partial b} = \left[w_1 h_1(1-\tau) + b\right]^{\eta} - \rho\left[w_2 h_2(1-\tau) + b(1+r)\right]^{\eta}(1+r) = 0$$

where, given the new wage equation (48) we now have $dw_2 / dh_1 = \alpha - \alpha h_1 / 200$, and so:

$$\frac{dP}{dh_1} = \tau\left\{w_1 + \frac{h_2}{1+r}\frac{dw_2}{dh_1}\right\}\frac{1+r}{2+r} = \tau\left\{w_1 + \frac{h_2}{1+r}w_2\left(\alpha - \frac{\alpha}{200}h_1\right)\right\}\frac{1+r}{2+r} \qquad \frac{dP}{dh_2} = \tau\frac{w_2}{2+r}$$

There is also a new first order condition describing the problem of the government:

(57) $$\frac{\partial V}{\partial \tau} = \lambda f'(P)\left(1+\rho\right)\frac{\partial P}{\partial \tau} + C_1^{\eta}\frac{\partial C_1}{\partial \tau} + \rho C_2^{\eta}\frac{\partial C_2}{\partial \tau} = 0$$

As before I assume $\rho(1+r)=1$ in order to simplify the problem and focus on the key issues. In this case, and with no borrowing constraints, we get that $C_1 = C_2 = C$. Then, (57) reduces to:

$$\lambda f'(P)\left(1+\rho\right)\frac{\partial P}{\partial \tau} + C^{\eta}\left(1+\rho\right)\frac{\partial C}{\partial \tau} = 0$$

And, as $\partial P/\partial \tau = -\partial C/\partial \tau$, we just have that:

(58) $$\lambda f'(P) = C^{\eta}$$

This says that the benevolent government (or social planner) sets the tax rate so as to equate the marginal utility of private consumption to that of consumption of the public good.

To complete the model we must specify the functional form of $f(P)$. The curvature of $f(P)$ determines the relative size of the public sector as workers grow wealthier. I consider three alternative forms: $f(P) = \log(P)$, $f(P) = 2P^{.5}$ and $f(P) = P$. These correspond to cases where P/C declines, is stable or grows as C increases. One might try to calibrate $f(P)$ by looking at how tax rates evolve as countries become richer. Fortunately, my main results are not very sensitive to this assumption – at least qualitatively – so I do not pursue this.

I consider several variants of the model of equations (52)-(58). Those equations describe a social planner version of the model in which workers, in deciding on hours, consider how increased labor supply leads to increased provision of the public good. I also consider a "free rider" version of the model where there are many identical workers, and each assumes his/her own actions have a trivial impact on provision of the public good (dP/dh =0). Then the first term in equations (54)-(56) drops out. And I consider a version of the model with borrowing constraints. In that case set b=0 in (54)-(55), and drop equation (56). In that model, I assume the government can still borrow/lend across periods, so (52) still holds.

All models are calibrated with η = -.50. The scaling parameter λ is set so the optimal tax rate is 40% when there is no human capital accumulation (i.e., when α = 0). Tables 9-10 show how optimal income tax rates vary with γ and α. Note that as α increases people become wealthier (because, *ceteris paribus*, their t=2 wage is higher).

In Table 9, we have $f(P) = \log(P)$. Thus, utility has more curvature in the public good than the private good, so the optimal tax rate falls as α increases. For instance, if we increase α to .008, and adopt my preferred value of γ=.50, the optimal tax rate falls to 33.9%.[24]

Now consider the second case, where $f(P) = 2P^{.5}$ and η = -.50. In this case (58) is simply $\lambda P^{-0.5}=C^{-0.5}$. As the curvature of the utility function in the public and private goods is equal, the optimal rate is always 40%.[25] However, as we will see later, while the optimal tax rate is invariant to α and $(1/\gamma)$, the welfare cost of a proportional tax is increasing in both.

Table 10 reports results for the case of $f(P) = P$ and η = -.50. In this case (58) reduces to $\lambda = C^{\eta}$. So the government sets the tax rate so as to keep the marginal utility of private consumption constant, meaning the optimal tax rate in increasing in α. For example, when α=.008 and γ=.50, the optimal tax rate is 51.9% in the free-rider version of the model, and even higher in the social planner version.[26]

Finally, Tables 11-13 report welfare costs of proportional income taxes, and how this is influenced by preference parameters (γ and η) and the importance of human capital (α). I only report results for the free-rider version of the model with no borrowing constraints.

I report two measures of welfare loss. To obtain them, I also solve a version of the model in which a lump sum tax is used to finance the public good. The lump sum tax is set to the level that would fund the same level of the public good obtained in the solution to the proportional tax version of the model.

The first measure of welfare loss, denoted C^* in the tables, is the amount of extra consumption that consumers in the proportional tax world must be given to enable them to attain the same utility level (more precisely, the same level of the optimized value function) they enjoy in the lump-sum tax world, expressed as a fraction of consumption in the proportional tax world. The second measure, C^{**}, is the loss in consumption in the lump-sum

[24] The effect of α of the optimal tax rate is stronger if γ is smaller (i.e., intertemporal substitution is greater). It is also stronger in the free-rider version of the model. The bottom panel of Table 9 reports results for the version of the model with borrowing constraints. Interestingly, the results hardly differ from those in the top panel.

[25] As $C=I(1-\tau)$ and $P=I\tau$, where I is $(1+r)/(2+r)$ times the PV of lifetime income, we have $P/C = \tau/(1-\tau)$. This implies $\lambda^2=\tau/(1-\tau)$ so $\tau= \lambda^2/(1+\lambda^2)$. Thus the optimal tax rate is a constant that only depends on λ. With borrowing constraints, this argument for a constant optimal tax rate does not hold, as C is no longer equal in both periods. But in the simulations the optimal rate still never deviates from 40% by more than 0.8%.

[26] Again, results are little different in the borrowing constrained case.

tax world that would bring the consumer down to the utility level he/she has in the proportional tax world, expressed as a fraction of consumption in the lump-sum tax world.

Table 11 reports results for the $f(P)$=log(P) case. The top panel reports results for the η = -.75 case and the bottom panel reports results for η = -.50. Each panel gives results for γ ranging from 0.25 to 4 and for α from 0 to 0.012. I also report the uncompensated labor supply elasticity (in the α=0 case) $e=(1+\eta)/(\gamma-\eta)$, because it helps put the results in context. For instance, in a static model without human capital, and abstracting from income effects, Saez et al (2009) give the simple formula that for a <u>flat rate tax</u> the marginal excess burden is $-e\tau/(1-\tau-e\tau)$.[27] Thus, the utility cost of taxation is increasing in e in that framework.[28]

It is evident in both panels of Table 11 that the welfare losses from the proportional tax are strongly inversely related to γ. Perhaps the most interesting comparison is between the "conventional" case of (α = 0, γ = 4), which corresponds to the typical results from prior studies that estimate the Frisch elasticity ignoring human capital, and my preferred setting of (α = .008, γ = 0.5), which I have argued are plausible values once one accounts for human capital. In the "conventional" setting, welfare losses are 3 to 4% of consumption, regardless of the value of η or the welfare measure used. But in the (α = .008, γ = 0.5) case the welfare losses are much larger, ranging from 8.3% to 11.4% of consumption.

By combining the results from Tables 9 and 11, we can also express the burden of the tax as a fraction of the revenue raised. Letting R denote revenue we have $R = C \cdot \tau/(1-\tau)$, and letting W denote the welfare loss we have $W = b \cdot C$, where b is the percentage loss figure reported in Table 11. Thus, the welfare loss to consumers as a fraction of revenue raised is $W/R = b \cdot \tau/(1-\tau)$. For the "conventional" (α = 0, γ = 4) case, using η = -.50 and taking $b=C^*$ we have W/R = (3.90)(1-.40)/(.40) = 5.8%. For my preferred case of (α = .008, γ = 0.5) we have W/R = (11.38)(1-.347)/(.347) = 21.5%. Thus, using parameter values that account for human capital, the utility cost to consumers as a fraction of revenue is 4 times greater.[29]

These calculations are not very sensitive to η. For α in the plausible range of .008 to .010, welfare losses are very similar in the η = -.75 and η = -.50 cases (for all values of γ). But a notable pattern in Table 11 is that, for η = -.75, welfare losses are roughly invariant to the level of α. In contrast, if η = -.50, the welfare cost of the tax falls sharply as α increases. It is

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[27] That is, for each extra dollar of tax collected, the utility cost to consumers in dollar terms is $-e\tau/(1-\tau-e\tau)$. For example, if τ=0.40 and e=0.50 this gives 0.50, meaning the cost is 50 cents for each dollar raised.

[28] Similarly, Saez (2001) shows that, in general, optimal tax rates in the Mirrlees (1971) model depend on both compensated and uncompensated elasticities, but his equation (9) shows that only the <u>uncompensated</u> elasticity, and government tastes for redistribution, matter for the optimal <u>flat rate</u> tax (i.e., set the Pareto parameter a=1).

[29] It seems appropriate to use $b = C^*$ in these calculations as it is a fraction of the consumption C that consumers actually receive under the proportional tax, and that C is what appears in the revenue formula $R = C \cdot \tau/(1-\tau)$.

only in the plausible range for α that the two roughly coincide. The reason for this behavior is that when η = -.75 we are not too far from log(C) utility, so the curvature of utility for P is only slightly greater than that for C. As a result, optimal tax rates do not fall much as α increases. But in the η = -.50 case there is much less curvature in C, and so the optimal P/C falls faster as α increases.[30] So the reason the welfare cost of taxation falls as α increases in the η = -.50 case is simply that taxes themselves are falling.

Now consider Table 12, which reports results for the case where $f(P) = 2P^{.5}$. As we discussed earlier, if η = -.50 the optimal tax rate is constant at 40%. If η = -.75 the optimal tax rate is slightly increasing in α.[31] In both cases, the welfare cost of the proportional tax is increasing as human capital becomes more important. The increase is much more pronounced when η = -.75 (the case where taxes are increasing). But as before, for α in the plausible range of .008 to .010, welfare losses are very similar for different values of η. If γ=0.5 and η = -.75, the welfare loss from the tax is 12 to 15% of consumption, depending on the measure used.[32]

Now consider losses as a percent of revenue. For the "conventional" (α=0, γ=4) case we again have 5.9%. (As the tax rate is unchanged and α=0 there is no reason for results to change). For my preferred case of (α=.008, γ=0.5) we have W/R = (18.11)(1-.40)/(.40) = 27.2%. So, using parameter values that account for human capital, we obtain a utility cost to consumers about 4.5 times greater than in the "conventional" case.

Finally, Table 13 reports results for $f(P) = P$. As we saw in Table 10, in this case the optimal tax rate rises substantially with α because as people become wealthier they demand more of the public good. For example, if γ = 1/2 then when α increases from 0 to .008 the optimal tax rate increases from 40% to 51.9%. The welfare losses from distortionary income taxation become quite substantial in this case. For example, when α = .008, η = -.75, γ = 1/2, the welfare loss is 19 to 28% of consumption, depending on the measure.

We can again express the utility losses as a percent of revenue. For my preferred case of (α=.008, γ=0.5), and with η = -.5, we have W/R = (41.62)(1-.511)/(.511) = 39.8%. This is 6.7 times greater than the cost calculated using "conventional" parameter values that ignore human capital. In summary, while results differ in detail for different specifications of $f(P)$, the basic pattern is similar: Utility losses, as a fraction of revenue, are 4 to 7 times greater if we use parameter values that are plausible in a human capital version of the life-cycle model.

[30] For instance, at γ = 0.5, as α goes from 0 to .008 the optimal tax rate falls from 40% to 33.9% in the η = -.50 case (see Table 9). But it only falls from 40% to 38.1% in the η = -.75 case (not reported in Table 9).

[31] For example, when γ = 0.5 the optimal tax rate increases to only 42.7% when α increases to .008.

[32] The welfare losses from the income tax would appear even greater if the lump sum tax were chosen optimally (achieving the 1st best). For example, in the η=-.50, γ=1/2, α=.008 case, welfare losses are 12-18% compared to the constrained lump sum tax (that raises the same revenue), but 17-25% compared to the optimal lump sum tax.

VI. Conclusion

When human capital is added to the standard life-cycle labor supply model, the wage is no longer the opportunity cost of time (OCT). Rather, the OCT becomes the wage *plus* the return on human capital investment. Here, I show this has important implications for how workers respond to tax changes, and for estimation/interpretation of labor supply elasticities.

One key result is that permanent tax changes can have larger effects on *current* labor supply than transitory tax changes. This contradicts the conventional wisdom that transitory tax changes should have larger short-run effects. The intuition is that a transitory tax change only alters the current after-tax wage (and hence only a part of the OCT), while a permanent tax change alters the return to human capital investment as well.

Using a simple two-period model, I showed that the condition for permanent tax changes to have larger effects (than transitory) is that returns to work experience must be sufficiently large relative to income effects. I also showed that this condition can hold for quite plausible values of preference parameters and returns to work experience.

An important motivation for looking at permanent tax changes is that Imai and Keane (2004), who structurally estimate a life-cycle model with human capital, focussed only on the intertemporal (or Frisch) elasticity. Their preference parameter estimates implied that, *in a model without human capital*, the Frisch elasticity would be 3.8. But, their simulations show transitory tax effects much smaller than an elasticity of 3.8 would lead one to expect. Human capital <u>dampens</u> the transitory tax response, because a transitory tax only alters the current after-tax wage (and not the human capital component of the OCT). But this leaves a key question unanswered: How does human capital affect responses to permanent tax changes?

Consistent with results in Imai-Keane, simulations of our simple two-period model show that, as returns to work experience increase, elasticities with respect to transitory tax changes drop rapidly (for given preference parameters). But, in sharp contrast, the impact of permanent tax changes declines only slightly as human capital becomes more important. This is because permanent tax changes do affect future returns to human capital investment.

Furthermore, in a multi-period setting, human capital appears to <u>magnify</u> the response to permanent tax changes. The preference parameters in the Imai and Keane (2004) model would imply a compensated (Hicks) elasticity of about 1.0 in a world with no human capital. But simulation of their model generates a compensated elasticity of 1.3 – a 30% increase. This is because a permanent tax increase at t has a <u>cumulative</u> effect: it leads to less labor supply at t, which lowers wages at t+1, further reducing labor supply at t+1, etc..

Another key result is that even a "small" return on human capital investment (in a sense made precise in the paper) can lead to severe downward bias in *conventional* methods of estimating the intertemporal (or Frisch) elasticity – where, by "conventional," I mean methods that ignore human capital. In the standard life-cycle model, the Frisch elasticity is an upper bound on the Marshall and Hicks. Thus, the low estimates of the Frisch elasticity in most prior work (i.e., about 0.10 to 0.30) have contributed to a consensus that the Hicks and Marshall elasticities must be small as well. But the simulations presented here show that compensated labor supply elasticities can be several times larger than the upper bound implied by conventional methods of estimating the Frisch elasticity.

I also showed that use of exogenous tax regime changes to identify labor supply elasticities does not resolve the bias problem in conventional estimation methods, and can even make the bias greater. The point is that, in a model with human capital, any tax change will induce changes in the incentive to acquire human capital. Thus, any change in the time path of after-tax wages induced by exogenous tax changes will nevertheless be endogenous – as the wage path is influenced by changes in human capital investment decisions. The only solution to this problem is to model the joint labor supply/human capital investment process, as in Heckman (1976) and Imai and Keane (2004).

I went on to use the simple life-cycle labor supply model with human capital to study welfare effects of proportional (i.e., flat-rate) income taxation. In the model, the benevolent government sets the tax rate optimally to equate marginal utility of consumption of the public and private goods. I consider a range of values for the curvature in consumers' utility from the public good. Returns to work experience are set so the wage rate grows by roughly 1/3 over the first 20 years of the working life (which I argue is a conservative value).

The key free parameters in these experiments are the utility curvature parameters for consumption (η) and hours (γ). For example, Imai and Keane (2004) estimate $\eta \approx -.75$ and $(1/\gamma) \approx 4$. These values imply uncompensated and compensated labor supply elasticities with respect to permanent tax hikes of 0.176 and 0.698, respectively. At these values, the welfare losses from proportional income taxation are substantial, ranging from 13% to 35% of consumption, depending on degree of curvature in utility from the public good.

It is important to note that the compensated and uncompensated elasticities mentioned here are not the traditional Marshall and Hicks elastictities. Instead, they are generalizations that apply in the dynamic case with human capital, as given by equations (40) and (49). Presentation of these new elasticity formulas is the main technical contribution of the paper.

The welfare losses from taxation can also be expressed as a fraction of revenue. It is interesting to compare losses in a "conventional" life-cycle model with no human capital, and a typically small value of the Frisch elasticity (i.e., $(1/\gamma) \approx 0.25$) to those in a model that accounts for human capital and, thus, has a correspondingly higher value of $(1/\gamma)$, such as 2. Consistent with conventional wisdom, I find welfare losses as percent of revenue are less than 6% in the "conventional" model. But in the model with human capital they are 4 to 7 times greater (depending on the degree of curvature in utility from the public good).

Earlier, I noted that, in a multi-period setting, a permanent tax hike has a cumulative effect that grows over time. By reducing labor supply, the rate of accumulation of human capital is also reduced, which further reduces labor supply, and so on. This suggests that static models that focus on the effect of taxes on labor supply *holding work experience fixed* may be missing an important channel through which welfare costs of taxation arise. As noted by Keane and Wolpin (2000, 2010), changes in the tax/transfer system that reduce rewards to working will also reduce educational attainment. Accounting for this additional channel would presumably magnify the tax effects on human capital found here.

This paper is part of an emerging literature exploring mechanisms that may have caused prior work to understate labor supply elasticities. Besides the human capital mechanism studied here, other potentially important mechanisms that have been considered include liquidity constraints (Domeij and Floden, (2006)), uninsurable wage risk (Low and Maldoom (2004)), corner solutions in labor supply (Rogerson and Wallenius (2007), French (2005), Kimmel and Kniesner (1998)) and fixed costs of adjustment (Chetty (2010)). An important task for future research is to sort out the relative importance of these mechanisms. Suffice it to say, while the conventional wisdom still suggests that labor supply elasticities are small, more dissent from that position is now emerging – see Keane and Rogerson (2010) for a detailed survey.

Historically, Mirrlees (1971) expressed surprise that optimal tax rates were so low (about 20 to 30%) in his model, but Stern (1976) noted that that optimal tax rates would be much higher (i.e., well over 50%) if utility parameters were set to values that implied much less elastic labor supply. He argued this was more consistent with existing empirical work. But, given the downward bias in elasticity estimates induced by failure to account for human capital (or other factors noted above), the very low elasticity estimates used by Stern may be suspect, while the higher elasticities in Mirrlees' original paper may be more plausible.

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Table 1: Baseline Simulation

α	γ	η = -.75						η = -.5						η = -.25					
		0	**0.25**	**0.5**	**1**	**2**	**4**	**0**	**0.25**	**0.5**	**1**	**2**	**4**	**0**	**0.25**	**0.5**	**1**	**2**	**4**
0	h_1		100	100	100	100	100		100	100	100	100	100		100	100	100	100	100
	h_2		100	100	100	100	100		100	100	100	100	100		100	100	100	100	100
	w_2		100	100	100	100	100		100	100	100	100	100		100	100	100	100	100
0.001	h_1	99	102	101	101	101	100	101	103	102	101	101	100	105	105	103	102	101	101
	h_2	109	103	102	101	101	100	113	104	103	102	101	101	127	107	104	102	101	101
	w_2	103	103	103	103	103	103	103	103	103	103	103	103	103	103	103	103	103	103
0.003	h_1	104	105	105	103	102	101	109	109	107	105	103	102	130	116	111	106	104	102
	h_2	120	110	107	104	103	101	135	115	110	106	103	102	211	127	115	108	104	102
	w_2	111	111	111	110	110	110	111	111	111	111	110	110	114	112	112	111	110	110
0.005	h_1	110	109	108	106	104	102	120	116	112	108	105	103	158	130	120	111	106	103
	h_2	132	117	112	108	105	103	166	128	118	110	106	103	390	157	129	115	107	104
	w_2	120	120	119	119	118	118	122	121	120	119	118	118	130	125	122	120	119	118
0.007	h_1	116	114	112	109	106	103	133	124	118	112	107	104	175	146	130	117	109	105
	h_2	147	126	118	111	106	104	209	145	128	116	108	104	670	202	148	122	111	105
	w_2	131	130	129	128	127	126	137	134	132	129	127	126	147	141	**136**	132	128	126
0.008	h_1	120	117	114	110	107	104	140	128	121	114	108	105	181	153	136	120	111	106
	h_2	155	130	121	113	108	104	236	155	133	118	110	105	849	229	160	127	112	106
	w_2	138	137	**135**	133	132	130	146	141	**139**	135	133	131	156	150	**144**	138	134	131
0.010	h_1	127	122	118	113	109	105	152	138	128	119	111	106	188	166	147	127	114	107
	h_2	174	141	128	117	110	105	299	178	146	125	113	106	1306	295	187	137	116	108
	w_2	154	151	**149**	145	142	140	166	159	**154**	149	144	141	175	170	164	154	146	142
0.012	h_1	135	128	123	117	111	106	162	147	136	124	114	107	192	175	158	135	118	109
	h_2	196	152	135	121	112	106	372	206	161	132	116	108	1937	374	220	150	121	109
	w_2	173	168	165	160	155	151	188	180	174	165	157	152	196	193	186	173	161	153

Table 2: Baseline Simulation, Borrowing Constraint

		η = -.75						η = -.5						η = -.25					
α	**γ**	**0**	**0.25**	**0.5**	**1**	**2**	**4**	**0**	**0.25**	**0.5**	**1**	**2**	**4**	**0**	**0.25**	**0.5**	**1**	**2**	**4**
0	h_1	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	h_2	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	w_2	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
0.001	h_1	104	103	102	102	101	101	106	104	103	102	101	101	112	106	104	102	101	101
	h_2	101	101	101	100	100	100	103	102	102	101	101	100	112	105	103	102	101	101
	w_2	103	103	103	103	103	103	103	103	103	103	103	103	104	103	103	103	103	103
0.003	h_1	111	108	107	105	103	102	117	112	109	106	103	102	138	118	112	107	104	102
	h_2	104	103	102	101	101	101	112	108	105	103	102	101	152	120	112	106	103	102
	w_2	112	111	111	111	110	110	112	112	111	111	110	110	115	113	112	111	110	110
0.005	h_1	119	114	111	108	105	103	130	120	115	110	106	103	163	133	121	112	107	103
	h_2	107	105	104	103	102	101	125	114	110	106	103	102	221	140	123	112	106	103
	w_2	122	121	120	119	118	118	125	122	121	120	119	118	130	125	123	120	119	118
0.007	h_1	126	119	116	111	107	104	141	128	121	114	108	104	178	147	131	118	110	105
	h_2	110	107	106	104	102	101	140	122	115	109	105	103	318	168	137	118	109	104
	w_2	135	132	131	129	127	126	140	135	133	130	128	126	147	141	**137**	132	129	127
0.008	h_1	129	122	118	113	108	105	147	132	124	116	109	105	183	154	137	121	111	106
	h_2	112	109	106	104	103	101	148	127	118	111	106	103	379	185	145	122	110	105
	w_2	142	139	**137**	135	132	131	148	143	140	**136**	133	131	156	150	**145**	138	134	131
0.010	h_1	136	128	122	116	110	106	157	140	131	120	112	106	190	166	147	128	115	107
	h_2	117	111	109	106	103	102	167	137	125	114	108	104	537	223	164	130	114	106
	w_2	158	154	**151**	147	143	140	167	161	156	**150**	144	141	175	171	164	154	146	142
0.012	h_1	142	133	126	119	112	107	165	148	137	125	114	108	194	175	157	135	118	109
	h_2	121	114	111	107	104	102	189	149	132	118	110	105	755	268	186	139	117	108
	w_2	178	171	167	161	156	151	189	181	174	166	158	152	196	193	186	173	161	153

Table 3: Labor Supply Response to Tax Change, Case of η = -.75

		Tax reduction in period 1						Tax reduction in both periods					
		γ						γ					
α	**Elasticity**	**0**	**0.25**	**0.5**	**1**	**2**	**4**	**0**	**0.25**	**0.5**	**1**	**2**	**4**
0	Total		1.570	0.835	0.445	0.235	0.122		0.249	0.199	0.142	0.090	0.052
	Compensated.		2.059	1.222	0.721	0.410	0.223		0.990	0.792	0.566	0.361	0.209
	Frisch		4.060	2.010	1.000	0.499	0.249		4.060	2.010	1.000	0.499	0.249
0.001	Total	7.784	1.278	0.733	0.408	0.220	0.116	0.212	0.236	0.194	0.140	0.090	0.052
	Compensated	8.192	1.731	1.104	0.675	0.392	0.215	0.841	0.935	0.770	0.558	0.358	0.208
	"Frisch"	-5.203	-0.839	-0.478	-0.263	-0.141	-0.074	3.200	0.430	0.221	0.109	0.053	0.025
0.003	Total	2.267	0.891	0.572	0.341	0.192	0.103	0.223	0.220	0.186	0.137	0.089	0.052
	Compensated	2.663	1.297	0.917	0.596	0.357	0.200	0.883	0.874	0.739	0.546	0.354	0.207
	"Frisch"	0.814	0.197	0.086	0.027	0.003	-0.002	1.404	0.390	0.208	0.099	0.045	0.020
0.005	Total	1.185	0.645	0.450	0.285	0.166	0.091	0.231	0.213	0.181	0.135	0.088	0.052
	Compensated	1.571	1.020	0.773	0.528	0.326	0.185	0.913	0.843	0.719	0.538	0.352	0.206
	"Frisch"	0.874	0.314	0.162	0.065	0.020	0.005	1.019	0.359	0.195	0.090	0.038	0.015
0.007	Total	0.714	0.473	0.353	0.236	0.142	0.079	0.234	0.208	0.178	0.134	0.088	0.052
	Compensated	1.079	0.820	0.657	0.469	0.297	0.171	0.920	0.822	0.705	0.532	0.350	0.206
	"Frisch"	0.817	0.350	0.190	0.079	0.024	0.005	0.836	0.332	0.183	0.082	0.032	0.011
0.008	Total	0.565	0.405	**0.312**	0.214	0.131	0.074	0.232	0.205	**0.176**	0.133	0.088	0.052
	Compensated	0.913	0.738	**0.606**	0.441	0.283	0.164	0.911	0.811	**0.698**	0.530	0.350	0.206
	"Frisch"	0.791	0.358	**0.198**	0.083	0.025	0.004	0.774	0.319	**0.177**	0.079	0.029	0.009
0.010	Total	0.358	0.295	**0.241**	0.174	0.111	0.064	0.221	0.198	**0.173**	0.132	0.088	0.052
	Compensated	0.663	0.597	**0.515**	0.391	0.257	0.151	0.865	0.783	**0.683**	0.525	0.349	0.206
	"Frisch"	0.752	0.370	**0.210**	0.088	0.024	0.002	0.682	0.296	**0.165**	0.072	0.024	0.006
0.012	Total	0.229	0.211	0.183	0.139	0.092	0.054	0.200	0.188	0.168	0.131	0.088	0.052
	Compensated	0.479	0.478	0.434	0.344	0.233	0.139	0.784	0.741	0.663	0.520	0.349	0.207
	"Frisch"	0.727	0.380	0.220	0.091	0.023	-0.001	0.615	0.276	0.154	0.065	0.019	0.003

Note: η = -.75 is the Imai and Keane (2004) estimate. The "Total" elasticity is the uncompensated. The "Frisch" elasticity refers to the estimate obtained using the conventional method of regressing the log hours change on the log earnings change. In the left panel this Frisch estimate is obtained using data that contain a tax cut at t=1, while in the right panel the Frisch estimate is obtained from data where the tax rate is equal in the two periods. The figures in bold type are the values obtained using values of the return to human capital investment in the plausible range (α = .008 to .010), and for my preferred value of γ=0.5.

Table 4: Labor Supply Response to Tax Change, Case of η = -.5

		Tax reduction in period 1						Tax reduction in both periods					
		γ						γ					
α	**Elasticity**	**0**	**0.25**	**0.5**	**1**	**2**	**4**	**0**	**0.25**	**0.5**	**1**	**2**	**4**
0	Total		1.844	1.030	0.568	0.305	0.160		0.666	0.499	0.332	0.199	0.111
	Compensated		2.279	1.353	0.783	0.434	0.231		1.326	0.994	0.663	0.398	0.221
	Frisch		4.060	2.010	1.000	0.499	0.249		4.060	2.010	1.000	0.499	0.249
0.001	Total	7.854	1.518	0.915	0.526	0.289	0.153	0.650	0.634	0.487	0.329	0.198	0.110
	Compensated	8.265	1.923	1.226	0.735	0.415	0.223	1.289	1.262	0.971	0.656	0.396	0.220
	“Frisch”	-3.734	-0.713	-0.431	-0.249	-0.138	-0.073	3.865	0.480	0.238	0.114	0.054	0.026
0.003	Total	2.306	1.083	0.732	0.451	0.257	0.139	0.700	0.599	0.472	0.324	0.197	0.110
	Compensated	2.703	1.447	1.022	0.651	0.379	0.207	1.384	1.191	0.940	0.646	0.393	0.220
	“Frisch”	1.502	0.346	0.147	0.046	0.009	-0.001	2.064	0.503	0.251	0.112	0.048	0.021
0.005	Total	1.176	0.795	0.589	0.386	0.228	0.125	0.710	0.579	0.462	0.321	0.196	0.110
	Compensated	1.541	1.127	0.861	0.578	0.347	0.192	1.399	1.149	0.920	0.640	0.392	0.220
	“Frisch”	1.438	0.493	0.242	0.092	0.028	0.006	1.686	0.509	0.256	0.110	0.043	0.016
0.007	Total	0.654	0.583	0.472	0.329	0.202	0.113	0.641	0.552	0.452	0.319	0.196	0.110
	Compensated	0.945	0.878	0.724	0.513	0.316	0.177	1.259	1.095	0.898	0.635	0.392	0.220
	“Frisch”	1.334	0.553	0.287	0.114	0.034	0.007	1.536	0.507	0.257	0.107	0.038	0.013
0.008	Total	0.489	0.495	**0.420**	0.303	0.189	0.107	0.578	0.532	**0.445**	0.318	0.197	0.110
	Compensated	0.732	0.768	**0.661**	0.482	0.302	0.171	1.134	1.054	**0.884**	0.633	0.392	0.220
	“Frisch”	1.297	0.574	**0.304**	0.121	0.036	0.007	1.500	0.505	**0.256**	0.105	0.036	0.011
0.010	Total	0.275	0.349	**0.327**	0.254	0.165	0.095	0.436	0.475	**0.424**	0.315	0.197	0.111
	Compensated	0.431	0.570	**0.541**	0.423	0.274	0.157	0.854	0.938	**0.841**	0.626	0.393	0.221
	“Frisch”	1.246	0.610	**0.334**	0.134	0.038	0.005	1.470	0.502	**0.254**	0.102	0.032	0.008
0.012	Total	0.162	0.240	0.249	0.210	0.143	0.084	0.315	0.400	0.390	0.309	0.197	0.111
	Compensated	0.255	0.407	0.431	0.367	0.248	0.144	0.618	0.789	0.774	0.614	0.393	0.222
	“Frisch”	1.213	0.642	0.362	0.145	0.039	0.003	1.470	0.501	0.251	0.098	0.029	0.005

Note: η = -.5 is the Keane and Wolpin (2001) estimate**.** The “Total” elasticity is the uncompensated. The “Frisch” elasticity refers to the estimate obtained using the conventional method of regressing the log hours change on the log earnings change. In the left panel this Frisch estimate is obtained using data that contain a tax cut at t=1, while in the right panel the Frisch estimate is obtained from data where the tax rate is equal in the two periods. The figures in bold type are the values obtained using values of the return to human capital investment in the plausible range (α = .008 to .010), and for my preferred value of γ=0.5.

Table 5: Labor Supply Response to Tax Change, Case of $\eta = -.25$

		Tax reduction in period 1						Tax reduction in both periods					
		γ						γ					
α	**Elasticity**	**0**	**0.25**	**0.5**	**1**	**2**	**4**	**0**	**0.25**	**0.5**	**1**	**2**	**4**
0	Total		2.393	1.356	0.741	0.391	0.202		1.504	1.000	0.599	0.332	0.176
	Compensated		2.721	1.572	0.870	0.463	0.240		2.002	1.332	0.798	0.443	0.234
	Frisch		4.060	2.010	1.000	0.499	0.249		4.060	2.010	1.000	0.499	0.249
0.001	Total	8.040	2.006	1.222	0.693	0.373	0.194	2.058	1.450	0.985	0.595	0.331	0.176
	Compensated	8.454	2.314	1.431	0.819	0.443	0.231	2.726	1.929	1.311	0.793	0.442	0.234
	"Frisch"	0.191	-0.474	-0.356	-0.230	-0.133	-0.072	5.927	0.580	0.268	0.121	0.055	0.026
0.003	Total	2.163	1.460	1.003	0.608	0.338	0.179	2.043	1.381	0.965	0.591	0.331	0.176
	Compensated	2.493	1.734	1.199	0.729	0.406	0.215	2.687	1.835	1.285	0.788	0.441	0.234
	"Frisch"	3.121	0.632	0.246	0.073	0.015	0.000	4.464	0.734	0.324	0.131	0.052	0.022
0.005	Total	0.707	1.033	0.816	0.532	0.306	0.164	1.139	1.259	0.941	0.590	0.332	0.176
	Compensated	0.844	1.261	0.997	0.649	0.372	0.200	1.492	1.669	1.251	0.786	0.442	0.234
	"Frisch"	2.672	0.841	0.374	0.130	0.037	0.008	4.983	0.839	0.362	0.138	0.049	0.018
0.007	Total	0.254	0.665	**0.642**	0.461	0.276	0.150	0.538	1.006	**0.884**	0.587	0.333	0.177
	Compensated	0.299	0.826	**0.800**	0.572	0.340	0.185	0.706	1.331	**1.175**	0.781	0.444	0.235
	"Frisch"	2.363	0.955	**0.453**	0.163	0.046	0.010	6.030	0.939	**0.391**	0.143	0.047	0.015
0.008	Total	0.164	0.515	**0.557**	0.427	0.261	0.144	0.385	0.852	**0.836**	0.583	0.334	0.177
	Compensated	0.190	0.642	**0.700**	0.535	0.325	0.178	0.505	1.126	**1.110**	0.776	0.445	0.236
	"Frisch"	2.237	0.999	**0.487**	0.176	0.049	0.010	6.626	0.992	**0.405**	0.144	0.045	0.013
0.010	Total	0.075	0.303	0.401	0.360	0.233	0.131	0.213	0.575	0.703	0.567	0.336	0.178
	Compensated	0.085	0.375	0.510	0.459	0.295	0.164	0.280	0.760	0.932	0.754	0.447	0.237
	"Frisch"	2.025	1.068	0.552	0.202	0.054	0.009	7.935	1.106	0.431	0.147	0.043	0.010
0.012	Total	0.038	0.181	0.276	0.295	0.207	0.119	0.127	0.381	0.550	0.536	0.337	0.179
	Compensated	0.041	0.220	0.352	0.382	0.266	0.151	0.167	0.504	0.729	0.712	0.448	0.239
	"Frisch"	1.857	1.113	0.614	0.228	0.059	0.008	9.440	1.222	0.460	0.149	0.041	0.008

Note: The "Total" elasticity is the uncompensated. The "Frisch" elasticity refers to the estimate obtained using the conventional method of regressing the log hours change on the log earnings change. In the left panel this Frisch estimate is obtained using data that contain a tax cut at t=1, while in the right panel the Frisch estimate is obtained from data where the tax rate is equal in the two periods. The figures in bold type are the values obtained using values of the return to human capital investment in the plausible range (α = .007 to .008), and for my preferred value of γ=0.5.

Table 6: Labor Supply Response to Tax Change, η = -.5, Borrowing Constraint

		Tax reduction in period 1						Tax reduction in both periods					
		γ						γ					
α	Elasticity	0	0.25	0.5	1	2	4	0	0.25	0.5	1	2	4
0	Total	1.000	0.666	0.499	0.332	0.199	0.111	1.000	0.666	0.499	0.332	0.199	0.111
	Compensated	1.990	1.326	0.994	0.663	0.398	0.221	1.990	1.326	0.994	0.663	0.398	0.221
	"Frisch"	1.000	0.666	0.499	0.332	0.199	0.111	1.000	0.666	0.499	0.332	0.199	0.111
0.001	Total	0.943	0.635	0.478	0.320	0.193	0.107	0.996	0.665	0.498	0.332	0.199	0.111
	Compensated	1.877	1.265	0.954	0.639	0.385	0.214	1.985	1.328	0.998	0.666	0.400	0.222
	"Frisch"	-1.248	-0.906	-0.709	-0.493	-0.306	-0.174	-0.597	-0.433	-0.338	-0.235	-0.146	-0.083
0.003	Total	0.823	0.575	0.439	0.298	0.181	0.101	0.959	0.656	0.496	0.332	0.199	0.111
	Compensated	1.636	1.145	0.876	0.594	0.361	0.202	1.926	1.321	1.000	0.671	0.404	0.224
	"Frisch"	-0.498	-0.433	-0.367	-0.275	-0.180	-0.106	-0.337	-0.300	-0.257	-0.195	-0.129	-0.076
0.005	Total	0.690	0.513	0.401	0.277	0.169	0.095	0.879	0.636	0.489	0.331	0.199	0.111
	Compensated	1.372	1.023	0.800	0.552	0.338	0.190	1.777	1.289	0.993	0.673	0.407	0.226
	"Frisch"	-0.243	-0.293	-0.276	-0.226	-0.157	-0.096	-0.159	-0.201	-0.193	-0.161	-0.114	-0.071
0.007	Total	0.553	0.450	0.364	0.256	0.159	0.090	0.760	0.602	0.477	0.329	0.200	0.111
	Compensated	1.101	0.897	0.725	0.512	0.317	0.179	1.548	1.226	0.974	0.672	0.409	0.228
	"Frisch"	-0.059	-0.194	-0.215	-0.196	-0.145	-0.092	-0.031	-0.124	-0.141	-0.132	-0.101	-0.065
0.008	Total	0.489	0.418	**0.345**	0.247	0.154	0.087	0.693	0.579	**0.469**	0.327	0.200	0.111
	Compensated	0.973	0.833	**0.687**	0.492	0.307	0.174	1.419	1.183	**0.958**	0.670	0.410	0.229
	"Frisch"	0.019	-0.151	**-0.189**	-0.183	-0.141	-0.091	0.020	-0.092	**-0.119**	-0.120	-0.095	-0.063
0.010	Total	0.376	0.354	**0.307**	0.227	0.144	0.082	0.562	0.524	**0.447**	0.323	0.200	0.111
	Compensated	0.749	0.706	**0.612**	0.454	0.288	0.164	1.163	1.076	**0.916**	0.663	0.411	0.230
	"Frisch"	0.156	-0.072	**-0.141**	-0.161	-0.133	-0.089	0.102	-0.039	**-0.081**	-0.097	-0.084	-0.058
0.012	Total	0.287	0.295	0.269	0.208	0.135	0.077	0.448	0.462	0.418	0.317	0.199	0.112
	Compensated	0.573	0.588	0.537	0.416	0.270	0.155	0.940	0.953	0.859	0.651	0.410	0.230
	"Frisch"	0.273	0.001	-0.096	-0.141	-0.127	-0.088	0.166	0.003	-0.050	-0.077	-0.074	-0.054

Note: η = -.5 is the Keane and Wolpin (2001) estimate**.** The "Total" elasticity is the uncompensated. The "Frisch" elasticity refers to the estimate obtained using the conventional method of regressing the log hours change on the log earnings change. In the left panel this Frisch estimate is obtained using data that contain a tax cut at t=1, while in the right panel the Frisch estimate is obtained from data where the tax rate is equal in the two periods. The figures in bold type are the values obtained using values of the return to human capital investment in the plausible range (α = .008 to .010), and for my preferred value of γ=0.5.

Table 7: Effect of Different Types of Tax Increases on Labor Supply in a Model with Human Capital (Imai-Keane)

	Transitory	Permanent (Unanticipated)	
Age		Uncompensated	Compensated
20	-1.5%	-0.7%	-3.2%
25	-1.8%	-0.6%	-2.7%
30	-2.2%	-0.6%	-2.4%
35	-2.6%	-0.5%	-2.3%
40	-3.2%	-0.7%	-2.3%
45	-3.8%	-1.0%	-2.8%
50	-4.7%	-2.3%	-4.2%
60	-8.6%	-9.4%	-10.5%

Note: All figures are contemporaneous effects of a 5% tax increase. The "transitory" increase only applies for one year at the indicated age. The "permanent" tax increases take effect (unexpectedly) at the indicated age and last until age 65. In the "compensated" case the proceeds of the tax (in each year) are distributed back to agents in lump sum form.

Table 8: Effects of Permanent Tax Increases on Labor Supply At Different Ages in a Model with Human Capital (Imai-Keane)

Age	Age 25			Age 30 (unexpected)			Age 35 (unexpected)	
	Hours	Wage		Hours	Wage		Hours	Wage
25	-2.7%							
30	-2.9%	-0.4%		-2.4%				
35	-3.2%	-0.7%		-2.7%	-0.3%		-2.3%	
40	-3.8%	-1.0%		-3.3%	-0.6%		-2.7%	-0.2%
45	-5.1%	-1.3%		-4.4%	-0.9%		-3.8%	-0.5%
50	-7.9%	-2.0%		-7.0%	-1.4%		-6.2%	-1.0%
55	-13.3%	-3.6%		-12.2%	-2.9%		-11.0%	-2.3%
60	-19.3%	-7.5%		-18.4%	-6.6%		-17.4%	-5.8%
65	-29.2%	-11.6%		-28.1%	-10.7%		-26.9%	-9.7%

Note: The tax increase is 5%. It takes effect (unexpectedly) at the indicated age and lasts until age 65. The proceeds of the tax (in each year) are distributed back to agents in lump sum form.

Table 9: Optimal Tax Rates: $f(P) = \log(P)$, $\eta = -.5$

Borrowing/Lending									
		α							
γ		**0**	**0.001**	**0.003**	**0.005**	**0.007**	**0.008**	**0.010**	**0.012**
0.25	Social planner	0.400	0.396	0.386	0.374	0.361	0.354	0.337	0.318
	Free-rider	0.400	0.392	0.374	0.354	0.331	0.319	0.294	0.270
0.5	Social planner	0.400	0.396	0.388	0.379	0.368	0.362	0.349	0.335
	Free-rider	0.400	0.394	0.381	0.365	0.348	**0.339**	0.320	0.300
1	Social planner	0.400	0.397	0.390	0.383	0.375	0.370	0.361	0.350
	Free-rider	0.400	0.396	0.386	0.375	0.363	0.357	0.344	0.329
2	Social planner	0.400	0.397	0.392	0.386	0.380	0.377	0.369	0.362
	Free-rider	0.400	0.397	0.390	0.382	0.374	0.370	0.360	0.351
4	Social planner	0.400	0.398	0.393	0.388	0.383	0.381	0.375	0.369
	Free-rider	**0.400**	0.397	0.392	0.386	0.380	0.377	0.371	0.363
No Borrowing/Lending									
		0	**0.001**	**0.003**	**0.005**	**0.007**	**0.008**	**0.010**	**0.012**
0.25	Social planner	0.400	0.396	0.386	0.376	0.365	0.360	0.347	0.334
	Free-rider	0.400	0.392	0.376	0.359	0.341	0.333	0.316	0.300
0.5	Social planner	0.400	0.396	0.388	0.380	0.371	0.366	0.356	0.345
	Free-rider	0.400	0.394	0.381	0.368	0.354	0.347	0.332	0.318
1	Social planner	0.400	0.397	0.390	0.384	0.376	0.372	0.365	0.356
	Free-rider	0.400	0.396	0.386	0.376	0.366	0.361	0.350	0.338
2	Social planner	0.400	0.397	0.392	0.387	0.381	0.378	0.372	0.365
	Free-rider	0.400	0.397	0.390	0.383	0.375	0.372	0.364	0.355
4	Social planner	0.400	0.398	0.393	0.389	0.384	0.382	0.377	0.371
	Free-rider	0.400	0.397	0.392	0.387	0.381	0.378	0.372	0.366

Note: The figures in bold correspond to the (α=0, γ=4) case, which conforms closely to the conventional wisdom for the value of γ in models without human capital, and the (α=.008, γ=0.5) case, which represents my preferred value based on estimates that account for human capital. These figures will be used later to calculate the welfare losses from taxation as a fraction of revenues.

Table 10: Optimal Tax Rates: $f(P) = P$, $\eta = -.5$

Borrowing/Lending									
		α							
γ		**0**	**0.001**	**0.003**	**0.005**	**0.007**	**0.008**	**0.010**	**0.012**
0.25	Social planner	0.400	0.460	0.584	0.704	0.805	0.844	0.904	
	Free-rider	0.400	0.415	0.447	0.480	0.514	0.532	0.566	0.601
0.5	Social planner	0.400	0.433	0.502	0.574	0.646	0.680	0.746	0.803
	Free-rider	0.400	0.414	0.443	0.473	0.503	**0.519**	0.550	0.582
1	Social planner	0.400	0.420	0.461	0.504	0.548	0.571	0.616	0.661
	Free-rider	0.400	0.412	0.437	0.463	0.489	0.503	0.530	0.558
2	Social planner	0.400	0.413	0.441	0.469	0.498	0.513	0.543	0.574
	Free-rider	0.400	0.410	0.432	0.453	0.476	0.487	0.510	0.534
4	Social planner	0.400	0.410	0.431	0.452	0.473	0.484	0.506	0.529
	Free-rider	**0.400**	0.409	0.427	0.446	0.465	0.475	0.494	0.514
No Borrowing/Lending									
		0	**0.001**	**0.003**	**0.005**	**0.007**	**0.008**	**0.010**	**0.012**
0.25	Social planner	0.400	0.459	0.579	0.702	0.816	0.861	0.925	0.945
	Free-rider	0.400	0.415	0.446	0.476	0.506	0.521	0.551	0.580
0.5	Social planner	0.400	0.433	0.501	0.572	0.645	0.682	0.753	0.816
	Free-rider	0.400	0.414	0.442	0.469	0.497	0.511	0.539	0.567
1	Social planner	0.400	0.420	0.461	0.504	0.548	0.571	0.617	0.664
	Free-rider	0.400	0.412	0.436	0.461	0.486	0.499	0.524	0.550
2	Social planner	0.400	0.413	0.441	0.469	0.499	0.514	0.545	0.577
	Free-rider	0.400	0.410	0.431	0.453	0.474	0.485	0.508	0.530
4	Social planner	0.400	0.410	0.431	0.452	0.474	0.486	0.509	0.533
	Free-rider	0.400	0.409	0.427	0.446	0.465	0.474	0.494	0.514

Note: The figures in bold correspond to the (α=0, γ=4) case, which conforms closely to the conventional wisdom for the value of γ in models without human capital, and the (α=.008, γ=0.5) case, which represents my preferred value based on estimates that account for human capital. These figures will be used later to calculate the welfare losses from taxation as a fraction of revenues.

Table 11: Welfare Losses from Proportional Income Tax, f(P) = log(P)

$\eta = -.75$

γ	$\frac{1+\eta}{\gamma-\eta}$		α: 0	0.001	0.003	0.005	0.007	0.008	0.010	0.012
0.25	0.25	C*	13.40	13.39	13.46	13.51	13.45	13.35	13.03	12.50
		C**	-10.66	-10.65	-10.68	-10.69	-10.63	-10.56	-10.33	-9.97
0.5	0.20	C*	11.25	11.27	11.34	11.41	11.44	**11.42**	11.31	11.07
		C**	-9.26	-9.27	-9.31	-9.34	-9.35	**-9.33**	-9.24	-9.06
1	0.14	C*	8.56	8.60	8.69	8.79	8.89	8.92	8.97	8.97
		C**	-7.35	-7.38	-7.45	-7.52	-7.58	-7.60	-7.63	-7.61
2	0.09	C*	5.81	5.86	5.95	6.06	6.16	6.22	6.32	6.42
		C**	-5.23	-5.27	-5.34	-5.42	-5.51	-5.55	-5.63	-5.70
4	0.05	C*	**3.56**	3.59	3.66	3.74	3.83	3.87	3.96	4.05
		C**	**-3.33**	-3.36	-3.42	-3.49	-3.56	-3.60	-3.68	-3.75

$\eta = -.5$

γ	$\frac{1+\eta}{\gamma-\eta}$		0	0.001	0.003	0.005	0.007	0.008	0.010	0.012
0.25	0.67	C*	20.02	19.23	17.66	15.73	13.40	12.16	9.77	7.74
		C**	-12.63	-12.26	-11.47	-10.47	-9.22	-8.54	-7.15	-5.89
0.5	0.50	C*	15.56	15.16	14.32	13.33	12.09	**11.38**	9.83	8.25
		C**	-10.74	-10.51	-10.03	-9.44	-8.71	**-8.29**	-7.35	-6.34
1	0.33	C*	10.83	10.69	10.38	10.02	9.57	9.30	8.68	7.92
		C**	-8.28	-8.17	-7.96	-7.70	-7.39	-7.21	-6.78	-6.26
2	0.20	C*	6.79	6.75	6.68	6.60	6.49	6.43	6.28	6.09
		C**	-5.69	-5.66	-5.60	-5.52	-5.43	-5.38	-5.26	-5.10
4	0.11	C*	**3.90**	3.90	3.90	3.89	3.89	3.88	3.87	3.85
		C**	**-3.51**	-3.51	-3.51	-3.50	-3.49	-3.48	-3.46	-3.44

Note: C* = consumption gain needed to compensate for tax distortion
(starting from proportional tax world)
C** = equivalent consumption loss (moving from lump sum tax to distorting tax world)

Table 12: Welfare Losses from Proportional Income Tax, f(P) = 2$P^{.5}$

η = -.75

γ	$\frac{1+\eta}{\gamma-\eta}$		α = **0**	**0.001**	**0.003**	**0.005**	**0.007**	**0.008**	**0.010**	**0.012**
0.25	0.25	C*	13.40	13.88	15.09	16.54	18.16	19.03	20.83	22.70
		C**	-10.66	-10.98	-11.75	-12.65	-13.63	-14.14	-15.18	-16.23
0.5	0.20	C*	11.25	11.63	12.54	13.60	14.81	**15.47**	16.86	18.34
		C**	-9.26	-9.53	-10.14	-10.86	-11.65	**-12.06**	-12.94	-13.85
1	0.14	C*	8.56	8.83	9.46	10.18	11.01	11.46	12.43	13.50
		C**	-7.35	-7.56	-8.03	-8.56	-9.15	-9.47	-10.14	-10.87
2	0.09	C*	5.81	5.99	6.38	6.83	7.34	7.62	8.23	8.91
		C**	-5.23	-5.38	-5.70	-6.06	-6.46	-6.68	-7.15	-7.67
4	0.05	C*	**3.56**	3.66	3.89	4.14	4.43	4.59	4.93	5.31
		C**	**-3.33**	-3.42	-3.62	-3.84	-4.09	-4.23	-4.52	-4.84

η = -.5

γ	$\frac{1+\eta}{\gamma-\eta}$		α = **0**	**0.001**	**0.003**	**0.005**	**0.007**	**0.008**	**0.010**	**0.012**
0.25	0.67	C*	20.02	20.36	21.31	22.27	22.93	23.08	23.01	22.57
		C**	-12.63	-12.79	-13.19	-13.58	-13.85	-13.92	-13.90	-13.73
0.5	0.50	C*	15.56	15.84	16.51	17.23	17.86	**18.11**	18.41	18.40
		C**	-10.74	-10.88	-11.21	-11.55	-11.86	**-11.98**	-12.12	-12.13
1	0.33	C*	10.83	11.04	11.51	12.03	12.57	12.83	13.32	13.72
		C**	-8.28	-8.40	-8.67	-8.97	-9.27	-9.42	-9.69	-9.91
2	0.20	C*	6.79	6.92	7.21	7.53	7.89	8.07	8.46	8.87
		C**	-5.69	-5.79	-5.99	-6.21	-6.45	-6.58	-6.83	-7.09
4	0.11	C*	**3.90**	3.97	4.14	4.32	4.51	4.62	4.84	5.08
		C**	**-3.51**	-3.57	-3.70	-3.85	-4.00	-4.09	-4.26	-4.45

Note: C* = consumption gain needed to compensate for tax distortion
(starting from proportional tax world)
C** = equivalent consumption loss (moving from lump sum tax to distorting tax world)

Table 13: Welfare Losses from Proportional Income Tax, f(P) = P

$\eta = -.75$

γ	$\frac{1+\eta}{\gamma-\eta}$		α = 0	0.001	0.003	0.005	0.007	0.008	0.010	0.012
0.25	0.25	C*	13.40	15.06	19.21	24.56	31.33	35.33	44.70	56.16
		C**	-10.66	-11.75	-14.31	-17.33	-20.75	-22.60	-26.53	-30.69
0.5	0.20	C*	11.25	12.55	15.68	19.66	24.64	**27.57**	34.41	42.76
		C**	-9.26	-10.16	-12.25	-14.71	-17.54	**-19.09**	-22.43	-26.05
1	0.14	C*	8.56	9.45	11.57	14.21	17.46	19.36	23.79	29.17
		C**	-7.35	-8.03	-9.58	-11.40	-13.52	-14.70	-17.28	-20.15
2	0.09	C*	5.81	6.36	7.63	9.19	11.08	12.18	14.72	17.79
		C**	-5.23	-5.68	-6.71	-7.92	-9.33	-10.12	-11.88	-13.89
4	0.05	C*	**3.56**	3.86	4.56	5.40	6.40	6.98	8.31	9.89
		C**	**-3.33**	-3.60	-4.21	-4.93	-5.77	-6.24	-7.30	-8.53

$\eta = -.5$

γ	$\frac{1+\eta}{\gamma-\eta}$		α = 0	0.001	0.003	0.005	0.007	0.008	0.010	0.012
0.25	0.67	C*	20.02	22.71	29.73	39.23	51.71	59.27	77.46	100.47
		C**	-12.63	-13.86	-16.71	-19.98	-23.58	-25.47	-29.36	-33.33
0.5	0.50	C*	15.56	17.49	22.30	28.61	36.74	**41.62**	53.27	67.86
		C**	-10.74	-11.75	-14.08	-16.78	-19.82	**-21.46**	-24.92	-28.57
1	0.33	C*	10.83	12.05	14.98	18.71	23.43	26.23	32.87	41.14
		C**	-8.28	-9.03	-10.75	-12.76	-15.07	-16.34	-19.09	-22.11
2	0.20	C*	6.79	7.46	9.05	11.02	13.47	14.90	18.27	22.43
		C**	-5.69	-6.18	-7.30	-8.62	-10.16	-11.02	-12.92	-15.07
4	0.11	C*	**3.90**	4.25	5.05	6.02	7.20	7.88	9.46	11.38
		C**	**-3.51**	-3.80	-4.45	-5.22	-6.12	-6.62	-7.75	-9.06

Note: C* = consumption gain needed to compensate for tax distortion
(starting from proportional tax world)
C** = equivalent consumption loss (moving from lump sum tax to distorting tax world)