

Problem Set 6
Due in lecture Tuesday, November 13

1. Consider the continuous-time consumption problem discussed in lecture: an individual lives from 0 to T ; has initial wealth $A(0)$; and a path of labor income given by $Y(t)$. The path of the instantaneous interest rate is given by $r(t)$. There is no uncertainty.

Suppose the individual's instantaneous utility function is logarithmic. That is, lifetime utility is $\int_{t=0}^T e^{-\delta t} \ln[C(t)] dt$. Derive an expression for $C(t)$ as a function of things the individual takes as given.

2. Consider a household that lives for two periods and has constant absolute risk aversion utility, $U = -e^{-\gamma C_1} - \beta e^{-\gamma C_2}$, $\gamma > 0, \beta > 0$. Assume the household's initial wealth is zero, and that its labor incomes in the two periods, Y_1 and Y_2 , are certain.

a. Suppose the only asset is an asset earning rate of return $\bar{r}$ for sure. The household can borrow or lend any amount at this interest rate (but, as usual, any borrowing in period 1 has to be paid off in period 2). What is the household's budget constraint? What are the conditions characterizing the solution to its optimization problem?

b. Consider the same set-up as in part (a). Suppose Y_1 falls by X ($X > 0$) and Y_2 rises by $(1 + r)X$. Without doing any math, explain whether C_1 will rise, fall, or be unchanged (or whether it is not possible to tell).

c. Suppose that in addition to the safe asset, there is an asset with return that is distribution normally with mean r_1 ($r_1 < \bar{r}$) and variance σ^2 ($\sigma^2 > 0$). Without doing any math, explain whether the household will hold a positive, negative, or zero amount of the risky asset (or whether it is not possible to tell).

d. Consider the same set-up as in part (c), except that to make it simpler, $Y_2 = 0$ and the household only gets utility from period 2 consumption, so $U = -e^{-\gamma C_2}$. Find an expression for the amount of the risky asset the household will hold. (Useful fact: If x is normally distributed with mean μ and variance V , then $E[e^x] = e^{\mu + V/2}$.)

3. Romer, Problem 8.10.

(OVER)

4. (The risk-free rate puzzle.) There is considerable evidence that individuals are quite impatient and quite risk averse. In light of this, consider the standard Euler equation relating consumption in periods t and $t+1$ under certainty: $U'(C_t) = [(1+r)/(1+\rho)]U'(C_{t+1})$. Suppose that ρ is 5 percent, the coefficient of relative risk aversion is 4, and that the growth rate of consumption is 1.5 percent. What must r be for consumers to be satisfying their Euler equation?

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

5. In their abstract, Parker, Souleles, Johnson, and McClelland report that in response to the 2008 Economic Stimulus Payments, “there was ... a significant increase in spending on durable goods, in particular”:

- A. “vehicles”.
- B. “housing”.
- C. “energy-efficient appliances”
- D. “electronics”.

6. An individual lives for 3 periods. In period 1, his or her objective function is $U(C_1) + \beta U(C_2) + \gamma U(C_3)$. In period 2, his or her objective function is $U(C_2) + \delta U(C_3)$. The individual's preferences are not time consistent if:

- A. $\delta \neq \beta$.
- B. $\delta \neq \gamma$.
- C. $\delta \neq \beta/\gamma$.
- D. $\delta \neq \gamma/\beta$.

7. Romer, Problem 8.14.

8. Romer, Problem 8.15.