

## Economics 202A, Problem Set 3

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1. *OG Model for the Open Economy.* Consider the overlapping generations model with the following twists: population is constant, the path of output  $\{y_t^y, y_t^o\}$  is exogenous with  $y_t^o \equiv 0$ , the country may borrow from foreigners or lend to them at the fixed real interest rate  $r$ , and  $U(c_t^y, c_{t+1}^o) = \log c_t^y + \beta \log c_{t+1}^o$ . (a) If taxes on the young (old) are  $\tau_t^y$  ( $\tau_t^o$ ), calculate the consumption functions of the young and the old. (b) What is the intertemporal budget constraint of the government. (c) Assume that initially government debt and taxes are zero. Now consider the following fiscal policy: on date 0, the government makes a gift of  $d/2$  in government bonds to the date-0 young and the same gift to the date-0 old. These bonds begin to pay interest (at rate  $r$ ) on date 1. Taxes on date 0 population do not change, but taxes on everyone rise by  $rd/2$  from  $t = 1$  onward. Show that this policy is consistent with the government's intertemporal budget constraint. (d) Calculate the effect of the fiscal policy (as a function of  $d$ ) on *aggregate* consumption for every date  $t = 0, 1, 2, 3$ , etc. (e) What is the effect on long-run aggregate consumption? This effect cannot be due to the crowding out of capital, because there is no capital. Explain intuitively what has happened.
2. *Barro versus Feldstein.* In a critique of Barro's famous paper "Are Government Bonds Net Wealth?" Martin Feldstein (*JPE*, April 1976) argued that government debt may be net wealth in a growing economy. His case went as follows: Suppose the government gives an amount of debt  $D_0$  to people and taxes them for all interest paid on this and other government debt issued in the future. Let the interest rate in the economy be fixed at  $r$ , the growth rate of *total* (not per capita) output  $g$ , and suppose  $r > g$  (ruling out Ponzi games). Suppose the government taxes people to cover  $(r - g)D(t)$  each period but simply issues new debt in the amount  $gD(t)$  to cover the balance of the total interest bill  $rD(t)$ . Then, Feldstein argued, the public debt-to-output ratio will remain constant, the government's intertemporal budget constraint will still hold, but the people who receive the gift  $D_0$  from the

government will enjoy a net increase in wealth equal to  $(g/r)D_0$ , which is proportional to the portion of the debt rolled over each period. [Use the continuous-time government budget constraint

$$D_0 = \int_0^{\infty} e^{-rt} T(t) dt$$

for this one, where  $T(t)$  denotes total taxes collected on date  $t$ .] (a) Do you agree with Feldstein's analysis for an economy with a constant number of identical immortal agents? (b) How would this argument fare if the demographics were different? Specifically, assume that people are immortal, but that *new* immortal people are born each period and are taxed to pay for past government liabilities as well as those incurred during their own lifetimes.

3. *Intertemporal Tax Smoothing.* Here is a problem related to F. P. Ramsey's *other* great contribution to economics, his paper on optimal taxation. (The ramifications of the Ramsey tax principle throughout economics are many.) The government seeks to maximize the utility of a typical consumer,

$$U = \int_0^{\infty} e^{-rt} \{ \log[c(t)] - \phi \ell(t) + v [g(t)] \} dt,$$

where  $\ell$  is the labor that the individual devotes to production and  $g$  is a public good that the government provides (perhaps national defense). The path of  $g$  is exogenous. (I am assuming here that the subjective discount rate equals the market real rate of interest  $r$ .) Output is given by  $y = A\ell$ ; however, the government uses a tax on labor to finance itself, so that the worker's perceived return to working is instead

$$y = (A - \tau) \ell.$$

This results in a divergence between the private and social returns to effort, a tax *distortion*. Assume that initially the government has no debt (this is not an essential assumption, of course). One can view the government as solving the intertemporal problem

$$\max_{\{c(t), \tau(t)\}} U$$

subject to the worker's first-order condition for labor supply [where does it come from? – this is part (a) of this problem],

$$\frac{A - \tau(t)}{c(t)} = \phi, \quad (1)$$

the output constraint

$$c(t) + g(t) = y(t) = A\ell(t), \quad (2)$$

and the intertemporal government budget constraint

$$\int_0^{\infty} e^{-rt} [g(t) - \tau(t)\ell(t)] dt = 0.$$

This last budget condition can, alternatively, be represented by the set of equations for government debt

$$\dot{d}(t) = rd(t) + g(t) - \tau(t)\ell(t), \quad d(0) = 0, \quad \lim_{t \rightarrow \infty} e^{-rt}d(t) = 0. \quad (3)$$

(b) Explain how equation (1), written as

$$c = \frac{A - \tau}{\phi},$$

allows the government to control consumption through tax policy. Why doesn't  $g$  enter this consumption expression, and do you think this simplification would hold in general? [Hint: See the answer to part (c) below.]

(c) Show why (2) implies that labor supply can be controlled by

$$\ell = \frac{A - \tau}{\phi A} + \frac{g}{A}.$$

(d) Now show that the government's problem can be expressed *without direct reference to the labor-income tax rate  $\tau$*  as

$$\max_{\{\tau(t)\}} \int_0^{\infty} e^{-rt} \left( \log [c(t)] - \phi \left[ \frac{c(t) + g(t)}{A} \right] + v [g(t)] \right) dt$$

subject to

$$\dot{d}(t) = rd(t) + g(t) - [A - \phi c(t)] \left[ \frac{c(t) + g(t)}{A} \right].$$

plus initial and terminal conditions on government debt.

(e) Apply the Maximum Principle to the preceding problem, in which consumption is the only state variable. Show that the costate variable  $\lambda$  is constant over time. The variable has the interpretation of the *shadow price* of resources to the government. With lumpsum taxation  $\lambda = 1$ . With distorting taxes  $\lambda > 1$ . The constancy of this marginal distortion (the marginal deadweight burden of taxation) across different time periods is Ramsey's principle.

(f) Show that the first-order condition for the control  $c$  takes the form

$$\frac{1}{c} = \text{a decreasing linear function of } c.$$

Graph the two sides of this equation in a diagram where  $c$  is on the horizontal axis. What do you make of the possibility of two intersections? (Hint: look up "Laffer curve.") Which intersection represents an *optimal* tax rate?

(g) Let's imagine that  $A = 1$ ,  $\phi = 0.1$ ,  $g = 0.5$ ,  $\lambda = 1.2$ . (We basically can validate any level of  $\lambda$  by adjusting the value of  $d(0)$  – can you see why?) What values of consumption satisfy the first-order condition of part (f)? Use your analytical first-order condition for consumption to calculate numerically the optimal response  $dc/dg$  at the "good" tax solution. Then use the expression  $\tau = A - \phi c$  to calculate the derivative  $d\tau/dg$ . Do taxes rise by as much as the increase in  $g$ , or by less?