Economics 202A: Macroeconomics

Problem Set 6

Due: Date to be determined

1. *Capital asset pricing model* (*"Classic" CAPM*). Assume a two-period model in which each investor i maximizes the expected value of a quadratic function of next period's random wealth, W^i:

$$U^i = \mathrm{E}\left\{\alpha W^i - \frac{\gamma}{2}\left(W^i\right)^2\right\}.$$

Next period's wealth W^i depends on current wealth, W_0^i, the realized (net) returns on the N risky assets in which wealth can be held, $\{r_j\}_{j=1}^N$, the nonrandom return r_F on a riskless asset, and the investment shares $\left\{x_j^i\right\}_{j=1}^N$ of initial wealth that investor i selects for the available risky assets:

$$W^i = W_0^i\left[\sum_{j=1}^N x_j^i(1+r_j) + \left(1 - \sum_{j=1}^N x_j^i\right)(1+r_F)\right].$$

(a) Show how to write the last equation as

$$W^i = W_0^i\left[(1+r_F) + \sum_{j=1}^N x_j^i(r_j - r_F)\right].$$

(b) Derive investor i's first-order optimum condition with respect to x_j^i.

(c) Sum these optimum conditions over all M investors i to derive an equilibrium condition involving aggregate second-period wealth, $W \equiv \sum_{i=1}^M W^i$.

(d) Define the coefficient

$$\rho \equiv \frac{\gamma \mathrm{E}\{W\}/M}{\alpha - \gamma \mathrm{E}\{W\}/M}.$$

Show that we can interpret ρ as a measure of "average" relative risk aversion.

(e) Define the (gross) "return on the market" as

$$1 + r_M = \frac{W}{W_0},$$

where $W_0 \equiv \sum_{i=1}^{M} W_0^i$. Show that the equilibrium condition from part c, above, can be put into the form:

$$\mathrm{E}\{r_j - r_F\} = \frac{\rho \mathrm{Cov}\{r_j, r_M\}}{\mathrm{E}\{1 + r_M\}}.$$

(f) What is the intuitive interpretation of the last condition?

(g) Show how to write the condition from part e, above, as

$$\mathrm{E}\{r_j - r_F\} = \beta_j \mathrm{E}\{r_M - r_F\},$$

where

$$\beta_j \equiv \frac{\mathrm{Cov}\{r_j, r_M\}}{\mathrm{Var}(r_M)}.$$

The CAPM framework predicts that a risky asset's "beta," as defined here, determines the degree to which it can be expected to outperform the market as a whole. (This form of the model can be tested from market returns data alone, without assumptions on the degree of risk aversion.)

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