UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2011
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Problem Set 7

Due in lecture, Tuesday, November 22, 2011

1. S***addle-path of the q-theory model.*** (Special ground rules for this problem: (1) You must try it first under exam conditions – no readings, no notes, no help. (2) You may then look at lecture notes and readings, but not consult with others or look at the midterm solutions. (3) You may then look at the midterm solutions and/or consult with others.)

Consider the 2 equations of the q-theory model,

$$\dot{q}(t) = rq(t) - \pi\big(K(t)\big), \qquad \dot{K}(t) = f\big(q(t)\big).$$

a. Define the steady state of the model, $(\bar{q}, \bar{K})$. Show that the model's linear (Taylor) approximation in the neighborhood of the steady state takes the form:

$$[\dot{q}, \dot{K}]' \approx \mathrm{G}[q - \bar{q}, K - \bar{K}]',$$

where G =

A	B
C	0

Be sure to show how A, B, and C depend on exogenous parameters, the steady-state values of q and K, and/or the properties of $\pi(\cdot)$ and $f(\cdot)$.

b. Show that the characteristic roots of the preceding 2x2 matrix are:

$$\lambda_1, \lambda_2 = \frac{r \mp \sqrt{r^2 - 4f'(1)\pi'(\bar{K})}}{2},$$

where $\lambda_1 > 0$ and $\lambda_2 < 0$. Please indicate why the second condition holds.

c. Show that the eigenvectors of the matrix G are proportional to the matrix

X =

λ_1/f'(1)	λ_2/f'(1)
1	1

d. Define $[\tilde{q}, \tilde{K}]' \equiv \mathrm{X}^{-1}[q - \bar{q}, K - \bar{K}]'$, and note that this implies that $[\dot{\tilde{q}}, \dot{\tilde{K}}]' = \mathrm{X}^{-1}[\dot{q}, \dot{K}]'$. Explain how that change of variables enables us to write the solution of our differential equation system in the form $[\tilde{q}(t), \tilde{K}(t)]' = [\tilde{q}(0)e^{\lambda_1 t}, \tilde{K}(0)e^{\lambda_2 t}]'$ for arbitrary initial conditions $\tilde{q}(0)$ and $\tilde{K}(0)$.

e. From this last relationship deduce that:

$$q(t) - \bar{q} = \tilde{q}(0)(\lambda_1/f'(1))e^{\lambda_1 t} + \tilde{K}(0)(\lambda_2/f'(1))e^{\lambda_2 t},$$

$$K(t) - \bar{K} = \tilde{q}(0)e^{\lambda_1 t} + \tilde{K}(0)e^{\lambda_2 t}.$$

(OVER)

f. Recalling that $\lambda_1 > 0$ and $\lambda_2 < 0$, identify the initial condition that will ensure the economy is on the convergent saddle-path in the usual phase diagram with K on the horizontal axis and q on the vertical axis.

g. For our linear approximation above, express the (linear) equation for the saddle-path in the form

$$q(t) - \bar{q} = \Omega[K(t) - \bar{K}]$$

for an appropriate constant slope $\Omega < 0$. Be sure to show how Ω depends on the model parameters, the steady-state values of q and K, and/or the properties of $\pi(\cdot)$ and $f(\cdot)$.

h. Recall that

$$\lambda_2 = \frac{r - \sqrt{r^2 - 4f'(1)\pi(\bar{K})}}{2}.$$

Discuss which parameters and steady state values affect the slope of the saddle-path. How do they impact the slope? Why?

2. (A variable investment subsidy in the q theory model.) Consider the q theory model of investment considered in lecture and in the book. Assume that initially K and q are at their long-run equilibrium values. Now suppose the government (unexpectedly and permanently) adopts a policy of subsidizing investment. Specifically, the government pays each firm φ for each unit of investment it undertakes (and firms pay the government φ for each unit of disinvestment they undertake). The subsidy paid at a point in time depends on the aggregate capital stock at that time: $\varphi = \varphi(K(t))$, with $\varphi(\$) > 0$, $\varphi'(\$) < 0$.

a. How, if at all, does the $\dot{K} = 0$ locus change when the government adopts the subsidy?
b. How, if at all, does the $\dot{q} = 0$ locus change when the government adopts the subsidy?
c. How, if at all, does the capital stock change at the moment the government adopts the subsidy?

3. Romer, Problem 9.7.

4. Consider an individual maximizing $E_0[\sum_{t=0}^{\infty} \beta^t u(C_t)]$, with our usual assumptions. If r^i denotes the return on some asset, asset i, from period 0 to period 1, then optimization requires:

A. $\beta E_0[u'(C_1)] = E_0[(1 + r^i)]E_0[(u'(C_0)] + \text{Cov}_0[r^i, u'(C_0)]$.
B. $E_0[u'(C_1)] = \beta E_0[(1 + r^i)]E_0[(u'(C_0)] - \text{Cov}_0[r^i, u'(C_0)]$.
C. $E_o[1 + r^i] = E_0[\frac{\beta u'(C_1)}{u'(C_0]}$.
D. $E_0\left[\frac{\beta u'(C_1)(1+r^i)}{u'(C_0)}\right] = 1$.

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

Romer, Problems 9.9, 9.12, 8.8, 8.9.